1 Introduction

During the 18th century in France and Germany a rather new kind of textbooks, which we will call the *Cours* and *Anfangsgründe* literature, was published. These textbooks were created, on the one hand, to assist the teaching of the mathematical sciences in higher education and, on the other hand, to make the reception of the teaching among students more effective. Interestingly, a structural point of view seems to be contained in the titles themselves. Indeed, "*cours*" means "course, lessons, classes", while *Anfangsgründe* means "elements, basics": the relation to teaching is explicit in French textbooks, whereas it remains implicit in the German ones. The textbooks in both languages were usually for beginners; the novelty was indeed that they were written in the respective national languages. The reformers of the Enlightenment fought for the dissemination of knowledge – what could be realized, for instance, using the national languages instead of Latin in education and books. Thereby, also those people who were not allowed to study, like women, had the possibility to learn the mathematical sciences.

The *Cours* and *Anfangsgründe* textbooks were adapted to the particular circumstances of the 18th century, namely to the way the mathematical sciences were taught at this time. Before this period, the main teaching method was rote memorization. Traditionally, knowledge was dictated without any closer examination. This changed in the context of Enlightenment, when autonomous thinking became the principal goal of education. For this purpose, the *Cours* and the *Anfangsgründe* were very useful due to a variety of respects. Indeed, they could be used as lecture notes, so that the relevant topics did not have to be dictated anymore, and, also, as memorandum, where students could afterwards search for contents.

Among the contemporary studies on the history of mathematics teaching, there is a lack of literature on the substance and development of these kinds of textbooks. In order to fill this gap, the present work[1] deals with some characteristics, with the structure, and with the contents of the *Cours* and the *Anfangsgründe* literature. Of peculiar inter-

1 This work provides some results of the project "*Traditionen der schriftlichen Mathematik und Mathematikvermittlung im deutschen und im französischen Sprachraum zwischen 1650 und 1820-Herausbildung und Differenzierung von wissenschaftlichen Disziplinen in nationalen Kontexten*", supported by the Deutsche Forschungsgemeinschaft (DFG) at the Bergische Universität, Wuppertal. The final aim of the project was not only to establish a comparison between the French and German textbooks that were used during the 18th century to teach the mathematical sciences in higher education, but also to eventually analyze the emergence of teaching traditions by retracing their possible origins in the textbooks written in Latin, especially by the Jesuits. To this purpose, together with Dagmar Mrozik, we moreover worked on a comprehensive database based on the software Archiv-Editor, developed by the DFG project "*Personendaten-Repositorium*", at the Berlin-Brandenburgische Akademie der Wissenschaften.

est in these textbooks might be the educational transposition – that is the systematic and didactic editing of research contents (cf. Chevallard 1991, pp. 39 ff.). On the one hand, this implies some differences between research and school knowledge, which were already institutionally split during the 18th century. On the other hand, it seems that the authors of this textbooks wanted to combine these two elements, which accounts for an interaction between scientific disciplines and didactics. In addition to that, our knowledge about the educational system in the 18th century is not thorough enough. Our study of the textbooks could enlighten the situation: it could contribute to clarify the educational circumstances of how mathematical sciences were taught, and also the role of mathematics in higher education during this period. Despite the fact that there were many differences between the French and the German educational system during the 18th century, the *Cours* and the *Anfangsgründe* present several similarities, so that it is fully profitable to discuss and compare them.

First of all, we need to deal with two preliminary points to clear the ground from any possible misunderstanding. During the 18th century, as well as in the preceding centuries, the terms "*mathématique(s)*" and "*Mathematik*" were still understood in a much wider sense than we nowadays do. The French and German authors of the textbooks of this period only rarely used the terms "*sciences mathématiques*" and "*mathematische Wissenschaften*". Nevertheless, we prefer to employ, when needed, the term "mathematical sciences" rather than "mathematics" to underline its comprehensive meaning and to recall that, at that time, among the mathematical sciences were included not only the pure disciplines such as geometry, arithmetic, algebra, and analysis, but also the applied ones, such as, for instance, mechanics, optics, astronomy, civil and military architectures. In general, the pure mathematical sciences were regarded as the doctrine of the magnitudes, that is, what can be measured or calculated; whereas their instantiation in some concrete bodies were considered within the applied mathematical sciences (for more details, cf. Section 2.1.2 on page 10 and Section 3.6 on page 37).

The second preliminary point concerns the choice of the textbooks. We consider the following criteria. Firstly, the textbooks must have been written with a teaching purpose for higher education. Secondly, they must have meant to provide a complete presentation of the mathematical sciences. Whatever "complete" means depends not only on each single author, but also on the time span. Indeed, there were some shifts in Germany during the 18th century concerning the framework of the mathematical sciences. Thirdly, we only consider textbooks that are written in a national language, namely French or German. Fourthly, we focus on the 18th century.

For the case of France, two facts must be taken into account: that the first textbooks not in Latin appeared in the first half of the 17th century and that the educational system underwent some major changes during the French Revolution. Therefore, we rather consider the textbooks written within this time lapse. With these limitations , we found around sixty textbooks. Clearly, we cannot provide a satisfactory account of all of them in this paper, so we choose a small selection including the most used ones, according to the number of their editions and to the secondary literature. In the end, this selection includes the textbooks by Bernard Forest de Bélidor (1698-1761), Nicolas-Louis de La Caille (1713-1762), Charles Étienne Louis Camus (1699-1768), Étienne Bézout (1739-1783), and Charles Bossut (1730-1814).

For the case of Germany, the *Anfangsgründe* tradition begins with Christian Wolff (1679-1754), who published his *Anfangsgründe aller mathematischen Wissenschaften* in 1710. It was often used and reprinted until 1800, a long time after his death. It was without any competition for almost fifty years, until the next generation of mathematicians published their textbooks in the second half of the 18th century: Abraham Gotthelf Kästner (1719-1800), Johann Andreas von Segner (1704-1777), Wenceslaus Johann Gustav Karsten (1732-1787), Heinrich Wilhelm Clemm (1725-1775), and Georg Simon Klügel (1739-1812). The textbooks of these authors were also the most used ones in the 18th century – as it is shown by the number of their editions and by comments in the secondary literature (cf. Kühn 1987, pp. 72 ff.). After this period, the *Anfangsgründe* seemed to be out of use, which can be explained by the changes within the German educational system. In 1810, there was the popular educational reform in Prussia. The reformers established different kinds of schools, reworked the curricula, and required new adapted textbooks. In addition to that, there was a vast increase of knowledge, mainly in analysis (differential and integral calculus) which should have been included in the new textbooks.

In the second place, it is worth to spend a few words on the peculiarities of the 18th century in Europe. For sure, one of the most distinctive features of this period is the encyclopedism, that is, the attitude of human beings towards knowledge characterized by the wish to satisfy curiosity in completest way possible, and to classify the results. Encyclopedias are the tangible embodiment of this attitude. While during the Middle Ages the clergy had the monopoly over knowledge, starting from Renaissance – and in particular from the 17th century – it became more widespread and, at the same time, more specialized. The production of scholar works increased and got fragmented into many topics. This caused, on the one hand, the creation of specific tools and, on the other hand, determined their success. These were: catalogs, bibliographies, indexes,

analytical tables, and – above all – dictionaries and encyclopedias. Particular attention was paid to the technical knowledge that, mainly orally conveyed, was now displayed, theorized, and visually represented. Among the dictionaries and encyclopedias we count, for instance, the *Lexicon Technicum, or an Universal English Dictionary of Arts and Sciences* (1704) by Harris, the *Universal Lexicon aller Wissenschaften und Künste* (1732-1750) by Zedler, and, of course, the *Encyclopédie ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers* (1751-1772) by D'Alembert and Diderot. With the latter, the encyclopedic attitude reaches without any doubt its highest point. It is composed of seventeen volumes of text and eleven volumes of figures, so that Voltaire judged it as a "*monument des progrès de l'esprit humain*". In the *Encyclopédie*, sciences, arts, and crafts – that is, the scientific and technical knowledge – play a major role. At the beginning of the publication, D'Alembert has already been a member of the *Académie des sciences* and an author of a renown mechanical treatise, while Diderot had worked in the domains of medicine and mathematics. Moreover, a good number of collaborators were the savants of the time, working in scientifical domains. D'Alembert himself wrote most of the mathematical parts (helped by La Chapelle), the mechanical parts (in which he was a specialist), and the astronomical parts (helped by Formey and Jacourt). One of the main peculiarities of this encyclopedia are the parts about the technical crafts: the authors wanted to reestablish them, since they could no longer be ignored due to technical progress. In 1777, four volumes of text and one volume of figures were added to the *Encyclopédie* by Panckoucke, who is also author of the *Encyclopédie Méthodique* (1832).

The present work is divided into three main sections. In the first two, we give a parallel overview of the *Cours* and of the *Anfangsgründe* literature, respectively. Firstly, we take a look at the circumstances in education in France and Germany in order to contextualize the textbooks. Then, we give a description of a selection of textbooks. We take care to answer the following questions: When were these textbooks published? Who were the authors? For whom were the textbooks written? What was the intention of the authors? What are the peculiar contents and structures of these textbooks? What do we know about their usage and dissemination? Unfortunately, due to the lack of sources, we cannot always give a completely satisfactory account; nevertheless, our work provides solid basis for further studies. In the third section, we give an insight into some specific topics of the *Cours* and of the *Anfangsgründe.* For this purpose we focus on four case studies: negative numbers, Pythagoras theorem, ballistics, and fortification.

2 The *Cours* Literature

2.1 Educational Circumstances in France during the 18th Century

With regard to institutionalized science teaching, we need at first to consider that, as Brockliss points out,

> [i]n the context of seventeenth- and eighteenth-century France the term "higher education" is an anachronism. It implies the existence of a carefully articulated system of educational provision, functionally differentiated and age-specific. But at this date no such system pertained anywhere in Europe, let alone in France (cf. Brockliss 1987, p. 2).

In the following, we summarize the main characteristics of the French educational system during the 18th century.[2] We moreover specify which knowledge fields defined the term "mathematical sciences" in this period.

2.1.1 Institutions

The mathematical sciences were taught in a variety of contexts in France during the 18th century. From the most to the least attended establishments, there were: universities, colleges, military schools, technical schools, and *maisons particulières*. The mathematical sciences were also taught by private teachers. Since the latter were completely independent from the institutional context (which makes the data gathering extremely difficult), we do not report on it.

The foundation of universities in France dated back to the 12th century. On the eve of the French Revolution, there were about 25 universities with 300-400 students on average each. They were financed by private endowments (in particular by the Crown) and by the students' fees, so that they were quite prosperous overall. They had the monopoly of granting degrees in one of the three faculties, namely medicine, law, and theology. The mathematical sciences were also dealt with at universities, even though they were not directly taught. Since the colleges had absorbed the teaching of the former faculty of arts, this last faculty had been reduced to the function of delivering the necessary degrees for entering one of the three faculties. More precisely, some boards had been created to evaluate the students in the propedeutical subjects – among which mathematics was – in order to admit them.

2 The main source of the following paragraph is (Brockliss 1987).

The colleges had generally been created more recently and in a larger number than universities. Such institutions had of course existed for a long time, but they were intended to serve as residences where underprivileged students were lodged and fed. These institutions were directly attached to universities. The colleges, as we know them during the 18th century, began to evolve during the second half of the 15th century, when some of the residences in Paris started to run classes in competition with the Faculty of Arts. Shortly before the French Revolution, of the 348 colleges only 171 offered a complete teaching that included also the last year of philosophy: they were called *collèges de plein exercice* (cf. Brockliss 1987, p. 22). During the 17th century, they were generally more often attended than universities, but afterwards, due to the overall increase of the educational offers, a period of decline started. The colleges were run both by seculars (cf. Brockliss 1987, pp. 23 and 481) and by teaching orders, like the Jesuits (until their banishment in 1762), the Oratorians, the Benedectines, and, to a lesser extent, by some others. Most of the colleges in the 18th century inherited the boarding schools feature of their predecessors. They were mainly intended for those students who wanted to continue their studies at university and could only award a degree if they were affiliated to a university. As already mentioned above, the liberal arts (in particular languages) and philosophy were their major teaching subjects. Mathematical contents were implemented only during the last philosophy year, when the main topics of elementary pure mathematics were taught to students without prior mathematical knowledge. This also included a large amount of physics, so that a considerable number of topics had to be dealt with within a short time span.

The military schools were instead far more recently created, namely, around the second half of the 18th century. They derived from the practice common in each company of the army to train a certain number of young nobles to become the future army officers. This instruction was then taken over by the government. The military schools were founded in small towns, usually where a college previously existed, and were the *élite* basis of the future *académies*. Since they were in most cases restructured colleges, they inherited some of their characteristics: they were, for instance, boarding schools run by regular teaching orders, especially by the Benedectines. In contrast with the colleges, the whole range of the mathematical sciences was taught in the military schools. Classics and philosophy were also taught, as well as equitation and military tactics – an instruction that only these institutions delivered.

In technical schools, like the École Royale des Ponts et Chaussées (1775) and the École des Mines (1783), and in the *académies* or *maisons particulières* some mathematical teaching were also imparted. As the military schools, they were founded dur-

ing the second half of the 18th century and, complementary to them, they were meant to satisfy the demand for institutional instruction in the applied mathematical sciences.

To sum up, during the 18th century the mathematical sciences were thoroughly taught in two kinds of institutions, namely colleges and military schools. Nevertheless, only in the military schools the teaching included a wide spectrum of topics and only in some of these schools a high level of contents was reached. If we exclude elementary military schools, where only an elementary teaching in the mathematical sciences was delivered, there were mainly three kinds of military schools that instructed the future officers in higher mathematical topics: the one for the navy corps, the one for the artillery corps, and the one for the military engineer corp.

The military schools for the navy were a long-lasting institution. They were founded in 1689 by royal order and lasted almost until the French Revolution. Many professors and students were also part of the newly-founded Académie Royale de Marine in Brest (1752). Despite all the premises, the navy schools never attained an outstanding level. The students lacked discipline, their theoretical studies were often interrupted by the wars (and also by the sea service during peacetime), and in general, the navy in France at that time was not as renowned as the military corps on the mainland. In 1716, the *élite* corps of the Gardes du Pavillon was created within the navy. In 1763, after the defeat against England during the Seven Years' War, the teaching for the navy was reformed. Étienne Bézout was commissioned with the renewal. One year later, he wrote the first volume of his *Cours* and was appointed *examinateur*. Indeed, to guarantee a higher standard for the officers' education, examinations were established in order to admit the students and to let them pass to the subsequent years (as already was the case for the École du Génie). Nevertheless, this change did not have the expected positive impact. From 1771 to 1774, the navy school in Rochefort was closed and partially replaced by the school in Le Havre, which existed only for one year. Finally, in 1786, the Gardes de la Marine were suppressed, and two non-military schools, namely the colleges in Alais and Vannes, were put in charge of the navy officers' instruction.

The artillery schools lasted longer than the navy schools. They were established in 1720 through a royal order, motivated by the success of some prototype schools like the one in Douai (that subsequently moved to Metz and then to Strasbourg, before being closed), and outlived until the French Revolution. Their history is closely related to that of the military engineers schools. Without any doubt, the most innovative mathematics teaching was delivered at the École du Génie in Mézières. From the beginning in 1748 on, an entry examination was instituted and the *académicien* Charles Camus was appointed *examinateur*. In 1756, the École du Génie was unified with the École

d'Artillerie in La Fère and called École Royale des Élèves. This new institution was structured on the principles that had already inspired the École du Génie, and in particular, an entrance examination was set. Three years later, the two schools were split again. On the one hand, the École des Élèves was now reserved for artillery officers and was moved to Bapaume in 1765. It was closed in 1772.

On the other hand, the military engineer school was transfered back to Mézières and lasted until the Revolution. Moreover, a school (called the École Royale Militaire) was created in 1751 to train 500 nobles by birth who could not afford to pay for one of the existing schools. The best students were afterwards sent to the military engineers corp, the others to the artillery and, in the last instance, to the navy. In 1764, the Collège Henri IV in La Flèche, which previously belonged to the Jesuits, was merged with the École Royale Militaire and renamed École des Cadets; it was closed in 1776. Eleven military schools were founded in this year in order to fill this gap. With regard to these schools, Lacroix observed that they had been a "great experience that we did to refine the public teaching [*grande expérience que l'on fit pour perfectionner l'enseignement publique*]" (cf. Lacroix 1838, p. 50). We remark that the military school in Sorèze, previously a college run by the Benedectines, was especially renown for its pedagogical methods.

The list in Table 1 shows the most renowned military schools up to the French Revolution.

École Royale d'Hydrographie	1666	Le Havre
École d'Hydrographie	1673	Saint Malo
École d'Artillerie	1679	Douai
Écoles des Gardes du Pavillon et de la Marine	1689	Brest, Rochefort, Toulon
Écoles d'Artillerie	1720	Auxonne, Besançon, Grenoble, La Fère, Metz, Strasbourg, Valence
École du Genie	1748	Mézières
École Royale Militaire	1751	Paris
École Royale des Élèves	1756	La Fère
École Spéciale des Mineurs	1764	Verdun
École desCcadets	1764	La Flèche
École des Ponts et Chaussées	1747	Paris
École des Gardes du Pavillon et de la Marine	1773	Le Havre
École Royale de la Marine	1773	Le Havre

Écoles Militaires	1776	Auxerre, Beaumont-en-Auge, Brienne, Effiat, Pont-à-Mousson, Pontlevoy, Rebais, Sorèze, Thiron, Tournon, Vendôme
École des Mines	1783	Paris

Table 1: List of military schools in France up to the French Revolution

With the French Revolution, the educational system underwent radical changes. For this reason, we take this event as upper bound and details about the further development after this period will not be thoroughly covered. In short, the *collèges* were transformed into the Écoles Centrales, which lasted until 1802. In 1794, the newly founded École Centrale des Travaux Publics was suppose to replace all the military engineers schools. Actually, one year later the school was renamed École Polytechnique and provided the general and theoretical training necessary for entering the Écoles d'Applications, which originated from the unification of the artillery and military engineers schools. Additionally, the schools for navy officers were definitively closed and replaced in 1791 by the Écoles d'Hydrographie et de Mathématiques, founded in the most important harbor towns. These schools were meant for the working-class, while the officers were educated on two training ships in Brest and Toulon, called the Écoles Spéciales de la Marine.

Finally, a few words on the institutions that supported the scientific research. During the Renaissance, the center of gravity of the intellectual life gradually passed from the universities to another kind of institution. These were the academies, which developed everywhere in Europe during the 17th and 18th centuries (for instance, in Rome in 1603, in Florence in 1657, in London in 1645, in Paris in 1666, in Berlin in 1700, in Moscow in 1725, and in Stockholm in 1769). In particular, at the *Académie des Sciences* in Paris, the scientific dialogue was conducted through epistolary exchanges and public debates and conferences. The *Académie* provided its members with rooms, laboratories, a library, and funds for experiences and missions. The members got together weekly to evaluate the scientific value of articles, books, and new inventions, in the first place of the members themselves. At the end of the 17th century, the *Académie* had considerably grown: it counted seventy members, organized into *honoraires*, *pensionnaires*, *associés*, *adjoints*, one secretary, and one treasurer. In addition to these, 85 *corréspondants*, who lived outside of Paris, were appointed. In 1793, the *Académie* was closed and finally restored in 1816.