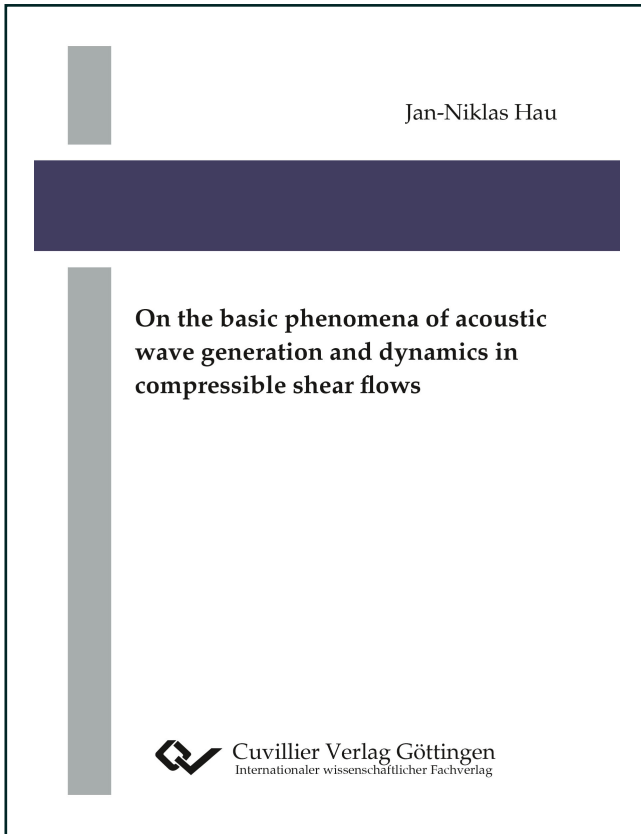




Jan-Niklas Hau (Autor)

**On the basic phenomena of acoustic wave generation and dynamics in compressible shear flows**



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