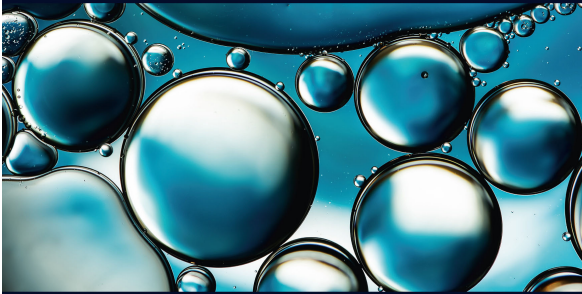




Christina Kallendorf (Autor)

**An Eulerian Discontinuous Galerkin Method for the Numerical Simulation of Interfacial Transport**



**An Eulerian Discontinuous Galerkin  
Method for the Numerical Simulation  
of Interfacial Transport**

Christina Kallendorf



Cuvillier Verlag Göttingen  
Internationaler wissenschaftlicher Fachverlag

<https://cuvillier.de/de/shop/publications/7480>

Copyright:

Cuvillier Verlag, Inhaberin Annette Jentsch-Cuvillier, Nonnenstieg 8, 37075 Göttingen, Germany  
Telefon: +49 (0)551 54724-0, E-Mail: [info@cuvillier.de](mailto:info@cuvillier.de), Website: <https://cuvillier.de>

# Contents

<b>List of Figures</b>	<b>x</b>
<b>Nomenclature</b>	<b>xi</b>
<b>List of Tables</b>	<b>xv</b>
<b>Nomenclature</b>	<b>xvii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation . . . . .	1
1.2 Thesis Outline . . . . .	4
<b>2 Equations of the Physical Model</b>	<b>7</b>
2.1 Transport Equations in Two-Phase Flow . . . . .	7
2.1.1 Mass Balance Laws in a Two-Phase Setting . . . . .	7
2.1.2 The Geometric Surface . . . . .	10
2.1.3 Transport Theorems for the Bulks . . . . .	12
2.1.4 Transport and Divergence Theorems for the Interface . . . . .	13
2.1.5 Mass Balance Laws of the Bulks and of the interface . . . . .	14
2.2 Conservation of Mass and Linear Momentum . . . . .	15
2.3 Surfactant Transport Equations . . . . .	17
<b>3 State of the Art of Numerical Methods</b>	<b>21</b>
3.1 Treatment of Fluidic Interfaces . . . . .	21
3.1.1 Particle Methods . . . . .	21
3.1.2 Volume of Fluid Methods . . . . .	22
3.1.3 Level Set Methods . . . . .	23
3.1.4 ALE Methods . . . . .	24
3.2 Treatment of Surface Problems . . . . .	24
3.2.1 Lagrangian Approaches . . . . .	25
3.2.2 Eulerian Approaches . . . . .	26
3.3 Interfacial Transport in Two-Phase Flows . . . . .	27
<b>4 An Introduction to DG Methods</b>	<b>31</b>
4.1 Discontinuous Galerkin methods – A Selected Overview . . . . .	31
4.1.1 The Discontinuous Galerkin Discretization . . . . .	31
4.1.2 Discrete Quantities . . . . .	34
4.1.3 DG Methods for Second Order PDEs . . . . .	36



4.2	Hamilton-Jacobi Equations . . . . .	39
4.3	Choice of Method . . . . .	40
4.3.1	The DG Method for Fluid Applications . . . . .	40
4.3.2	The Level Set Method for Representing Fluidic Interfaces . . . . .	41
4.3.3	An DG based Extension Approach for the Interfacial Transport Equation . . . . .	42
<b>5</b>	<b>New Conservation Laws of Surface Transport Equations</b>	<b>45</b>
5.1	Motivation for Constructing Conservation Laws . . . . .	45
5.2	Construction of Conservation Laws using the Direct Method . . . . .	47
5.3	Conserved Form of the Equations . . . . .	49
5.3.1	Conservation Laws of Surfactant Transport without Diffusion . . . . .	49
5.3.2	Conservation Laws of Surfactant Transport including Diffusion . . . . .	52
5.3.3	Conservation Laws of General Interfacial Balance Laws . . . . .	53
5.3.4	Conservation Laws in the Presence of Source Terms . . . . .	54
<b>6</b>	<b>Exact Solutions to the Interfacial Surfactant Transport Equation</b>	<b>57</b>
6.1	Motivation . . . . .	57
6.2	Governing Equations . . . . .	58
6.3	Exact Solutions . . . . .	62
6.3.1	Unsteady Solution . . . . .	62
6.3.2	Steady solution . . . . .	65
6.3.3	Solution of unsteady surfactant transportation without convection . . . . .	66
6.4	Discussion and Conclusions . . . . .	68
<b>7</b>	<b>A Narrow Band Method for the Eulerian Approach</b>	<b>71</b>
7.1	A Coordinate Based Narrow Band Approach . . . . .	71
7.2	The Pseudo-timestepping Method for Extending Data . . . . .	75
7.3	Numerical performance . . . . .	77
<b>8</b>	<b>The Eulerian DG Algorithm and Numerical Results</b>	<b>83</b>
8.1	Discretization of surface transport equations . . . . .	83
8.2	Surface Transport Equations without Diffusion . . . . .	86
8.2.1	Numerical Results . . . . .	86
8.3	Surface Transport Equations dominated by Diffusion . . . . .	93
8.3.1	Details of the numerical method . . . . .	95
8.3.2	Numerical Results . . . . .	98
8.4	Surface Transport Equations including both Convection and Diffusion . . . . .	103
8.5	Interfacial transport Stokes flow around a spherical droplet . . . . .	110
8.5.1	Purely diffusive transport . . . . .	110
8.5.2	Surfactant transport including advection . . . . .	110



<b>Contents</b>	vii
<b>9 Conclusion</b>	<b>115</b>
<b>A Appendix</b>	<b>133</b>
A.1 Point Symmetries of the Surfactant Transport Equations . . . . .	133