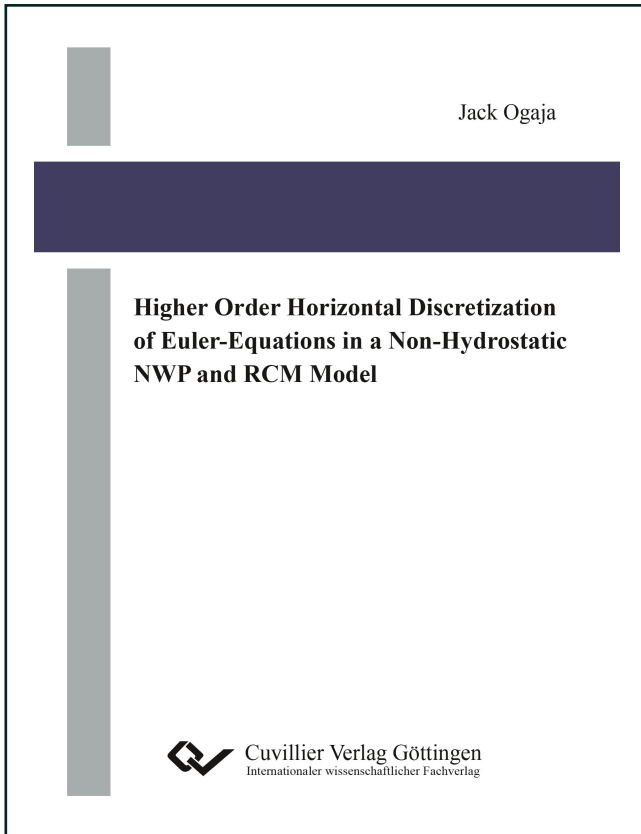




Jack Ogaja (Autor)

## Higher Order Horizontal Discretization of Euler-Equations in a Non-Hydrostatic NWP and RCM Model



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