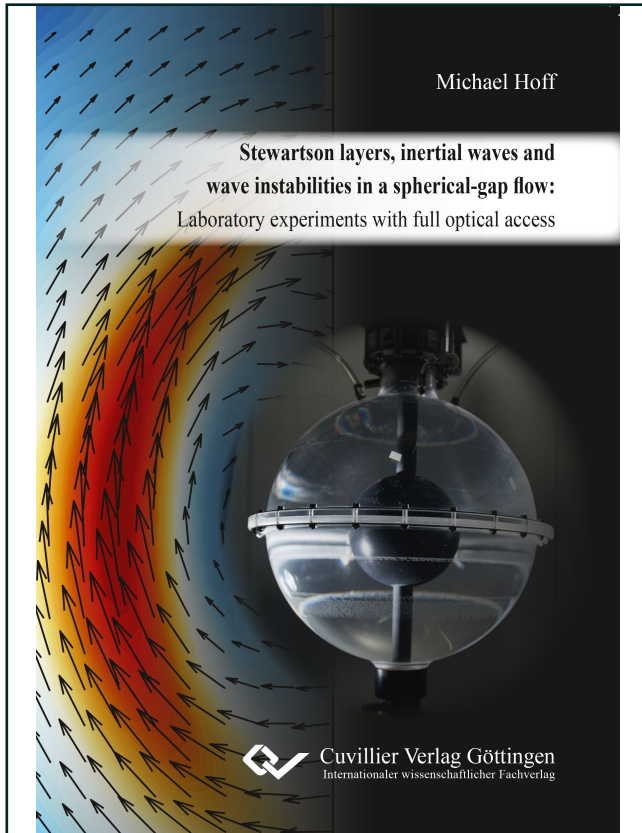




Michael Hoff (Autor)

Stewartson layers, inertial waves and wave instabilities in a spherical-gap flow

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