

# 1. The double roles of the winding in the electromechanical energy conversion

From the energy point of view, an electrical machine can be seen as an electromechanical energy converter. With the aid of the magnetic energy  $E_m$ , it changes the electrical energy  $E_e$  into mechanical energy  $E_\Omega$  and vice versa, where the process is followed by the generation of the dissipative energy  $E_d$  (heat energy).

The energy conservation law ensures that, for each infinite small time interval  $dt$ , for the motor mode, there is:

$$dE_e = dE_\Omega + dE_m + dE_d \quad (1.1)$$

while for the generator mode, there is:

$$dE_\Omega = dE_e + dE_m + dE_d \quad (1.2)$$

This phenomenon can be illustrated through Figure 1.1.

Since it is an electromechanical process, to describe the process completely, electrical and mechanical state variables are needed. Different choices of the electrical and mechanical state variables are possible with various abstractive level. For the following discussion, a multi-phase electrical machine specific and practical approach is chosen.

There are two different ways to describe the energy conversion process: lumped quantity approach and field quantity approach. The lumped quantities are understood as, the determination of such quantities happens only at defined location, while for the field quantities, the determination of such quantities (if possible) occurs through the whole space.

The following discussion is specific to the case of motor mode (equation 1.1) but can be extended for the generator mode without difficulty.

## 1.1. The lumped quantity approach

For the lumped quantity approach, the  $m$ -phase current is chosen as the electrical state variables, which is considered as an algebraic vector  $i$

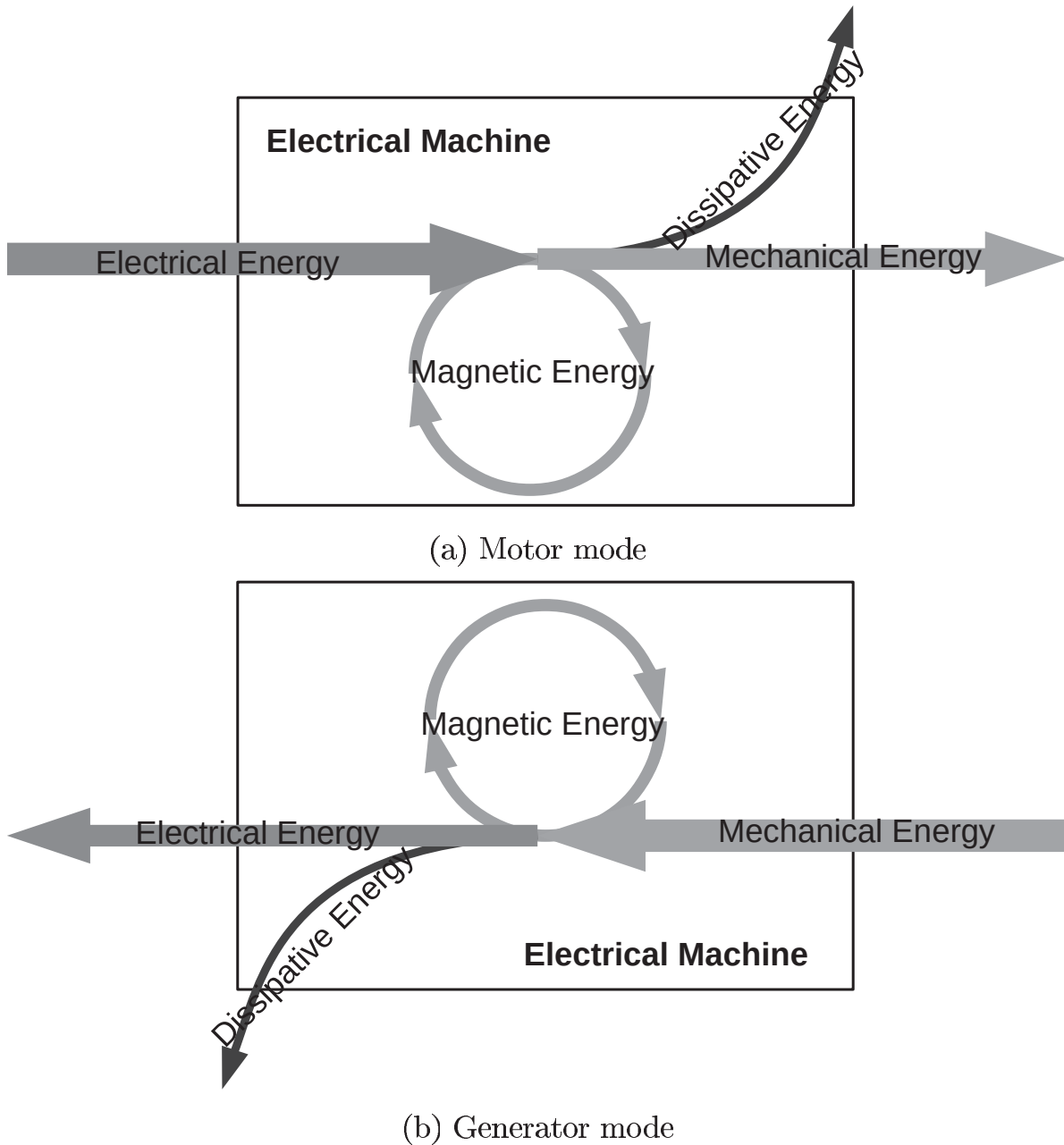


Figure 1.1.: The electrical machine as an electromechanical energy converter

with each phase current as its element. The rotor angular position is chosen as the mechanical state variable. To include the case of multiple rotors, just like magnetic gear [3] or electrical transmission system [4], the mechanical state variable is also represented with an algebraic vector  $\Omega$ , with the angular position of each rotor as its element.

According to the classical electromagnetic theory [82], the energy variation can be predicted through the following equations respective.



### 1.1.1. Separation of the electrical energy variation into dissipative and coupling electrical energy variation

For the electrical energy variation, there is:

$$dE_e = \mathbf{i}^T \mathbf{u} dt \quad (1.3)$$

where the elements of  $\mathbf{u}$  and  $\mathbf{i}$  are the instantaneous phase voltage and current of the m-phase winding. For a detailed analysis of the process, it is to separate the phase voltage into two parts, with the first part caused due to electrical resistance of the winding  $\mathbf{R}$  and the second part due to the change of the magnetic flux linkage acting on the winding  $\psi$ :

$$\mathbf{u} = \mathbf{R}\mathbf{i} + \frac{d\psi}{dt} \quad (1.4)$$

Equation 1.3 can then be formulated as:

$$dE_e = \mathbf{R}|\mathbf{i}|^2 dt + \mathbf{i}^T d\psi = dE_{e,d} + dE_{e,c} \quad (1.5)$$

with the first term of the dissipative electrical energy  $dE_{e,d}$  and the second term the coupling electrical energy  $dE_{e,c}$ .

A better understanding of the conversion process can be obtained, if the coupling electrical energy is further separated into two parts, which is mathematically to formulate the absolute differential part  $d\psi$  as a sum of several partial differential parts by using the state variables:

$$d\psi = \mathbf{J}_\Omega d\Omega + \mathbf{J}_i d\mathbf{i} \quad (1.6)$$

with the Jacobian matrix:

$$\begin{aligned} \mathbf{J}_{\Omega, nk} &= \frac{\partial \psi_n}{\partial \Omega_k} \\ \mathbf{J}_{i, nk} &= \frac{\partial \psi_n}{\partial i_k} \end{aligned} \quad (1.7)$$

The coupling electrical energy  $dE_{e,c}$  can then be formulated as:

$$dE_{e,c} = \mathbf{i}^T \mathbf{J}_\Omega d\Omega + \mathbf{i}^T \mathbf{J}_i d\mathbf{i} \quad (1.8)$$

with the first term is caused by the mechanical movement of the rotor and the second term is due to the variation of the current.

### 1.1.2. The relationship between the coupling electrical energy variation, the mechanical energy variation and the magnetic energy variation

For the mechanical energy variation, there is:

$$dE_{\Omega} = \mathbf{T}_q^T d\Omega \quad (1.9)$$

with  $\mathbf{T}_q$  the mechanical torque acting on the particular shaft with movement of  $\Omega$ .

For the magnetic energy variation, there is unfortunately no easy equation available. Nevertheless, by considering the magnetic energy as a state function, the following formulation can be used:

$$dE_m = E_m(\mathbf{i} + d\mathbf{i}, \Omega + d\Omega) - E_m(\mathbf{i}, \Omega) \quad (1.10)$$

with  $E_m$  is a state function of the state variables  $\mathbf{i}, \Omega$ . <sup>(1)</sup>

By considering the magnetic energy as function of the state variables  $\mathbf{i}, \Omega$ , a formulation of the total differential  $dE_m$  equation 1.10 as sum of partial differentials can be obtained:

$$dE_m = (\mathbf{J}_{\Omega})^T d\Omega + (\mathbf{J}_i)^T d\mathbf{i} \quad (1.11)$$

with the element of the Jacobian vector:

$$\begin{aligned} J_{\Omega,n} &= \frac{\partial E_m}{\partial \Omega_n} \\ J_{i,n} &= \frac{\partial E_m}{\partial i_n} \end{aligned} \quad (1.12)$$

According to the energy conservative laws (Equation 1.1) and by considering the particular energy variation (Equation 1.8, 1.9 and 1.11), for the electrical state variables, there is:

$$\mathbf{i}^T \mathbf{J}_i d\mathbf{i} = (\mathbf{J}_i)^T d\mathbf{i} \quad (1.13)$$

and for the mechanical state variables, there is:

$$\mathbf{i}^T \mathbf{J}_{\Omega} d\Omega = \mathbf{T}_q^T d\Omega + (\mathbf{J}_{\Omega})^T d\Omega \quad (1.14)$$

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<sup>(1)</sup>At this point, it is interesting to mention that, for the magnetic energy  $E_m$ , there is a quite simple formulation:  $E_m = \int \mathbf{i}^T d\psi$ , with the current hold zero and move the rotor to the end position, then ramp the current up to the end value [82]. This can be obtained by integrate  $dE_{e,c} = dE_m + dE_{\Omega}$ , resulting  $\int \mathbf{i}^T d\psi = E_m + \int \mathbf{T}_q^T d\Omega$ , if the current hold zero and move the rotor to the end position, the second term vanishes, leading to equation above. This can be done, only if the magnetic energy is a state function, which means its value is independent on the integration path!



### 1.1.3. The relationship between the coupling electrical energy variation, the mechanical energy variation and the magnetic co-energy variation

By using the equations above, there is a more complicate relationship for the mechanical state variables, which is however more important for the electromechanical conversion process. This can be easily solved, if a new state quantity named magnetic co-energy is introduced, which is:

$$E'_m = \int \boldsymbol{\psi}^T d\mathbf{i} = \mathbf{i}^T \boldsymbol{\psi} - E_m \quad (1.15)$$

With this new state quantity, the total differential of the magnetic energy can be formulated as: so that:

$$\begin{aligned} dE_m &= \boldsymbol{\psi}^T d\mathbf{i} + \mathbf{i}^T d\boldsymbol{\psi} - dE'_m \\ &= \boldsymbol{\psi}^T d\mathbf{i} + \mathbf{i}^T \mathbf{J}_\Omega d\boldsymbol{\Omega} + \mathbf{i}^T \mathbf{J}^i d\mathbf{i} - (\mathbf{J}'_\Omega)^T d\boldsymbol{\Omega} - (\mathbf{J}'_i)^T d\mathbf{i} \\ &= (\boldsymbol{\psi}^T + \mathbf{i}^T \mathbf{J}^i - (\mathbf{J}'_i)^T) d\mathbf{i} + (\mathbf{i}^T \mathbf{J}_\Omega - (\mathbf{J}'_\Omega)^T) d\boldsymbol{\Omega} \end{aligned} \quad (1.16)$$

with the new Jacobian vector:

$$\begin{aligned} J'_{\Omega,n} &= \frac{\partial E'_m}{\partial \Omega_n} \\ J'_{i,n} &= \frac{\partial E'_m}{\partial i_n} \end{aligned} \quad (1.17)$$

Equation 1.14 is then simplified to:

$$(\mathbf{J}'_\Omega)^T d\boldsymbol{\Omega} = \mathbf{T}_q^T d\boldsymbol{\Omega} \quad (1.18)$$

while a simple form of Equation 1.13 is still got:

$$\boldsymbol{\psi}^T d\mathbf{i} = (\mathbf{J}'_i)^T d\mathbf{i} \quad (1.19)$$

## 1.2. The field quantity approach

For the field quantity approach, the current density within the whole machine is chosen as the electrical state variables, which is considered as a physical vector  $\vec{\mathbf{J}}(x, y, z, t)$ .



To formulate the electromechanical energy conversion process can be formulated using the field quantities, the Poynting theorem is used, which supposes the electrical power being calculated as:

$$\frac{dE_e}{dt} = - \int \operatorname{div}(\vec{E} \times \vec{H}) dV \quad (1.20)$$

By considering the vector calculus:

$$\operatorname{div}(\vec{E} \times \vec{H}) = \vec{H} \cdot \operatorname{rot} \vec{E} - \vec{E} \cdot \operatorname{rot} \vec{H} \quad (1.21)$$

together with the maxwell equations <sup>(2)</sup>

$$\begin{aligned} \operatorname{rot} \vec{E} &= - \frac{d\vec{B}}{dt} \\ \operatorname{rot} \vec{H} &= \vec{J} \end{aligned} \quad (1.22)$$

and the material equation:

$$\vec{E} = \rho \vec{J} \quad (1.23)$$

Equation 1.20 changes to:

$$dE_e = \int \rho |\vec{J}|^2 dV dt + \int \vec{H} d\vec{B} dV = dE_{e,d} + dE_{e,c} \quad (1.24)$$

with the first term the dissipative electrical energy  $dE_{e,d}$  and the second term the coupling electrical energy  $dE_{e,c}$ .

To get a better understanding of the relationship between the lumped quantities and the field quantities, it is reasonable to introduce the magnetic vector potential, which is defined as:

$$\vec{B} = \nabla \times \vec{A} \quad (1.25)$$

By consideration of the vector calculus:

$$\vec{a} \cdot (\nabla \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \nabla \cdot (\vec{a} \times \vec{b}) \quad (1.26)$$

<sup>(2)</sup> Absolute differential is used instead of partial differential, since the stator and rotor are with different coordinate system, of which the speed of stator and rotor are zero. This assumption is valid for the case  $v \ll c$ , which is for electrical machine always the case.



the second term of Equation 1.24 changes to:

$$\int \vec{H} d\vec{B} dV = \int \vec{J} d\vec{A} dV - \int \nabla \cdot (\vec{H} \times \vec{A}) dV \quad (1.27)$$

which is reduced to:

$$\int \vec{H} d\vec{B} dV = \int \vec{J} d\vec{A} dV \quad (1.28)$$

since:

$$\int \nabla \cdot (\vec{H} \times \vec{A}) dV = \int \vec{H} \times \vec{A} d\vec{S} = 0 \quad (1.29)$$

if the integration surface is chosen at the infinite far place [71].

Equation 1.24 is then:

$$dE_e = \int \rho |\vec{J}|^2 dV + \int \vec{J} d\vec{A} dV = dE_{e,d} + dE_{e,c} \quad (1.30)$$

with the first term the dissipative electrical energy and the second term the coupling electrical energy. <sup>(3)</sup>

### 1.3. Winding: a “double-way bridge”

Comparing Equations 1.5 and 1.30 for the formulation of the coupling electrical energy  $dE_{e,c}$ :

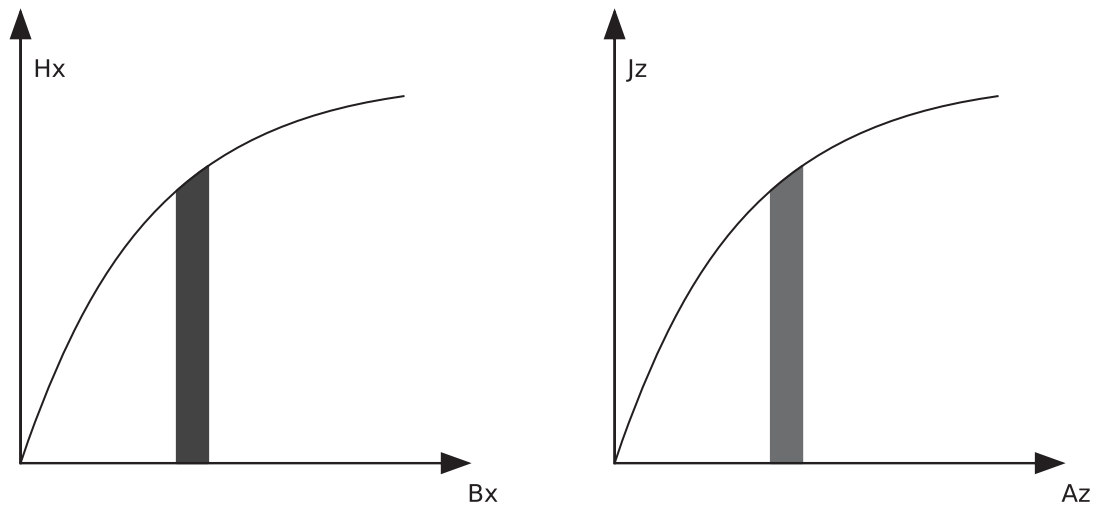
$$dE_{e,c} = i^T d\psi = \int \vec{J} d\vec{A} dV \quad (1.31)$$

It is clear that the lumped quantity  $i$  is corresponding with the field quantity  $\vec{J}$  and the lumped quantity  $\psi$  is corresponding with the field quantity  $\vec{A}$ .

Actually, in case of 2D, there are following simple relationship between these quantities:

$$\begin{aligned} i_k &= \int J_z ds_n \\ \psi_n &= \frac{w_k l_z}{S_n} \int A_z ds_n \end{aligned} \quad (1.32)$$

<sup>(3)</sup>The same consideration based on the energy conservative law yields the following formulation for the magnetic energy:  $E_m = \int_V \left( \int_{\vec{A}} \vec{J} d\vec{A} \right) dV$ , with the integration occurs when the rotor at the particular end position.



(a) Coupling energy calculated by using  $B_x$  and  $H_x$  (b) Coupling energy calculated by using  $A_x$  and  $J_x$

Figure 1.2.: Different methods for the calculation of the electrical coupling energy by fixed rotor position (for the illustration only one component of the field quantities is used)

where  $w_k$  and  $S_n$  are the total number of turns and cross-sectional area of winding  $k$  respective. The formulation for the phase flux linkage is widely used in 2D finite element software [85].

Obviously, there should be one component within the electrical machine, which plays the role of a “bridge” linking the lumped and field quantities. Since there are two electrical lumped quantities ( $\psi$  and  $i$ ) and two magnetic field quantities ( $\vec{A}$  and  $\vec{J}$ ), it should be two “bridges”, linking the quantities completely.

Fortunately, these two functionalities, namely:

- changing the phase current  $i$  into spatial distributed current density  $\vec{J}$ ,
- and changing the spatial distributed magnetic vector potential  $\vec{A}$  into phase flux linkage  $\psi$ ,

are realized by the same component: the multi-phase symmetrical winding. Therefore, the multi-phase symmetrical winding can be seen as the key component of the electrical machine during the electromechanical energy conversion process.





This fact can be clearly illustrated in figure 1.3, where a detailed discussion and mathematical modeling of the block “Symmetrical multi-phase Winding” is occurred in chapter 5.

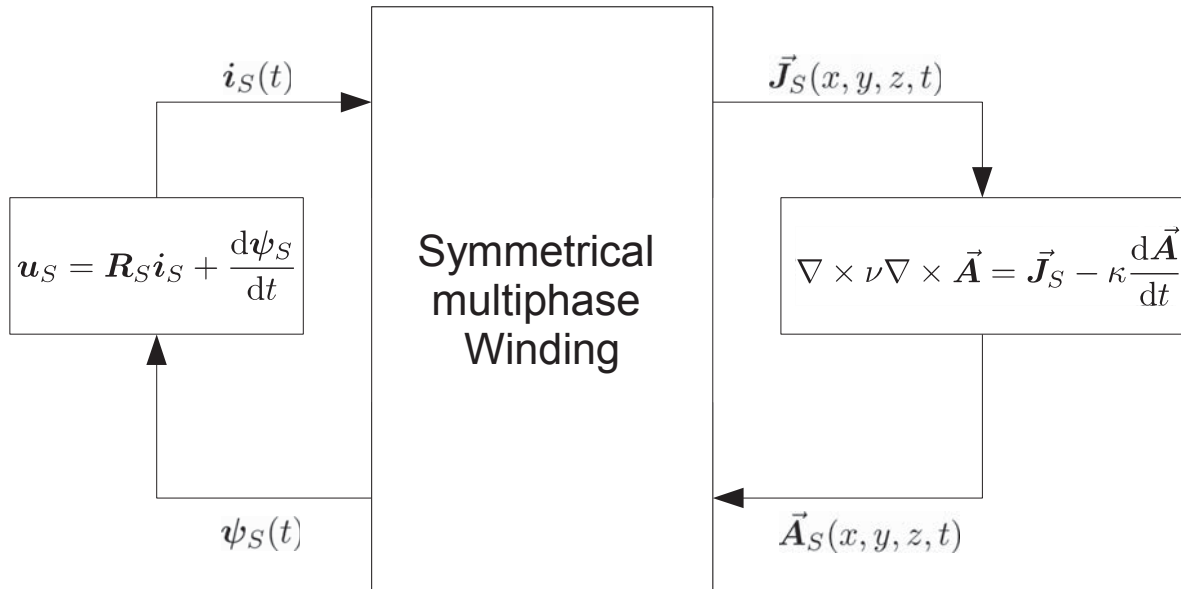


Figure 1.3.: The double roles of the multi-phase symmetrical winding

## 2. The impacts of the winding on the machine performance

As discussed in the previous chapter, the multi-phase symmetrical winding is the key component during the electromechanical energy conversion process. It impacts the machine performance through the following ways.

### 2.1. The winding insulation

The insulation of the winding specifies the operation voltage and temperature level of the winding. For the permanent magnet synchronous machine where the air-gap flux density is constant, the induced winding voltage is linearly related to the rotor speed ( $\vec{E}_v = \vec{v} \times \vec{B}$ ), and the current is directly proportional to the electromagnetic force ( $\vec{F} = i\vec{l} \times \vec{B}$ ). Because the winding temperature is linearly related to the winding current losses ( $\Delta T_w = R_{w,th} P_{w,loss}$ ) where the winding current losses is direct proportional to the square of the current ( $P_{w,loss} = R_{w,el} i^2$ ). The insulation of the winding defines the max. speed as well as the max. torque of the electrical machine and thus the max. power density. This can be seen from figure 2.1 which shows the impacts of the winding insulation on the power density and efficiency of the machine.

### 2.2. The number of turns

In general, a winding is a serial and/or parallel connection of coils with the same number of turns. For a given winding topology, the number of turns of the winding is linearly related to the number of turns of the coils. Unlike the winding insulation, changing the number of turns does not affect the max. power density of the machine. However, it has a significant impact on the shape of the speed-torque operation map. For a given max. phase current and voltage, an increasing of the number of turns increases the max. electromagnetic force linearly ( $\vec{F} = w_k i\vec{l} \times \vec{B}$ ). However, the max. rotor speed is decreased hyperbolically ( $u =$

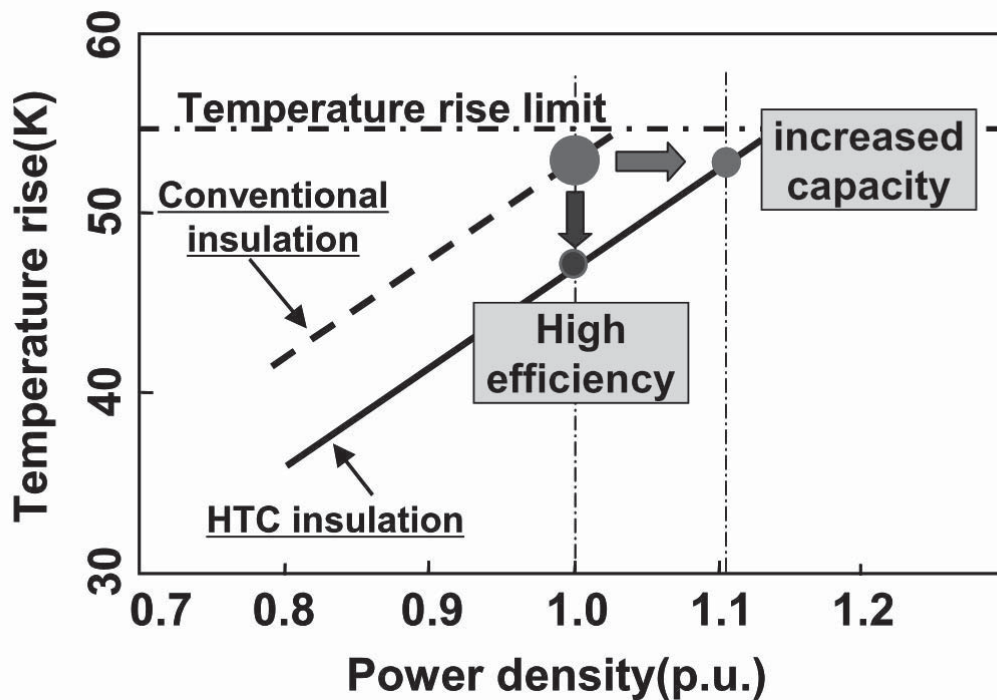


Figure 2.1.: impacts of the winding insulation on the power density and efficiency of the machine [32]

$w_k l \vec{v} \times \vec{B}$ ). This can be seen from figure 2.2 which shows the impacts of the number of turns on the max. speed-torque operation curve of the machine.

## 2.3. The winding production method

Two winding properties depend strongly on the winding production method: the slot filling factor (figure 2.3) and the end-winding (figure 2.4). All this has direct impacts on the machine performance.

Winding of high slot filling factor means larger copper cross section and therefore smaller electrical and thermal resistance. This results in a better efficiency and power density. The electrical resistance and the thermal conductivity (according to [64]) are given:

$$R_{c,el} = \rho \frac{l}{A}, \quad \kappa_{w,th} = \kappa_p \frac{(1 + f_c)\kappa_c + (1 - f_p)\kappa_p}{(1 - f_c)\kappa_c + (1 + f_p)\kappa_p}$$

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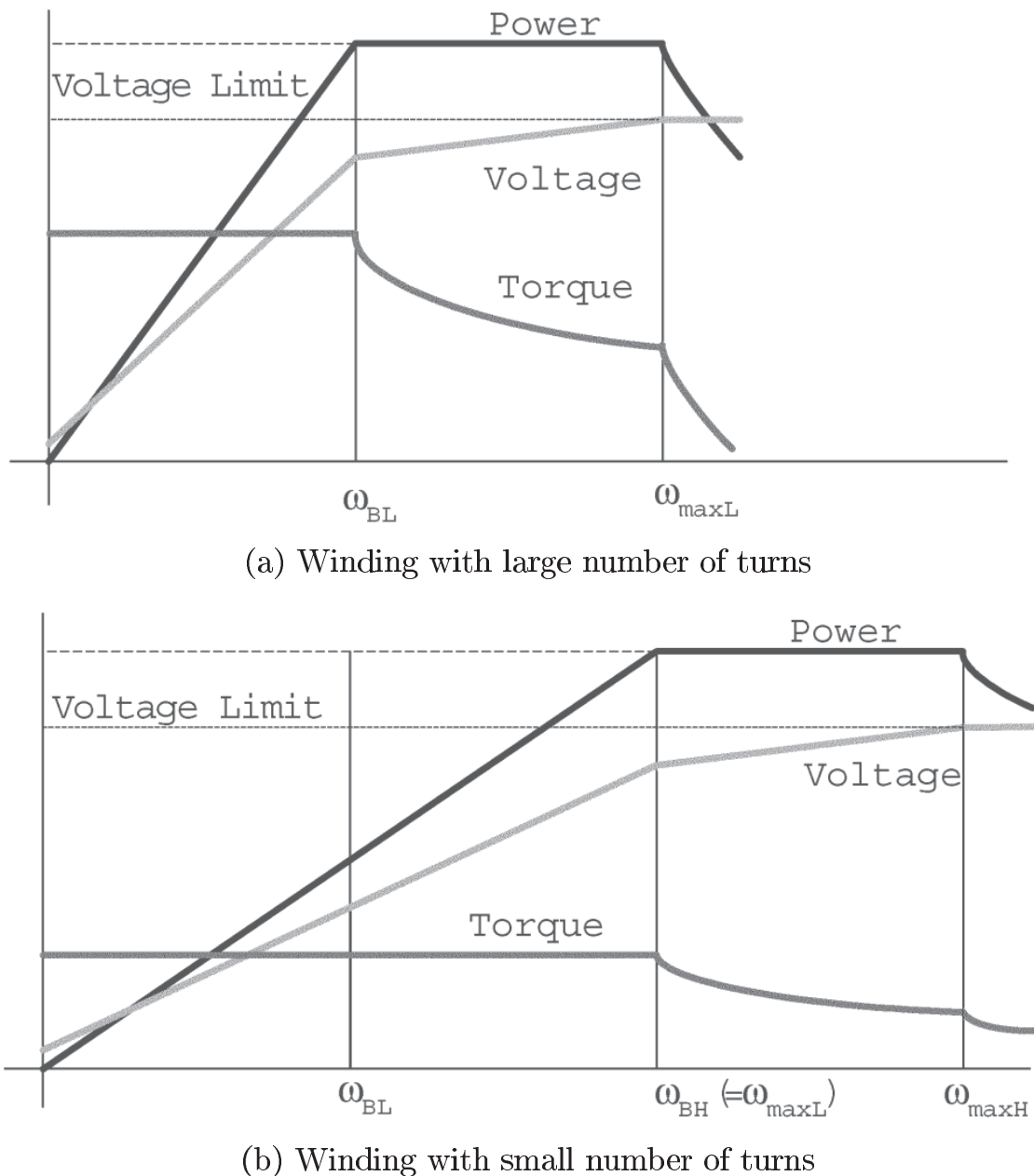


Figure 2.2.: impacts of the number of turns on the max. speed-torque operation curve [70]

where  $A$  is the copper cross section,  $\kappa_c$  and  $f_c$  are thermal conductivity and filling factor of copper,  $\kappa_p$  and  $f_p$  are thermal conductivity and filling factor of the insulation material.

For the same designed space, winding of short end-winding means more space of iron stack and therefore more area for the torque generation. This is because the electromechanical energy conversion occurs not in the air-gap area of the iron stack.

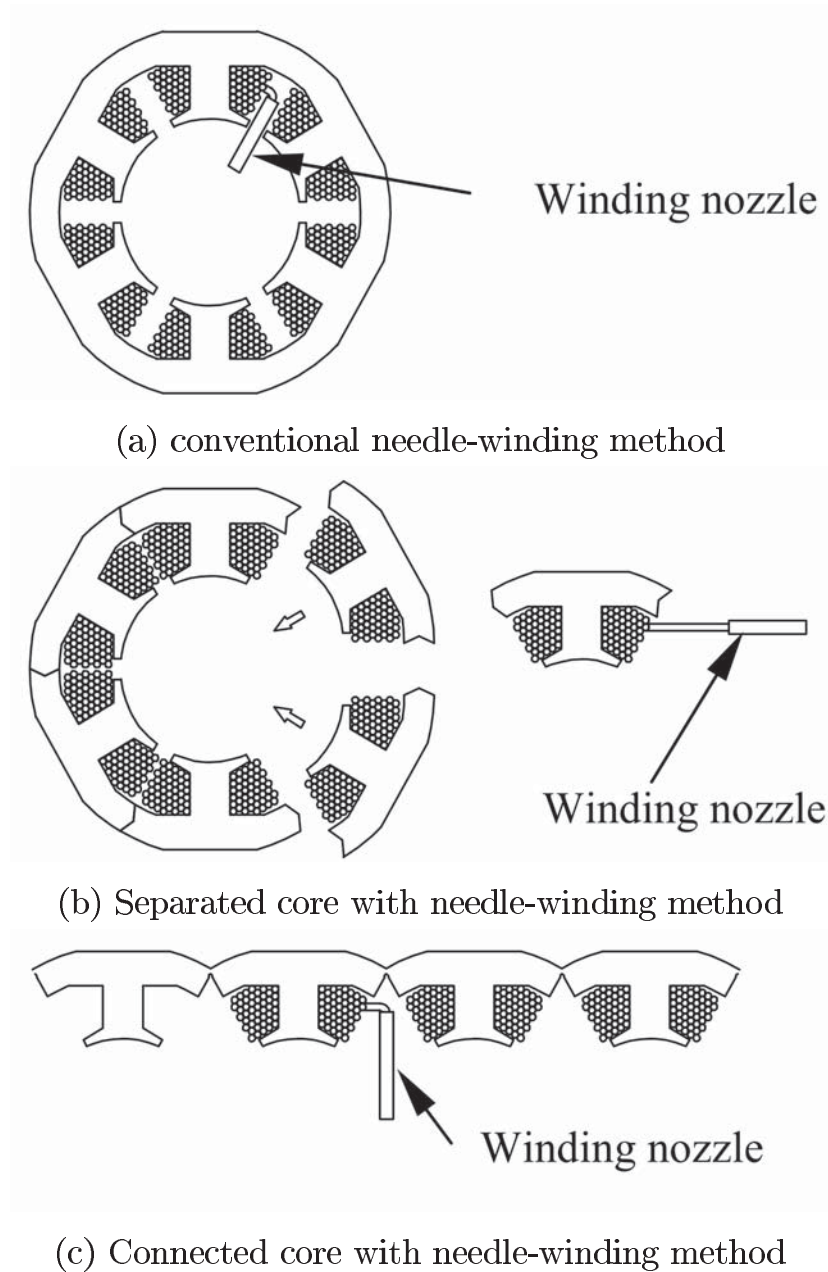
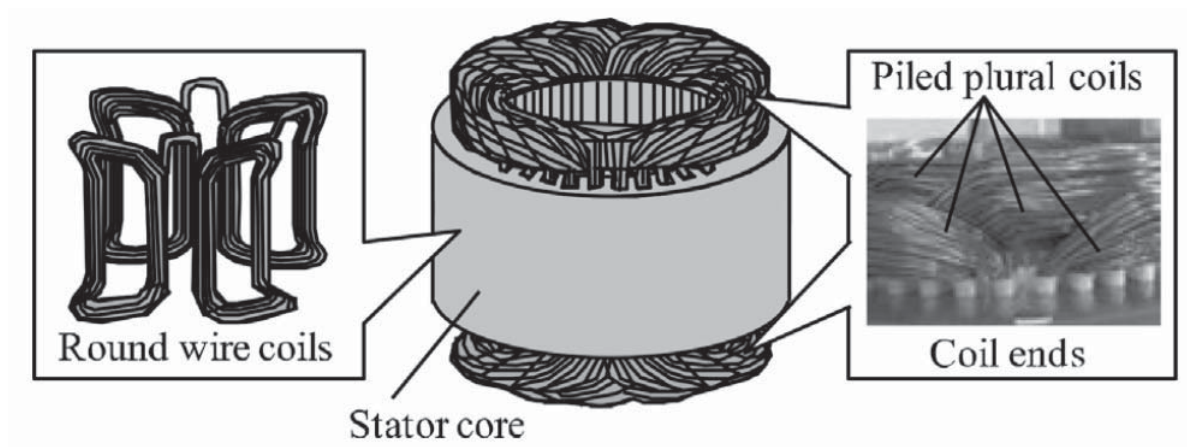
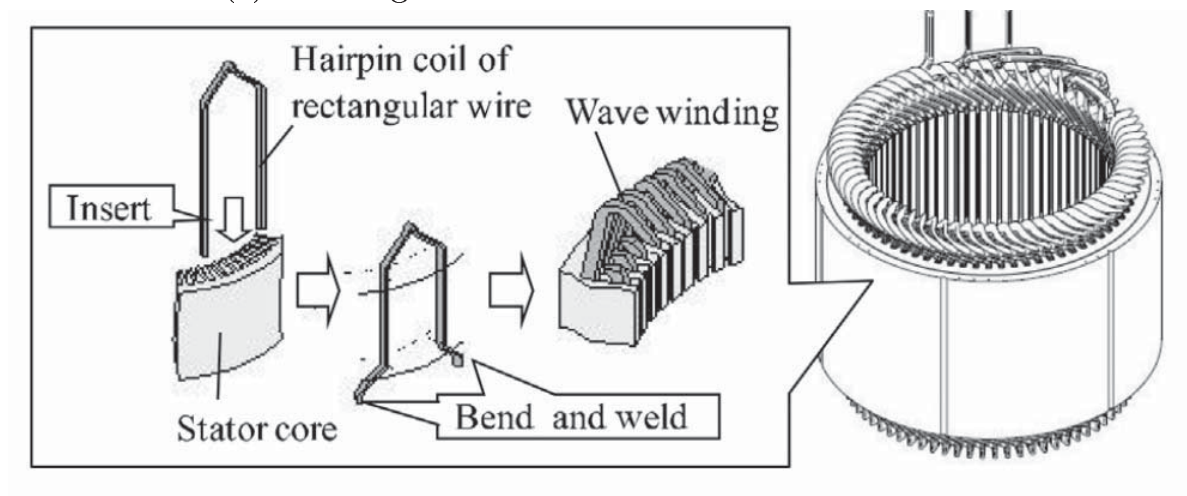


Figure 2.3.: Impacts of the winding production method on slot filling factor [1]



(a) Winding with conventional coils of round wire



(b) Winding with Hairpin-Coil of rectangular wire

Figure 2.4.: Impacts of the winding production method on the end-winding [38]

## 2.4. The winding topology

The winding topology describes how the winding is distributed among the stator circumference. A well-designed winding topology can convert a sinusoidally varied current system  $\underline{i}e^{j\omega t}$  into a sinusoidal MMF space harmonic  $\underline{\Theta}_k e^{j(kx+\omega t)}$  with a possible large amplitude of the working harmonic and possible small amplitude of sub- and over-harmonics. This is the precondition that the electrical machine supplies constant power (electrical and mechanical) with high efficiency. Thus the winding topology impacts the machine performance in various ways which is discussed in the next sections in detail.

### 2.4.1. Torque quality

The Impacts of the tooth coil winding of different layer (1-, 2- and 4-layer) on torque quality of a 12 slots/10 Poles interior PM machine is discussed by various authors [81, 55].

Wang et al. [81] show that for high current excitation, the machine with 4-layer winding performs the highest torque although the winding factor of the working harmonic of this winding topology is the lowest. This is contrary to the classical theory because it is claimed that higher winding factor of the working harmonic results in higher torque. It is to mention that the classical theory is valid for the fundamental harmonic winding topology (fundamental harmonic as working harmonic) without considering the saturation of the iron parts. Both of these assumptions are not met by the investigated winding since the working harmonic of the winding is the 5-harmonic and the existence of the sub-harmonic causes the saturation of iron part. As the 4-layer winding has the smallest sub-harmonics contents, its iron part is less saturated. The same Effect has been reported by Reddy et al. [55] which shows a 5.2% improvement of the torque density from a 2-layer winding to a 4-layer winding for the same peak current excitation.

Wang et al. [81] also show that for the peak current excitation, the torque ripple of the investigated 12 slots/10 poles interior machine can be reduced under 2% by using the 4-layer winding. For the single- and double-layer winding, this value is 3.9% and 5.0% respectively. The same effect is observed by Reddy et al. [55] who claim that the torque ripple of the investigated 12 slots/10 poles interior machine can be reduced

## 2. The impacts of the winding on the machine performance

from 18.5% (1-layer winding) to 5.0% (2-layer winding) towards 3.5% (4-layer) at base speed operation range and from 51.2% (1-layer winding) to 20.6% (2-layer winding) towards 8.9% (4-layer winding) at flux-weakening operation range.

### 2.4.2. Torque-speed operation range

The impacts of the winding topology on the torque-speed operation range is discussed by various authors [22, 70].

For the same rated load current and voltage conditions, Dajaku et al. [22] investigate the impacts of two different winding topologies on the torque-speed range of an interior PM machine with 10 poles. The first winding is a conventional 12 slots double-layer winding with coils of the same number of turns and the second winding is a novel 18 slots double-layer winding with coils of a different number of turns. The results show that even the new winding topology is with a lower winding factor of the working harmonic (0.760 vs. 0.933), with a well-chosen number of turns per phase (19/14 instead of 30), the new machine is with a wider torque-speed operation range. In the field weakening operation range, an increasing of the output power for about more than 20% can be achieved (figure 2.5).

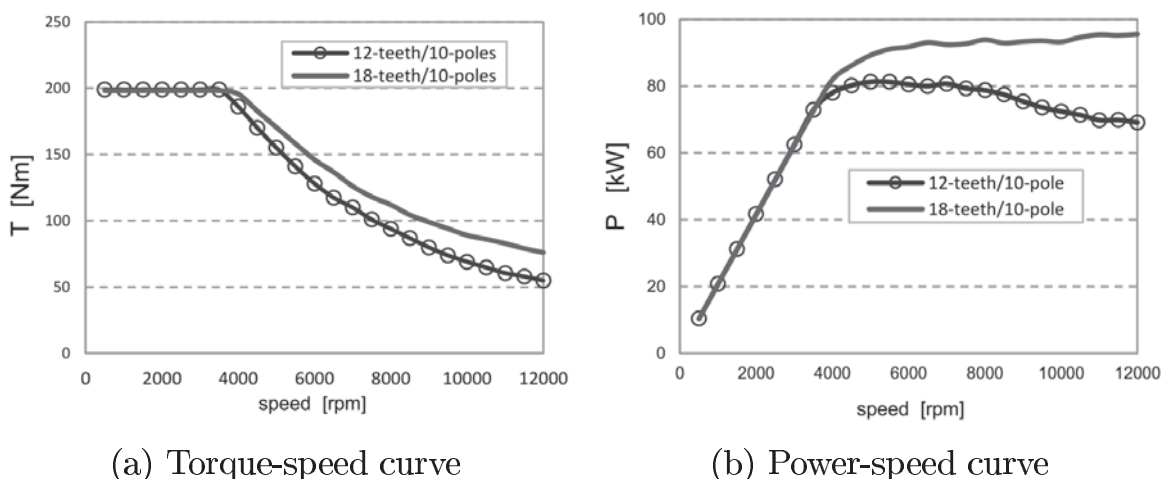


Figure 2.5.: Impacts of the winding topology on the torque-speed operation range [22]