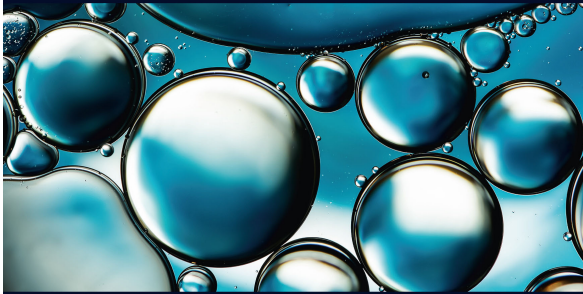




Christina Kallendorf (Autor)

**An Eulerian Discontinuous Galerkin Method for the
Numerical Simulation of Interfacial Transport**



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