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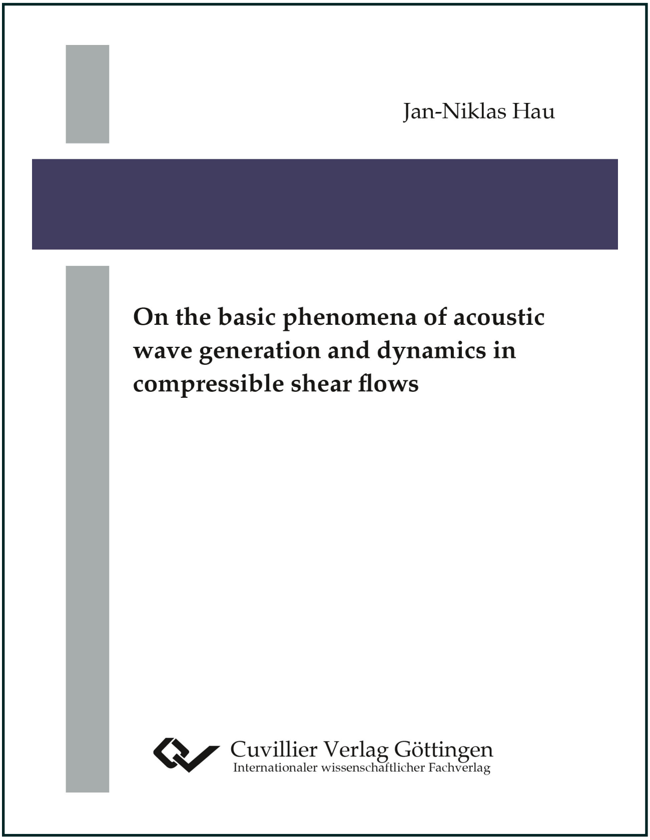
On the basic phenomena of acoustic wave generation and dynamics in compressible shear flows



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1 Introduction

This thesis summarizes my work of the past four years at the Institute of Fluid Dynamics at the Technische Universität Darmstadt. It is noted that it contains parts of the following papers in slightly modified form, which resulted from my work as a research assistant:

- Hau, J.-N., Chagelishvili, G., Khujadze, G., Oberlack, M., Tevzadze, A. (2015): A comparative numerical analysis of linear and nonlinear aerodynamic sound generation by vortex disturbances in homentropic constant shear flows. *Phys. Fluids.* 27, 11, 126101.

- Chagelishvili, G., Hau, J.-N., Khujadze, G., Oberlack, M. (2016): Mechanical picture of the linear transient growth of vortical perturbations in incompressible smooth shear flows. *Phys. Rev. Fluids* 1, 4, 043603.

- Hau, J.-N., Oberlack, M., Chagelishvili, G. (2016): On the optimal systems of subalgebras for the equations of hydrodynamic stability analysis of smooth shear flows and their group-invariant solutions. *J. Math. Phys.* under review.

- Hau, J.-N., Chagelishvili, G., Oberlack, M. (2016): On the mathematical and physical aspects of the linear generation of acoustic waves by vortex perturbations in homentropic compressible shear flows. *Phys. Rev. Fluids* under review.

The main objectives were to investigate the basic phenomena of wave generation and propagation in unbounded compressible shear flows in the light of (i) the breakthrough achieved by the hydrodynamic stability community in the 1990s in understanding shear flow phenomena and (ii) Lie symmetry analysis. For the reason that the incompressible dynamics are a limiting case of the full, compressible ones it is feasible to also study these to some extend, in order to gain a deeper understanding of the underlying physical mechanisms. The importance of such shear flow configurations can be understood when considering the problem of aerodynamic sound generation and its further propagation. This is summarized under the term *aeroacoustics*, involving both phenomena and being a major subject of fluid dynamics, with applications in wide areas of engineering problems such as aircraft jet engines, naval and automotive applications. The field of application even extends to the geophysical (e.g., atmospheric turbulence and dynamics) and astrophysical (e.g., helio- and astroseismology – pressure oscillations in the sun and stars) context. Consequently, it appears naturally that aerodynamic sound generation represents an enormous scientific and technological challenge.

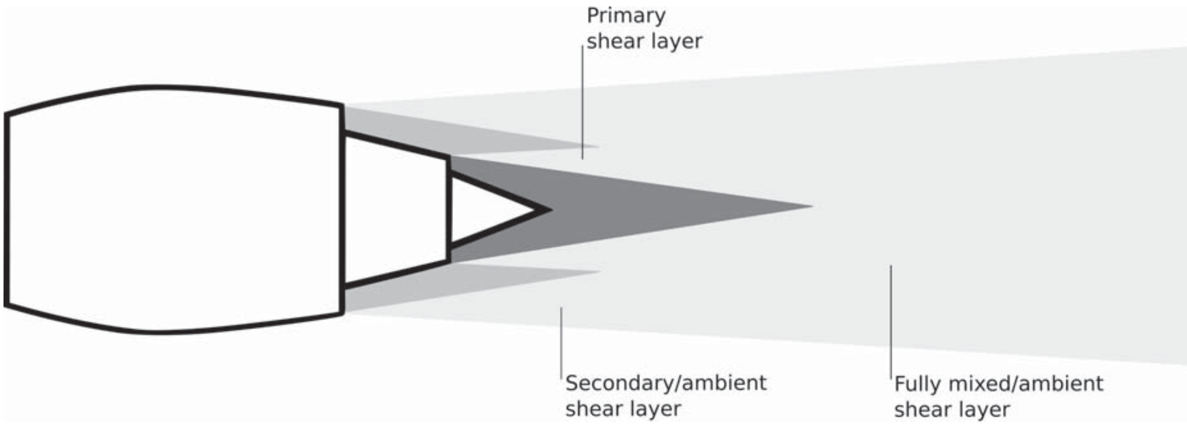

Figure 1.1: Sketch of different shear layers in a coaxial jet exhaust of a typical turbofan engine.

1.1 Motivation

Möhring, Müller & Obermeier (1983) summarize that flow acoustics/aeroacoustics differs from other acoustic disciplines in that flows play an essential role in the acoustic phenomena. Here, we can distinguish three essentially different processes, namely, (i) the generation of sound with essential participation of the flow, (ii) the propagation of sound through flow fields, and (iii) the generation of flow by sound. In the present thesis we concentrate on the first two issues.

A typical and popular engineering example, where the effect of shear in aerodynamic sound generation is ubiquitous, can be found in a coaxial jet exhaust of a turbofan engine, illustrated in figure 1.1. When it comes to the design process of modern turbojet engines, noise regulations make it imperative to take aeroacoustic properties into account. Indeed, the manufactures have put a considerable amount of effort into the reduction of noise emitted by their engines. This is usually defined in terms of the acoustic intensity, which is proportional to the mean square pressure. To achieve this aim, the temporal and spatial fluctuations of density and pressure about their respective ambient quantities that propagate in the form of waves, need to be reduced. In the past, this has often been achieved by increasing the bypass ratio. The slower co-moving secondary stream of air exhausting concentrically around the primary stream, the core jet, induces two annular mixing regions with significantly lower shear than that created by a single-stream jet at the same thrust (Rolls Royce 2015). It has been recognized that *shear* between the exhaust gases and the atmosphere has to be reduced in order to reduce the jet noise, which is a major source apart from combustion noise and flow–structure interactions.

As sketched in figure 1.1, several regions appear in the flow behind modern engines where shear layers are formed, i.e., where two parallel streams of fluids with different velocities meet at an interface. In such shear layers the velocity in the core region can be approximated to vary linearly (see figure 1.2). There are many more examples in engineering, where such flow configurations can be found, and where they are

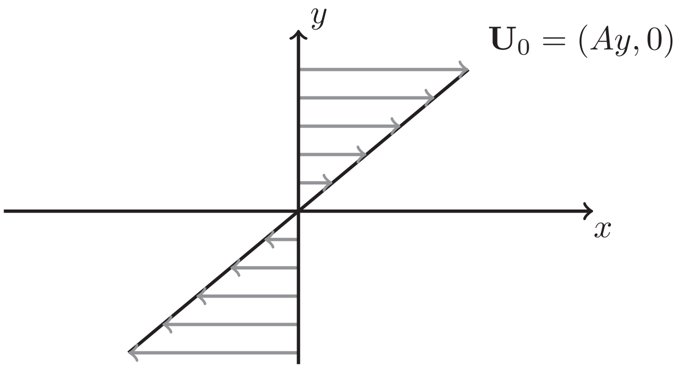

Figure 1.2: Sketch of the velocity profile of a two-dimensional plane shear flow, with velocity profile $\mathbf{U}_0 = (Ay, 0)$.

important in the sense of aeroacoustics, e.g., the flow around a vehicle. The present work mainly deals with this very flow region and we attempt to shed further light on the question asked by Ffowcs Williams (1977): *Where are the sources and how strong are they?*

In the context of the insight into aeroacoustic sound production mechanisms, Goldstein (2005) argues that, despite the various approaches, which have been undertaken to extract the *true sources of aerodynamic sound generation*, the *true sources of sound* still remain unknown. Partially, this might be related to the fact that some of the approaches oversimplify flow–acoustic (hydrodynamic and acoustic field) interaction (Sinayoko, Agarwal & Hu 2011), which are in general highly coupled (Goldstein 2005). The problems related to the answering of this question seem to be connected to the specificity of shear flows, which have received considerable attention both in the incompressible and compressible configurations.

Before going deeper into the mathematical difficulties that appear when treating the equations governing the dynamics of shear flows, we present these in the following and subsequently review the idea of an acoustic analogy as introduced by Lighthill (Lighthill 1952, Lighthill 1954) that still is the basis of many engineering approaches. Finally, we describe the breakthrough achieved by the hydrodynamic stability community in the 1990s, in which course the mentioned mathematical difficulties have been understood.

1.2 The governing system of equations

In fluid dynamics, gas or liquids are considered as a continuum. This means that a fluid particle can be defined as being larger compared to the molecular scales, yet, small compared to the other length scales of the considered problem. The governing set of partial differential equations – the laws of mass, momentum and energy conservation – are thus applied to an elementary fluid particle in order to describe the fluid motion. Dealing with small acoustic perturbations, the concept of linearization is useful (Rienstra & Hirschberg 2016).

The five scalar partial differential equations (PDEs), which are required in order to describe three-dimensional (3-D) fluid motions, are the compressible Navier-Stokes equations – the fundamental coupled PDEs describing the motion of an (in general) ideal gas. For the case of a 3-D problem the Navier-Stokes equations consist of the time-dependent continuity equation (conservation of mass), three time-dependent equations describing the conservation of momentum and an equation describing the time-dependent conservation of energy. This set of equations is presented below in cartesian tensor notation (Spurk & Aksel 2007, Kundu, Cohen & Dowling 2012) and reads for the conservation of mass

$$\frac{\partial \varrho}{\partial t} + \frac{\partial}{\partial x_i}\left(\varrho U_i\right) = 0 \, , \tag{1.1}$$

and for the conservation of momentum

$$\frac{\partial}{\partial t}\left(\varrho U_i\right) + \frac{\partial}{\partial x_i}\left(\varrho U_i U_j\right) = \varrho k_i + \frac{\partial}{\partial x_i}\tau_{ij} \, . \tag{1.2}$$

The stress tensor τ_{ij} is often split into $\tau_{ij} = -p\delta_{ij} + P_{ij}$, where P_{ij} is the friction stress tensor. The mass forces k_i are in the case of the earth's field $k_i = g_i$ – the gravity. The conservation of energy is given by the following equation:

$$\varrho U_i \frac{\partial U_i}{\partial t} + \varrho \frac{\partial e}{\partial t} = \varrho k_i U_i + U_i \frac{\partial \tau_{ij}}{\partial x_j} + \tau_{ij}\frac{\partial U_i}{x_j} - \frac{\partial q_i}{\partial x_i} \, . \tag{1.3}$$

In the above equations, t is the time, $\mathbf{x}$ is the position vector and $\mathbf{U}$ the instantaneous velocity vector; p and ϱ denote the pressure and density, respectively and q_i is the heat flux. After all, an equation of state, i.e., $p = p(\varrho, s)$, needs to be provided, with s denoting the entropy.

These equations can and need to be simplified in order to ease their handling. Given that there are no exact solutions for the general set of equations, making assumptions is a standard procedure in order to obtain, at least, approximate solutions. In the acoustic realm, viscous or turbulent stress terms τ_{ij} are usually neglected, as sound waves are not affected by those. Moreover, any perturbation is too fast to be affected by thermal conduction q_i, hence, it is considered of being as well of no importance (Rienstra & Hirschberg 2016). Thus, energy losses that are due to viscous and thermal diffusion between neighboring fluid particles are explicitly omitted in the analysis, i.e., each fluid particle is assumed to expand and contract adiabatically (Howe 2007). The equations simplify to the Euler equations of ideal gas dynamics. These are hyperbolic in contrast to the incompletely parabolic (parabolic and elliptic) Navier-Stokes equations and read for the conservation of mass

$$\frac{\partial \varrho}{\partial t} + \frac{\partial}{\partial x_i}\left(\varrho U_i\right) = 0 \, , \tag{1.4}$$

and momentum

$$\frac{\partial}{\partial t}\left(\varrho U_i\right) + \frac{\partial}{\partial x_j}\left(\varrho U_i U_j + p\delta_{ij}\right) = 0 \, , \tag{1.5}$$

where δ_{ij} denotes the Kronecker delta. External forces are neglected. Finally, the equation for energy conservation for an isentropic (constant entropy of a single particle) flow is given by

$$\frac{\partial s}{\partial t} + U_i \frac{\partial s}{\partial x_i} = 0 \; . \tag{1.6}$$

This approximation is reasonable for most applications, especially for the flow considered in the present work, as no boundaries are involved. Further, by assuming that the considered flow is homogeneous, which means that the entropy is uniform and constant ($\mathrm{d}s = 0$ and $p \propto \varrho^{\lambda}$), i.e., *homentropic*, the equation $s = s_0$ = const. provides a relation between the thermodynamic variables:

$$s = c_v \lg p - c_p \lg \varrho \; . \tag{1.7}$$

The specific heat capacity at constant pressure and at constant volume are denoted as c_p and c_v, respectively, and are related to each other by the ratio of specific heats $\lambda = c_p/c_v$. This enables to determine the motion solely from the equations of continuity and momentum together with the equation of state $p = p(\varrho, s)$, thus, the energy equation can be omitted. In more general situations, in which s is variable, it is necessary to retain the energy equation to account for coupling between macroscopic motions and the internal energy of the fluid (Howe 2007). However, in the framework of aeroacoustics, the branch of compressible flow where the velocity and pressure variations are small compared to the steady reference value, the pressure variations are in fact isentropic. In this regime, linear theory is often the most useful guide (Maslowe 1986). *De facto* the temporal and spatial fluctuations/variations about a base state spread in the form of (audible) waves – as a succession of compressions and rarefactions/expansions (Howe 1998). It is then justified to put $p = p(\varrho, s_0)$ and $P_0 = p(\varrho_0, s_0)$. The small linearized perturbations *ergo* must satisfy the relation

$$p = P_0 + p' = p(\varrho_0 + \varrho', s_0) \approx p(\varrho_0, s_0) + \left[\frac{\partial p}{\partial \varrho}(\varrho, s_0)\right] \varrho' = P_0 + c_s^2 \varrho' \; , \tag{1.8}$$

which is a Taylor expansion of the pressure about the reference thermodynamic state, denoted by the subscript 0 (neglecting higher order derivatives). Such an Ansatz, consisting of the separation of the dependent field variables, specifically

$$U_i = U_{0i} + u_i \; , \quad p = P_0 + p' \; , \quad \varrho = \varrho_0 + \varrho' \quad \text{and} \quad T = T_0 + T' \; , \tag{1.9}$$

is a basic concept in linear stability analysis (Schmid & Henningson 2001, Schmid 2007). Here, the fluctuating quantities u_i, ϱ', T' in velocity, density and temperature, are considered to be very small compared to their base/reference state. The derivative is evaluated at the thermodynamic reference state (P_0, ϱ_0, s_0) to

$$c_s = \sqrt{\left(\frac{\partial p}{\partial \varrho}\right)_s} \; , \tag{1.10}$$

and is known as the speed of sound, c_s, which is not necessarily a constant for non-isentropic flows (Howe 2007, Kundu et al. 2012).

Hence, rewriting equation (1.8), we find the linearized relation for the speed of sound for small perturbations:

$$\varrho' = p'/c_s^2\,. \tag{1.11}$$

Evaluating the equations for an isentropic process of a perfect gas that relate two states 1 and 2 with each other,

$$\frac{p_2}{p_1} = \left(\frac{\varrho_2}{\varrho_1}\right)^{\lambda}, \qquad \frac{T_2}{T_1} = \left(\frac{\varrho_2}{\varrho_1}\right)^{\lambda-1}, \tag{1.12}$$

together with the thermal equation of state, $p = \varrho RT$, at the local state of reference, moreover, gives

$$c_s^2 = \lambda\frac{p}{\varrho} = \lambda\frac{P_0}{\varrho_0} = \lambda RT_0\,. \tag{1.13}$$

Here, R is the gas constant and is defined as $R = c_p - c_v$ (see, e.g., Refs. (Howe 2007, Spurk & Aksel 2007, Kundu et al. 2012)). Generally, the speed of sound in air is taken to be about $340\mathrm{m/s}$. Ordinary sound levels in air, i.e., acoustic-pressure magnitudes, are of order $1\mathrm{Pa}$ or less (Kundu et al. 2012), so, the relation

$$\frac{p'}{\varrho c_s^2} \ll 1\,,$$

being the primary parametric requirement for the validity of acoustic theory, is justified, as it usually evaluates to values less than 10^{-5}. Due to the reason that the human ear has roughly a logarithmic sensitivity and that there are about six orders in amplitude between the hearing and pain threshold, noise is measured on a decibel scale:

$$20\,\mathrm{lg}\left(\frac{|p|}{p_{\mathrm{ref}}}\right) \qquad \text{with} \qquad |p| = \sqrt{\langle p'^2\rangle}\,.$$

The standardized reference pressure is given by $p_{\mathrm{ref}} = 2\cdot 10^{-5}\mathrm{Pa}$ (Howe 1998).

Preliminary, we study the acoustic wave generation process and their farther propagation in the 2-D parallel unbounded inviscid plane constant shear flow sketched in figure 1.2. This enables to grasp the phenomena that appear in the linearly sheared regions of more general situations, like mixing-layers and jet flows. Knowing the basic state of the investigated flow (its base velocity profile, density and pressure distribution) the idea of linearization can be used on the Euler equations (substituting the decomposition presented in equation (1.9) into the Euler equations (1.4)-(1.6), and neglecting higher order terms in the fluctuations).

These then have the following form

$$\left[\frac{\partial}{\partial t} + Ay\frac{\partial}{\partial x}\right]\varrho' + \varrho_0\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) = 0\,, \tag{1.14a}$$

$$\left[\frac{\partial}{\partial t} + Ay\frac{\partial}{\partial x}\right]u_x + Au_y = -\frac{1}{\varrho_0}\frac{\partial p'}{\partial x}\,, \tag{1.14b}$$

$$\left[\frac{\partial}{\partial t} + Ay\frac{\partial}{\partial x}\right]u_y = -\frac{1}{\varrho_0}\frac{\partial p'}{\partial y}\,, \tag{1.14c}$$

where p', ϱ' and $\mathbf{u} = (u_x, u_y)$ are the pressure, density and velocity disturbances, respectively. The base shear flow is homentropic – uniform in pressure and density ($P_0, \varrho_0 = \text{const}$) – and has a constant shear of velocity $\mathbf{U}_0 = (Ay, 0)$. The constant shear parameter A is assumed to be positive without the loss of generality and the disturbances to be adiabatically compressible with a constant speed of sound c_s, $p' = c_s^2\varrho'$ (Crighton 1981).

By simple manipulations of the set of equations (1.14), i.e., $\frac{\partial}{\partial y}$(1.14b)$-\frac{\partial}{\partial x}$(1.14c) $-A\cdot$ (1.14a) $= 0$, a linearized invariant of the system is derived

$$\left(\frac{\partial}{\partial t} + Ay\frac{\partial}{\partial x}\right)\left[\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x} - A\frac{\varrho'}{\varrho_0}\right] = 0\,. \tag{1.15}$$

This invariant can be associated to the potential vorticity (PV) as defined by Ertel, which is conserved in the absence of friction and heating (see, e.g., Ref. (McIntyre 2003)):

$$\mathrm{PV} = \underbrace{(\varrho_0 + \varrho')^{-1}\nabla\times\mathbf{U}}_{\substack{\text{absolute vorticity}\\ \text{on isentropic}\\ \text{surface}}}\cdot \underbrace{\nabla\Theta}_{\text{static stability}}\,, \tag{1.16}$$

where Θ is the potential temperature. Here, we have neglected rotation of the system (planetary vorticity). In the following, we derive the formulation given in equation (1.15) from the above formulation for the PV: In the considered 2-D case, we are interested in the conservation of PV on an isentropic surface. We can thus neglect the gradient of the potential vorticity, i.e., assuming constant static stability. So, the PV can be rewritten for the case of a barotropic fluid, where we are free to choose Θ to be the coordinate z, which is directed orthogonal to the regarded isentropic plane, as:

$$\begin{aligned}\mathrm{PV} &= (\varrho_0 + \varrho')^{-1}\nabla\times\mathbf{U}\cdot\nabla\Theta = (\varrho_0 + \varrho')^{-1}\nabla\times\mathbf{U}\cdot\nabla z = (\varrho_0 + \varrho')^{-1}\nabla\times\mathbf{U}\,\mathbf{e}_z\\ &= (\varrho_0 + \varrho')^{-1}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} - A\right)\,.\end{aligned} \tag{1.17}$$

Ertel's theorem is a differential statement of Kelvin's theorem, where the Kelvin contour is chosen in a surface, for which the baroclinic vector $\nabla\rho\times\nabla p$ lies in the surface and makes no contribution to the change in the circulation. For the reason that

$$\frac{\mathrm{D}}{\mathrm{D}t}\mathrm{PV} = 0 \tag{1.18}$$

holds, with $\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \mathbf{u}\frac{\partial}{\partial \mathbf{x}}$, the conservation of PV can be formulated as

$$\frac{\mathrm{D}}{\mathrm{D}t}\mathrm{PV} = \left[\frac{\partial}{\partial t} + u_x\frac{\partial}{\partial x} + Ay\frac{\partial}{\partial x} + u_y\frac{\partial}{\partial y}\right]\left(\frac{1}{\varrho_0 + \varrho'}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} - A\right)\right) . \tag{1.19}$$

Finally, neglecting higher order terms in the perturbation quantities as well as derivatives of order larger than one, yields the formulation given in equation (1.15). We stress here that this invariant (the PV) plays a central role in the present thesis as it enables to understand the nature of the imposed disturbances and their dynamics in the flow.

1.3 Lightill's acoustic analogy

Addressing the problem of noise emitted by a jet aircraft, i.e., comprehending how a freely evolving vortical flow occupying some finite region might emit sound waves, Lighthill formulated the idea of an acoustic analogy (AA) in his pioneering papers (Lighthill 1952, Lighthill 1954). This basic framework of aeroacoustic research rearranges the Navier-Stokes equations (1.1)-(1.2) into an equation for an aeroacoustic variable of interest, Ψ. The generalized form of the classical AA equation is given by a wave-equation in physical space

$$\mathcal{L}\Psi = \mathcal{S}(\Psi) , \tag{1.20}$$

where $\mathcal{L}$ is a simple wave equation operator and $\mathcal{S}$ a (mathematical) source term. By choosing the density disturbance ϱ' as the variable of interest, Lighthill's AA equation can be written as

$$\underbrace{\frac{\partial^2 \varrho'}{\partial t^2} - c_s^2\frac{\partial^2 \varrho'}{\partial x_i \partial x_i}}_{\text{sound propagation}} = \underbrace{\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}}_{'\text{source}'}, \quad \text{with } T_{ij} = \varrho U_i U_j + P_{ij} - \delta_{ij} c_s^2 \varrho, \tag{1.21}$$

where T_{ij} is the Lighthill stress tensor. The essence of this equation is the separation of the source of aerodynamic sound generation from the terms responsible for the sound propagation – the sound generation process is treated as the excitation of the linear acoustic field by the known source term $\mathcal{S}$ which is a complex combination of linear and nonlinear processes. The specificity of such an approach is that the results strongly depend on the form of the analogy equation as well as on the aeroacoustic variable chosen to analyze the process. This issue becomes even more crucial as the model – developed for engineering flows – should account for a background flow.

In the past several attempts have been undertaken to improve Lighthill's original idea to take into account: (i) the sound produced by vortices (Powell 1964) and fluid inhomogeneities (Howe 1975); (ii) the effects of arbitrary moving boundaries (Ffowcs Williams & Hawkings 1969); (iii) the duality of jet noise (Michalke & Fuchs 1975, Michalke 1977); (iv) flow–acoustic interactions (Phillips 1960, Lilley 1974). In Lilley's equation the propagator describes propagation/amplification of acoustic waves through a nonuniform base flow. Goldstein has further generalized Lighthill's idea, including a form of Lilley's equation for a homogeneous shear base-flow (Goldstein 1984, Goldstein 1987). The latter, assumed to be especially suitable for predicting sound generated by disturbances in parallel shear flows, however, entirely neglects sound production by linear sources for the benefit of a more sophisticated sound convection term. Since all the different modifications describe the sources in a different manner, there are difficulties in judging if one of them is superior to others (Samanta, Freund, Wei & Lele 2006). Despite the efforts, which have gone into the above cited extensions and corrections, in order to take the effect of an inhomogeneous background flow into account or to choose the aeroacoustic variable in a more sophisticated way, it is accepted (Tam 1995a, Colonius, Lele & Moin 1997, Self 2004, Kaltenbacher, Escobar, Becker & Ali 2008, Tam, Viswanathan, Ahuja & Panda 2008, Karabasov 2010) that a sensible empirical input is needed to predict reliably flow-generated sound. Moreover, in the context of the insight into aeroacoustic sound production mechanisms, Goldstein (2005) strongly argues that none of the suggested AAs is suitable to identify the *true sources of sound*. Even, he doubts that any arbitrary flow can be entirely decomposed into its acoustic and non-acoustic components – yet minimizing the impurity of one another – in a compressible medium, as the prevailing view assumes that the acoustic field is a (small) by-product of any compressible motion. In this sense the acoustic component includes all the pressure fluctuations but no vorticity, thus, the non-acoustic component is connected to purely vortical motion. This assumption holds for inviscid small-amplitude disturbances in a uniform base flow (Kovásznay 1953, Chu & Kovásznay 1958), and is the basis of modern aeroacoustic approaches (Ewert & Schröder 2003, Sinayoko et al. 2011). Still, this decomposition does not hold for simple shear flows, where vortices acquire a divergent nature, while the wave mode acquires curl in its evolution (Bodo, Chagelishvili, Murante, Tevzadze, Rossi & Ferrari 2005, Kalashnik, Mamatsashvili, Chagelishvili & Lominadze 2006, Tevzadze 2006). In general, this decomposition is the key problem of the determination of the source of acoustic waves and is discussed in detail in section 3.2.2.

The proclaimed second golden age of aeroacoustics establishes novel investigations (Lighthill 1992). The promising possibility of exploiting the recently advancing capabilities in computational fluid dynamics (CFD), which have arose and made an impressive progress past the last 60 years (Tam 1995a, Wang, Freund & Lele 2006), plays a major factor herein. Whilst hybrid computations are an alluring mean of computing the radiated sound by a possible source of aeroacoustic sound (Chyczewski & Long 1996, Ewert & Schröder 2003, Sandham, Morfey & Hu 2006, Bodony & Lele 2008) in engineering applications, direct numerical simulations (DNS) or large eddy simulations (LES) appear to be especially suitable tools to directly compute the sound field (Colonius & Lele 2004, Wang et al. 2006). The latter approaches help to unravel the flow physics and mechanisms of sound generation, as DNS is able to capture the dynamics in the full range of physical scales, and LES captures a range of energy-containing scales, while representing the smallest ones by a subgrid model (Colonius

& Lele 2004). Additionally, one has access to all flow quantities, both spatially and temporally. In spite of the comprehensive quantitative information, CFD alone is unable to provide a sufficient understanding of the physics that underlie aeroacoustic processes.

Against the background of the discussed issues that are related to the source term formulation in AAs, we propose to draw the attention towards the breakthrough of the hydrodynamic community in the 1990s, which brought understanding of phenomena induced by the non-orthogonality of the linear operators in nonuniform/shear flows. The essence of these issues also concerns compressible flow systems, i.e., sound-generating flow systems. The more so, most of the natural and engineering flows are nonuniform.

1.4 A change of paradigm in stability analysis of smooth shear flows

Since Reynolds' pioneering experiment in 1883, the transition from a quiescent laminar to a strongly turbulent state in non-uniform/shear flows has attracted the interest of generations of scientists. A classical approach to stability problems, aiming to analyze the onset of turbulence, consists of assuming a laminar base flow and superimposing a small perturbation. This analysis leads to a range of stable/unstable modes and respectively corresponding Reynolds numbers. The most famous and extensively used approach, the so-called normal mode approach (spectral expansion of disturbances in time and space, followed by eigenfunction analysis), was derived by Orr (1907). Here, periodic modes are considered, traveling in the streamwise direction. A large collection of literature exists on the application of this approach to homogeneous incompressible (Hopf 1914, Wasow 1953, Grohne 1954, Reid 1974, Bayly, Orszag & Herbert 1988) and compressible shear flows (Scott 1979, Möhring et al. 1983, Campos, Oliveira & Kobayashi 1999), as well as inhomogeneous stratified or magnetized shear flows (Miles 1961, Acheson 1976, Lindzen & Tung 1978, Maslowe 1986, Lindzen 1988). A comprehensive introduction to this field can be found in the book by Schmid & Henningson (2001). However, for numerous flows, especially the plane Couette and pipe flow, the results obtained by this approach, which captures only the asymptotic fate of the perturbation, can lead to deceptive conclusions (see, e.g., Ref. (Schmid 2007) and references herein), which do not coincide with experimental observations (Tillmark & Alfredsson 1992, Schmid 2007).

Specifically, the essence of the mentioned breakthrough in hydrodynamic stability theory in the 1990s is that the non-normal nature of shear flow systems was rigorously revealed and its consequences were well understood (Reddy & Henningson 1993, Trefethen, Trefethen, Reddy & Driscoll 1993, Criminale, Jackson & Jackson 2003, Trefethen & Embree 2005, Schmid 2007). The operators in the mathematical formalism of modal analysis of nonuniform/shear flows, such as for plane Couette and Poiseuille flows, are non-normal and the corresponding eigenmodes are non-orthogonal (Reddy & Henningson 1993, Trefethen & Embree 2005, Schmid 2007). In the course of the new understanding, the shortcomings of the classical/modal analysis for shear flows have been revealed. In fact, there is a strong interference among the eigenmodes due to the non-orthogonality. Consequently, a proper approach should fully analyze eigenmode interference. Although possible in principle, this is a formidable task in practice. These circumstances led to a change of paradigm in the mathematical approach of linear processes in smooth shear flows and shifted the focus from the long-time asymptotic to the short-time behavior. It was the so-called *nonmodal approach* that became well established and extensively used since the 1990s, resulting in the understanding and precise description of linear transient phenomena. The nonmodal analysis is a modification of the initial value problem and is capable of revealing several unexpected phenomena, which were overlooked by the spectral/modal analysis. There exist two different formulations of the nonmodal approach: the generalized stability theory (GST) (Farrell & Ioannou 1996) and the Kelvin mode approach. These approaches have enjoyed substantial success in furthering an adequate description of instabilities and in providing the missing linear and nonlinear dynamics in a variety of shear flows.

In front of the background of this breakthrough, we focus on the non-normality of shear flow induced dynamics to explain and describe aerodynamic sound generation. Here, the Kelvin mode approach, stemming from the paper by Lord Kelvin (1887), with a time-dependent shearwise wavenumber, has been extensively used since the 1990s (see, e.g., Refs. (Farrell & Ioannou 1993, Chagelishvili, Tevzadze, Bodo & Moiseev 1997b, Farrell & Ioannou 2000, Tevzadze, Chagelishvili & Goossens 2005, Bakas 2009, Kumar, Bertsch & Girimaji 2014, Hau, Chagelishvili, Khujadze, Oberlack & Tevzadze 2015)). It is the simplest formulation and involves the change of the independent variables from the laboratory to a moving frame and allows to quantitatively study the temporal evolution (the short-term behavior) of spatial Fourier harmonics (SFHs) of perturbations without any spectral expansion in time. *De facto*, the Kelvin mode represents the "simplest element" of shear flow physics (Chagelishvili, Chanishvili & Lominadze 1996) and has also been referred to as "flowing eigenfunctions" in expanding fluctuations (Yoshida 2005). This approach not only describes systems with constant shear flow, but also guides the understanding and qualitative description of smooth shear flow phenomena. There is various terminology for the Fourier mode with time-dependent wave vector in the different areas of fluid mechanics, e.g., "Kelvin mode", "shear wave", "shwave", "flowing eigenfunctions". Salhi & Cambon (2010) give a comprehensive survey of the method in the different communities (see sections I and IIC). There exist the first successful efforts leading to the revelation of a strong linear aerodynamic sound generation mechanism by vortex disturbances, intensively connected to the non-normality induced mode-coupling in shear flows (Chagelishvili et al. 1997b, Farrell & Ioannou 2000, Bakas 2009, Favraud & Pagneux 2014).

A linear coupling of vortex and acoustic wave modes in smooth shear flows was found by Chagelishvili et al. (1997b) and the consequences of the coupling were studied. This linear coupling leads to the abrupt emergence of acoustic waves from vortices, or, in other words, to spontaneous imbalance. The phenomenon is demonstrated on the simplest example of a 2-D unbounded compressible plane shear flow and is interpreted in the framework of a nonmodal approach by tracing the linear dynamics of initially imposed pure vortex mode SFHs in time. Farrell & Ioannou (2000) extended this linear analysis for viscous high Mach number flows integrating the viscous extension of the compressible shear wave solutions in unbounded constant shear flows. The inviscid growth is not sustained and all linear regime solutions in viscous unbounded constant shear flows are damped as time tends to infinity (because viscous damping rapidly increases as the wave number of the Kelvin-mode solution increases). Further, Bakas (2009) showed that at small Mach numbers the propagating acoustic waves are excited abruptly at a fixed time. Their spontaneous generation can be analyzed as an instance of a Stokes phenomenon in which the wave solution is switched on by the vortex perturbation when time crosses a Stokes line. Very recently, Favraud & Pagneux (2014) carried out an extensive analysis of the above problem and identified new details of the linear coupling phenomenon.

At this point we note that there does exist another branch of stability theory, where so called open/spatially developing flows are investigated, referred to as global stability theory. Here, modal solutions are extensively used (see for instance the reviews by Huerre & Monkewitz (1990) and Chomaz (2005)) aiming to find additional information, either locally (see, e.g., Ref. (Michalke 1984)) or globally, as pursued by Refs. (Åkervik, Ehrenstein, Gallaire & Henningson 2008, Lele, Mendez, Ryu, Nichols, Shoeybi & Moin 2010, Nichols & Lele 2011, Beneddine, Mettot & Sipp 2015). For the reason that these flows are not parallel anymore, as the base velocity profile also changes in streamwise direction now, the basic streamwise advection has to be taken into account by introducing an additional eigendirection and a non-normality, which differs from the classical (i.e., parallel shear flows) non-normality (Chomaz 2005). Åkervik et al. (2008) note in their study of a 2-D boundary layer flow that the non-normality of the linear evolution operator associated with open flows increases the more parallel the base flow becomes, which is in accordance with the results presented by Cossu & Chomaz (1997). Although, the phenomenon of non-normality is understood on the mathematical basis (see, e.g., Refs. (Schmid 2007, Schmid & Brandt 2014) and references therein), the underlying physics are still under investigation. For the reason that the physical mechanisms have neither been fully understood in the herein investigated temporally developing flow, we prefer to concentrate on this, believing that this simpler flow configuration can hold ready important insights. In fact, recently a first attempt has been made to gain new insights into the Ansatz functions used for classical stability analysis of constantly sheared flows. Nold & Oberlack (2013) found that the Ansatz functions for the Kelvin mode and modal approach rely on the symmetries of the governing set of linear PDEs. Moreover, a third Ansatz function has been presented, which indicates that there is a wider class of possible invariant solutions, which can serve as Ansatz functions.

Notably, the linear coupling revealed by Chagelishvili et al. (1997b) is a general phenomenon and is manifested not only for the vortex–acoustic wave system (the emission of acoustic waves is the simplest possible example of spontaneous imbalance). Having said that, McIntyre (2009) extensively discusses the spontaneous generation of waves from vortices that goes beyond any Lighthill-type scenario, where the author also points towards Lighthill's own awareness of possible exceptions in his theory. The linear coupling has been successfully used to describe different examples of spontaneous wave generation by vortex mode disturbances in atmospheric, astrophysical and magnetized shear flows (Tevzadze 1998, Tevzadze, Chagelishvili, Zahn, Chanishvili & Lominadze 2003, Heifetz & Farrell 2003, Vanneste & Yavneh 2004, Kalashnik et al. 2006, Vanneste 2008, Ólafsdóttir, Daalhus & Vanneste 2008, Bakas 2009, Lott, Plougonven & Vanneste 2010, Mamatsashvili, Avsarkisov, Chagelishvili, Chanishvili & Kalashnik 2010, Favraud & Pagneux 2013, Vanneste 2013), and the linear vortex–wave coupling as described in Refs. (Chagelishvili et al. 1997b, Farrell & Ioannou 2000, Bakas 2009, Favraud & Pagneux 2014) serves as a guide for the present investigation.

1.5 Outline of the thesis

The thesis is structured as follows: In chapter 2, we present a mechanical picture of the transient growth dynamics of vortex perturbations in incompressible shear flows both in two as well as in three dimensions. In fact, we reconstruct the linearized Euler equations on the basis of the proposed mechanical picture. Subsequently, in chapter 3 the linear generation of acoustic waves by initially pure vortex mode harmonics is analyzed. This involves the analysis of the mathematical and physical aspects of the wave generation process. Especially, the latter is a resumption of chapter 2. These results are fundamental for conducting the comparative analysis of linearly and non-linearly generated sound in a constantly sheared base flow in chapter 4. Here, we present the great importance of linear, anisotropic processes in non-normal flow systems up to rather large amplitudes of the initial disturbances inside the range of validity of Rapid Distortion Theory. Furthermore, we compare the topologies of the strong linear mechanism of acoustic wave generation and the source terms of various acoustic analogies, which appear to be incompatible with each other. In chapter 5 we present a symmetry analysis of the linearized compressible Euler equations and derive the one- and two-dimensional systems of subalgebras. This allows to find all possible group-invariant solutions/Ansatz functions, which involve, apart from the two classical – Kelvin mode and modal – approaches, a third Ansatz function, which generalizes the well-known ones. Further, we revisit the phenomenon of the critical layer in chapter 6. Analyzing its occurrence in the modal and nonmodal framework, we present an explanation in terms of Kelvin modes. These results are validated and enhanced by direct numerical simulations of a localized packet of wave harmonics propagating through a 2-D shear layer. Finally, we close with a summary and a final discussion of the results presented in this thesis in chapter 7.