1 Technical and Mathematical Background

1.1 Planning Optical Networks

In a general networking planning problem, we have some locations, possibly of different types, and would like to connect these in some specified way using a network. Usually, some locations have some form of demand. In optical networks, these locations are usually called the *customers*. There are many different kinds of networks, for example telecommunication networks, transport networks and road networks. In the telecommunications setting, reasons for the building of a network can be manifold and include access to the Internet, telephony, fax or specialized applications like dedicated, highly tolerant or high-speed networks.

Because the building of telecommunication networks usually involves high investments incurred by the deployment of network infrastructure, a careful planning of these networks is very important to minimize the potential waste of resources.

In network planning problems, we distinguish between *greenfield* and *brownfield* planning. The former means the planning for areas where no infrastructure exists so far. In brownfield planning, we have to consider the infrastructure already in place and account for any problems resulting from the technological migration.

In this thesis, we only explicitly consider greenfield planning problems, but the presented techniques can also be adapted for brownfield problems. Also, we usually assume for the purpose of simplification that we know exactly which customers want to connect to our network and how much bandwidth they need.

In real planning scenarios, the latter assumption is of course somewhat unrealistic. Here, stochastic and robust methods come into play, where one can consider wide ranges of possible customer scenarios and try to build a network that performes reasonably well in all the likely settings. While this also leads to interesting questions, we concentrate on exploring other aspects of the planning problems in greater detail. Therefore, we assume that no uncertainty in data is involved.

The goal of network planning is to find an appropriate network configuration, where appropriate refers to the fact that the resulting network should be feasible for the given traffic requirements. Configuration is a term encompassing the arrangement of hardware, the routing inside the network and also several other aspects which become especially important for optical networks. These include the problem of assignining some part of the optical spectrum to connections in the network, so called *lightpaths*, and possibly solving network embedding problems, depending on the structure of the bandwidth requests.

1.1.1 Network configurations

From the planning perspective, finding a network configuration for an optical network consists of two tasks depending on one another. One task is to place hardware devices with sufficient capacities at allowed locations. The other task is the establishment of lightpaths inside the optical network. This has to be done in such a way that the requested traffic can be transferred through the network while observing the capacity limits implied by the hardware limitations and actual routing. We now explain these tasks in some more detail.

Hardware configuration

From a somewhat abstract point of view, a physical network consists of a set of *nodes* and a set of *links* connecting pairs of nodes. The nodes can be used as pure transit nodes or represent access points of the network, where access points in this context refer to nodes where there is a connection to some other network or network layer and the node serves as a connection between them.

For the technical realization of a network, hardware devices need to be installed mainly on the nodes of the optical network.¹ Links in the network consist of optical fibers between nodes. Some hardware components are also necessary to enable transmission through the optical links. The hardware configuration of the network encompasses all decisions about which hardware devices should be installed at which locations. These devices include transmitters, receivers, fibers, switches, regenerators, splitters, couplers and filters. The number of transmitters and receivers is usually fixed in our problems as it is determined by the bandwidth requests. Installed devices offer *transmission capacity* on the links and the ability to switch, regenerate, convert or split at the nodes.

All these hardware components determine the available capacity; this part of the planning is called the *dimensioning* part. Sometimes, we express bandwidth demands as the number of optical channels assigned, usually when the channel size is fixed. In more dynamic networks like Flexgrid networks, we also describe them as widths of frequency intervals.

Lightpath configuration



Figure 1.1.1: Lightpaths in a network

¹Regenerator devices may also be placed on the links.

Connections in optical networks are realized by lightpaths, compare also Figure 1.1.1 on the previous page. The routing of the lightpaths determines the paths in the physical network. If we use optical technology that employs different wavelengths, a wavelength assignment has to be carried out as well, that is, a transmission wavelength needs to be assigned to each lightpath.

If the hardware configuration is fixed, the establishment of lightpaths is a pure software task since the hardware components can be reconfigured remotely. The problem of routing lightpaths through the network is called the *traffic engineering* part of the network planning problem.



1.1.2 Hierarchical network structure

Figure 1.1.2: Hierarchical structure of a network

Network layers

Optical networks usually have a hierarchical structure, see also the example in Figure 1.1.2. Subnetworks at certain geographical regions constitute partially autonomous networks and are interconnected to other subnetworks at certain access points. From the lower to higher levels, more and more traffic is aggregated. The network with the highest level of aggregation is called the *core* or *backbone* network. Because highly aggregated traffic in telecommunication networks typically needs very high bandwidth capacities, the core networks especially tend to be constructed using optical technology. The network layer with the lowest aggregation levels is called the *access* network. Because of the technical breakthroughs in the last decades, optical technology is now available and cheap enough to be also used in access networks. The business acronyms for these technologies include Fiber-to-the-Home (FTTH), Fiber-to-the-Building



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(FTTB) and Fiber-to-the-Curb / Cabinet (FTTC), depending on where the optical connection is terminated. From this termination point, some other technology is employed to connect to the customer on this so-called "last mile".

Virtual network layers

In telecommunication networks, a link generally corresponds to some kind of connection between nodes, for instance optical fibers or copper cables. This generally means some cables buried in the ground, so connections cannot be changed quickly or cheaply. To be able to provide computing networks with arbitrary topology quickly and flexibly, *network virtualization* introduces a virtual layer into the network.

This virtual layer corresponds to an overlay network on top of existing physical hardware. This reduces cost, because advanced network functionality can be moved to the overlay network, while using off-the-shelf switches and routers in the underlying physical network. These software-based solutions are typically much more portable and cost-effective than hardwarebased implementations.

Using this construction, links in the overlay layer could correspond to paths in the physical network, for example. If a physical node providing the resources for a virtual node fails, the network might be reconfigured to simply use another physical node without propagating the failure to the overlay network, which leads to more fault-tolerant networks.

1.1.3 Optical access networks

In a typical access network architecture, the connections originate at one end from the Central Offices (COs) where optical transmitters are placed. The connections pass along optical fibers and through intermediate nodes we call *Distribution Points*, where optical splitters and also other devices may be installed. These networks typically use a Point-to-Multipoint (P2MP) architecture. The termination points at the other end of the network are called the *customers*.

We distinguish between two types of optical access networks, the *Active* and the *Passive* Optical Access Networks.

Active Optical Networks (AONs) Active Optical Networks require optical-electro-optical conversion and Medium Access Control (MAC) switching in the distribution points. Their real-world use is quite limited, as they have higher operating expenses due their active use of electricity than their passive cousins. From a lightpath point-of-view, they are based on single-wavelength point-to-point links.

AONs can also be used without distribution points by deploying fiber links from the CO to each customer. As no part of the physical infrastructure is then reused for other connections, this is a very costly approach. While AONs are technologically feasible, the usage of Passive Optical Networks (PONs) is usually more cost-effective.

Passive Optical Networks (PONs) In contrast to the previously described AONs, no active components are used in the distribution points of PONs, so there is no need for electricity at intermediate points of the network. Therefore, they create less operational expenditure.



Figure 1.1.3: A passive optical network (PON)

In a typical PON architecture, an Optical Line terminal (OLT) at the Central Office (CO) is connected to several Optical Networks Units (ONUs) located at the customer end of the network. This is accomplished by splitting the lightpath at some Distribution Points (DPs). The connection between Optical Line Terminals and Distribution Points is called the Optical Feeder Network (OFN), while the connection between the Distribution Points and the Optical Network Units is called the Optical Distribution Network (ODN) or simply the Distribution Network. In the Central Office, each OLT services the OFN. To avoid collisions by frames sent by different ONUs in the Distribution Network at the same time, a protocol needs to be established to only allow one ONU to transmit, while the others have to wait for its transmission to complete.

Since the great advantage of PONs is the reuse of architecture, signals for and from each user need to be combined through multiplexing techniques in downstream and multiple access techniques in upstream. Time-Division Multiplexing (TDM) and Time-Division Multiple Access (TDMA) are the most commonly adopted solutions for these tasks. Wavelength-Division Multiplexing (WDM) accomplishes conflict avoidance by using different wavelengths, but also creates the need for more expensive hardware, as we will see later.

1.1.4 Collision avoidance

We now go into more detail concerning the collision avoidance techniques. While presented here for access networks, these techniqes are also employed in the other network layers. We concentrate on the most common approaches and thus distinguish between *Time-Division Multiplexing* and *Wavelength-Division Multiplexing*. While the former is based on sharing the available transmission time, the latter is based on sharing the available wavelength spectrum.

Time-Division multiplexing (TDM) This method of multiplexing was originally used for telegraphy and then employed for digital telephony, but is now also a prevalent way of sharing

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Figure 1.1.4: Standard WDM-PON architecture

an optical fiber between several parties. In TDM networks, time is divided into periodic time slots of fixed length, one for each transmission channel. For each channel, there is some time slot at which information for this channel can be transmitted.

On the upstream connections, Time-Division Multiple-Access (TDMA) is employed, which is a similar technology, but the signals come from different senders, multiple customers in our case. With *synchronous* TDMA, each sender gets a fixed time slot – more flexible approaches are called *asynchronous*. One asynchronous method, *Dynamic TDMA*, uses a scheduling algorithm that reserves a variable number of time slots in each transmission frame depending on the current traffic demand.

Wavelength-division multiplexing (WDM) In WDM networks, several optical signals may be transmitted on the same fiber. Conflict is avoided by using different wavelengths of light.

This method is very popular because it allows the network provider to increase the capacity without making changes to the fibers. This can be carried out simply by using better multiplexing hardware. On the other hand, additional costs are incurred by expensive equipment like tunable lasers which allow for transmission at varying frequencies of light.

As mentioned, an important advantage of PONs is that by splitting, less parallel fibers are needed to connect different customers; fibers can be reused, as long as the signals are not in conflict with each other. In WDM-PONs, these signals might use different wavelengths and could then be transmitted simultaneously on the same fiber. This also decreases the amount of optical fibers needed to achieve the same bandwidth capabilities compared to TDM-PONs.

1.2 Technical Background

In the first part of this section, we explain the physical basics of optical networks. We discuss some key hardware components used in optical telecommunication systems and identify the principle physical limits of optical communication. For further details and more in-depth explanations of the used technology and physical phenomena, we recommend the books by Mukherjee [2006] and Kazovsky [2011].

A prototypical optical network In a fiber-optical network, light waves are used to convey information between endpoints. The simplest prototype of a fiber-optical network, compare also Figure 1.2.1, consists of three components: an optical transmitter to convert some information-carrying electrical signal into an optical one, an optical fiber through which the signal is then sent and an optical receiver at the other end of the fiber to reconvert the signal to an electrical one. In this way, the information can be recovered after the transmission.

In this prototypical network, the optical transmitter and receiver are responsible for the conversion between the electrical and the optical signal, while the fiber-optic cable acts as an optical waveguide, in which the light is trapped and forced to move along the cable. We now explain the physical background of optical waveguides.

Optical reflection The *refractive index* of a material is defined as

$$n = \frac{c}{v}$$
,

where c is the speed of light in a vacuum and v is the *phase velocity* of light in the material, which is a material constant. The *phase velocity* is given in terms of the wavelength λ and period T as $v_P = \frac{\lambda}{T}$. When a light wave reaches the boundary between two materials with different refractive indices, a part of the wave will be refracted, while another part will be reflected. The angle of refraction is determined by *Snell's law*, which states that if θ_1 , θ_2 denote the angles measured from the normal directions of the surface boundary and n_1 , n_2 denote the respective refractive indices of the materials, we have that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

If $n_1 \gg n_2$ and $\sin \theta_1$ is close to one, we have

$$\frac{n_1}{n_2}\sin\theta_1 = \sin\theta_2 > 1$$

which cannot be fulfilled. In that case, a phenomenon called *Total Internal Reflection* occurs, in which the signal is not refracted, but totally reflected back into the original medium. The angle



Figure 1.2.1: A prototypical optical network

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Figure 1.2.2: Optical reflection phenomena

at which this begins to occur is called the *critical angle* and can be found as $\theta_c = \arcsin \frac{n_2}{n_1}$. An illustration of these phenomena can be found in Figure 1.2.2.

Optical fibers work by utilizing this phenomenon to guide the light inside themselves.

Encoding information on optical signals

While a regular sine wave does not carry information, it can be modified or more precisely *modulated* to encode digital signals. There are several basic methods to encode information on electromagnetic waves.



Figure 1.2.3: Signal modulation methods

In principle, modulation can be imposed on the phase, frequency, amplitude or polarization of the light beam, but most commonly, phase modulation is used. Amplitude modulation can be created via phase modulation with a device called a Mach-Zehnder interferometer, where the beam is split into two beams, one of them is phase-modulated and the beams are then recombined. By controlling the phase of the phase-modulated beam, interference will happen and can be constructive or destructive, thereby controlling the amplitude.

We do not go into any detail concerning these methods, but the ideas of these methods can be seen in a basic form in Figure 1.2.3 on the previous page.

1.2.1 Hardware components

We now introduce the main hardware components used to construct optical networks.

Optical fibers



Figure 1.2.4: Cross section of an optical fiber

An optical fiber consists of a transparent core surrounded by a dielectric cladding material that is itself surrounded by a polymer buffer coating for protective purposes, compare Figure 1.2.4. The core and the cladding are most commonly made of silica glass, but are also available as plastics or a combination of both materials. Plastic fibers usually lead to higher attenuation compared to glass fibers and therefore limit the possible transmission distance. In contrast, silica fibers exhibit low attenuation over a wide range of wavelengths. Advantages of silica also include that they are relatively easy to splice and that they offer more resistance against pulling and bending than plastic fibers, making them more resilient and thus easier to place.

To trap the light waves inside the core and make the fiber act as a waveguide, a material with a lower refractive index than the one of the core is used for the cladding, utilizing the *Total Internal Reflection* phenomenon described earlier.

We distinguish between two main types of optical fibers, *Single-Mode Fibers* and *Multi-Mode Fibers*. We now explain their main characteristica.

Single-Mode Fibers have a diameter comparable to the wave length of an optical signal. The most common type has a core diameter between 8 - 10 micrometers and is used in the near-infrared part of the spectrum. It supports only one optical signal at a time, but keeps the signal intact over a longer distance than a multi-mode fiber can. While the equipment needed to use single-mode fibers is more expensive than the equipment used for multi-mode fibers, the fibers themselves are cheaper to manufacture.

Due to less severe distortion phenomena, they support higher bandwidths and because of that, they are usually used for wide-area and metropolitan-area networks. Passive optical networks also employ them, achieving high data rates and long-distance transmission capabilities.

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Multi-Mode Fibers are usually found in local area networks and have the advantage that they can carry many modes of light simultaneously. In this context, a *mode* of a a signal refers to the way it takes through the fiber. While a single-mode fiber is relatively narrow and can be approximated as a one-dimensional guide, a multi-mode fiber is much thicker and therefore light rays can take a rather direct route through the fiber or zigzag off the cladding that has a different refractive index. The transition between the core and the cladding is either realized as a step-index profile or a graded-index profile. *Step-index profiles* mean uniform refractive index in the core and a sharp decrease in the cladding. They are more common in single-mode fibers. In contrast, *Graded-index profiles* are usually used for multi-mode fibers. This implies that the parts of the core that are closer to the fiber axis have a higher refractive index than the parts near the cladding. Most commonly, a nearly parabolic index profile is created that decreases modal dispersion and continuously refocuses rays.

Optical transmitters

Optical transmitters convert electrical signals carrying information into optical signals. A key component of these devices is a light source, most commonly a semiconductor laser. There are two ways of encoding information in the light signal. One variant is to use the laser as a continuous laser, on which an optical modular encodes a signal. A cheaper alternative is to direcly modulate the light source.

Optical modulators exploit electro-optic effects, by which the optical properties of a material can be changed by the application of an electric field. The changes in the optical properties are caused by forces resulting from the electric field that change the position, shape or orientation of molecules inside the modulator material. A very important effect in this context is the so-called *Pockels effect* or *Linear Electro-Optic Effect*. In crystals exhibiting this effect, the refractive index can be modified in a way that is proportional to the strength of the electric field. These materials then exhibit *Birefringence*, where the refractive index depends on the polarization and direction of light.

A widely used optical modulator type uses a lithium niobate crystal. As described, the refractive index can be changed by changing the strength of the electrical field – using a stronger field will make light travel slower through the crystal. As the phase of the light leaving the crystal is determined by the length of time it takes the light to pass the crystal, this can be used to create phase modulation.

Optical receivers

After the optical transmitter has performed an electro-optic conversion and the signal has travelled through the network, the task of the *Optical Receiver* is to convert the optical signal back into an electrical one. In that way, the transmitted information can be recovered. The main components of an optical receiver are a photodetector that generates an electrical current that is proportional to the optical power, several amplifiers and an electrical circuit that recovers the information.

Photodetectors absorb photons and generate electrical current proportional to the power of the optical signal. In optical systems, this job is usually executed by *photodiodes*. They can be classified into PN and PIN diodes.

Photodiodes are like regular semiconductor diodes in the sense that they conduct current in only one direction and offer a high resistance against conducting into the other direction. The difference between photodiodes and regular semiconductor diodes is that the sensitive area of the diode is exposed in such a way that light can reach it.



Figure 1.2.5: Schematic view of a p-n junction in an equilibrium state

Between resistant materials and insulators, there is a class of conducting materials called *semiconductors*, which can be crystalline or amorphous solids. They have the useful property that their conducting properties can be modified by introducing impurities into their highly pure base material, a process called *doping*.

The properties of semiconductors are explained using the movement of *charge carriers* inside a crystal lattice. Charge carriers can be electrons or the absence of an electron, where one could exist inside an atom or atomic lattice. The absense of an electron is called a *hole*, so there are two types of charge carriers in a semiconductor, electrons and holes.

The doping process increases the number of charge carriers. A semiconductor is said to be of p-type if its hole concentration is larger than its electron concentration, otherwise it is said to be of n-type.

The boundary area between a p-type and a n-type semiconductor is called a *p-n junction*. PN diodes are photodiodes based on p-n junctions, compare also Figure 1.2.5.

If no external voltage is applied, a p-n junction goes into an equilibrium state that exhibits a potential difference across the junction. This happens by the diffusion of electrons from the n-region into the p-region forming negatively charged ions in the p-region, while leaving behind positively charged ions in the n-region. This leaves no free charge carriers in the area of the junction, making it non-conductive. The area of non-conductivity is called the "depletion layer".

By the resulting potential difference, an electric field is created, counteracting the diffusion movement, until an equilibrium state is reached.