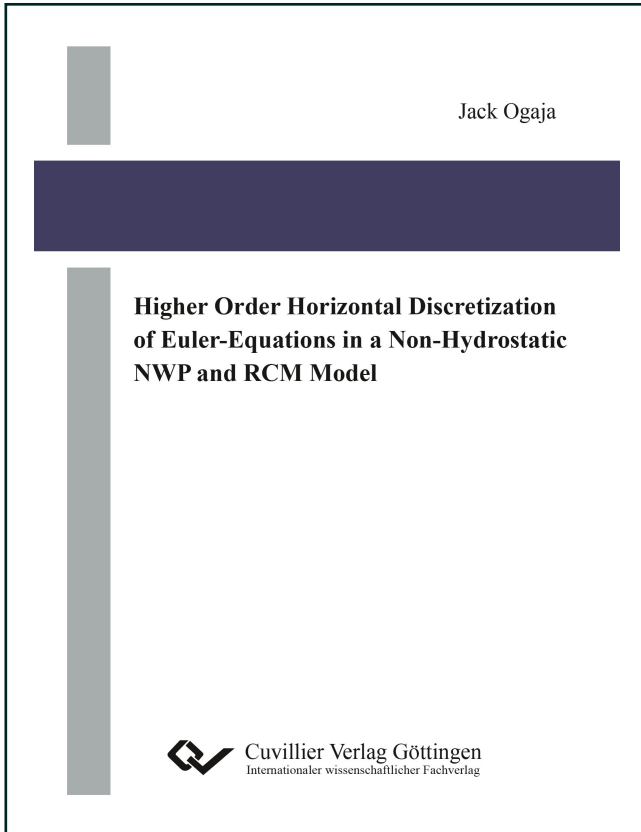




Jack Ogaja (Autor)

## **Higher Order Horizontal Discretization of Euler-Equations in a Non-Hydrostatic NWP and RCM Model**



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# TABLE OF CONTENTS

LIST OF TABLES . . . . .	xii
LIST OF FIGURES . . . . .	xiii
LIST OF FREQUENTLY USED ABBREVIATIONS . . . . .	xvi
NOTATION . . . . .	xvii
CHAPTER 1 INTRODUCTION . . . . .	1
1.1 Objectives . . . . .	5
1.2 Outline . . . . .	6
<b>I Fundamentals, Review and Theory</b>	<b>9</b>
CHAPTER 2 HORIZONTAL DISCRETIZATION ON A STAGGERED GRID . . . . .	10
2.1 Introduction . . . . .	10
2.2 Numerical Operators and Error Convergence . . . . .	12
2.2.1 Interpolation Operators . . . . .	12
2.2.2 Spatial Differencing Operators . . . . .	13
2.3 Discretization of Non-linear Terms . . . . .	14
2.3.1 Quasi-Higher Order Advection Schemes . . . . .	15
2.3.2 Higher Order Advection Schemes . . . . .	17
2.3.3 Quadratic Conserving Advection Schemes . . . . .	18
2.4 Summary . . . . .	23
CHAPTER 3 ERRORS AND STABILITY ANALYSIS OF ADVECTION SCHEMES	25
3.1 Introduction . . . . .	25
3.2 Error Analyses . . . . .	27
3.2.1 Discrete Dispersion Relation, Phase, and Phase Speed Approximations . . . . .	27
3.2.2 Group Velocity Approximations . . . . .	35
3.2.3 Alias Error . . . . .	36
3.3 Linear Stability Analysis . . . . .	40
3.4 Effective Resolution Analysis . . . . .	50
3.5 Summary . . . . .	53
<b>II The Model System and Code Implementation</b>	<b>61</b>
CHAPTER 4 SPATIAL SCHEMES IN A NON-HYDROSTATIC MODEL . . . . .	62



4.1	Introduction . . . . .	62
4.2	The Model System . . . . .	62
4.3	Compressible Euler Equations in Terrain-following Coordinates . . . . .	63
4.4	Boundary Conditions . . . . .	64
4.5	Quasi-Higher Order Advection Schemes . . . . .	65
4.6	Fourth Order Advection Scheme . . . . .	67
4.7	Fourth Order Quadratic Conserving Advection Scheme . . . . .	67
4.8	Fourth Order Discretization of Fast-waves Terms . . . . .	68
4.9	Numerical Schemes Configurations . . . . .	70
4.10	Summary . . . . .	70
<b>III Code Verification and Real Case Study</b>		<b>73</b>
CHAPTER 5 IDEALIZED NUMERICAL TESTS . . . . .		74
5.1	Introduction . . . . .	74
5.2	Euler Equations in Cartesian Coordinates . . . . .	75
5.3	Linear Analytic Solution . . . . .	76
5.4	The Model Configuration . . . . .	78
5.5	Error Norms . . . . .	79
5.6	Test's Quality . . . . .	80
5.7	2D Hydrostatic Linear Mountain Flow Case . . . . .	81
5.8	2D Non-hydrostatic Linear Mountain Case . . . . .	82
5.9	Vertical Flux of Horizontal Momentum . . . . .	84
5.10	2D Atmosphere at Rest Test Case . . . . .	88
	5.10.1 Numerical Stability Test . . . . .	89
	5.10.2 Convergence Study . . . . .	90
5.11	Summary . . . . .	91
CHAPTER 6 REAL CASE STUDY . . . . .		94
6.1	Configuration . . . . .	95
6.2	Simulation Stability . . . . .	96
6.3	Zonal Spectral Analysis . . . . .	97
	6.3.1 Kinetic Energy Spectra . . . . .	98
	6.3.2 Model's Effective Resolution . . . . .	101
	6.3.3 Precipitation Spectra . . . . .	103
6.4	Climatologies . . . . .	104
6.5	Summary . . . . .	109
<b>IV Summary and Outlook</b>		<b>113</b>
CHAPTER 7 SUMMARY AND OUTLOOK . . . . .		114
7.1	Summary . . . . .	114
7.2	Outlook . . . . .	117



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7.2.1	Physics-Dynamics Coupling . . . . .	117
7.2.2	Model Efficiency . . . . .	118
<b>V</b>	<b>Appendices and Bibliography</b>	<b>119</b>
APPENDIX A	FINITE DIFFERENCE APPROXIMATIONS . . . . .	120
APPENDIX B	NUMERICAL PROPERTIES OF THE MODEL'S SCHEMES . . . . .	122
B.1	Dissipation Property of Upwind Difference Schemes . . . . .	122
B.2	Conservative Property of Quadratic Conserving Schemes . . . . .	123
APPENDIX C	SUPPLEMENTARY FIGURES . . . . .	127
REFERENCES	. . . . .	129