

# 1 Introduction

## 1.1 Background

Fracture accidents are serious problems that human society has faced for a long time. Generally speaking, all components contain certain kinds of defects and micro-cracks, regardless of manufacturing and machining processes. A stress engineer confronts with an important question which kinds of defects, which size of defects, which loads and which environmental influence will affect the life of the designed components. Fracture mechanics is used to evaluate the strength of a structure or component in the presence of a crack or flaw. Development of fracture mechanics is helpful to eliminate some of the potential dangers due to increasing components complexity.

### 1.1.1 Fundamental of fracture mechanics

The typical process of solving fracture mechanics problem is to evaluate characteristic parameters of crack based on fracture criteria. The critical values of the characteristic parameters will be experimentally determined as a function of the crack length, specimen geometry and loading configurations.

The first successful analysis of a fracture problem was laid in 1920 by Griffith [1], who considered the propagation of brittle cracks in glass. He showed that the product of the far field stress, the square root of the crack length, and material properties governed crack extension. This product was shown to be related to the energy release rate,  $G$ , which represents the elastic energy per unit crack surface area required for crack extension.

Irwin [2, 3] later made significant advances by showing that the energy approach is equivalent to the stress intensity factor  $K$ . The fracture occurs when a critical stress distribution ahead of the crack tip is reached. The material property governing fracture may therefore be stated as a critical stress intensity,  $K_c$ , or in terms of energy as a critical value  $G_c$ . Demonstration of the equivalence of  $G$  and  $K$  provided the basis for development of the discipline of linear elastic fracture mechanics (LEFM).

Since no material could carry the infinite stresses derived by the elasticity assumption, a plastification extending from the crack tip must take place. The plastic deformations will cause stress redistribution and the elastic solution will be no longer valid for the whole inelastic zone. The concept of elastoplastic fracture mechanics (EPFM) is developed.

Under assumption of an elastic-perfectly plastic material, Irwin [4] presented a simplified model for determination of the plastic zone attending the crack tip under small-scale yielding (SSY). He argued that the plasticity at the crack tip causes the crack to behave as if it were longer than its true physical size. It means that the plastification reduce the structure carrying capacity. A cracked specimen with a plastic zone size  $r_p$  corresponds to a specimen with crack length  $a_{\text{eff}} = a + r_p$ . Correspondingly, the effective stress intensity factor  $K_{\text{eff}}$  is evaluated.

Wells [5] proposed a fracture criteria related to the displacement between crack surfaces which separate associated with plastic deformation. The separation distance led to the development of a new parameter which is known as crack tip opening displacement (CTOD). A standardized method firstly developed by Schwalbe [6] measures the CTOD on the specimen surface at the fatigue precrack tip over an original gauge length of 5 mm ( $\delta_5$ ).

In 1968, Rice [7] introduced an elastic-plastic fracture parameter to characterize nonlinear material behavior of crack tip,  $J$ -integral. It is the generalization of the energy release rate for nonlinear elastic materials. For stationary crack problems, if the remote applied load is only assumed monotonic, the  $J$ -integral is path-independent. It was firstly proposed as a fracture parameter by Begley and Landes [8].

Hutchinson [9], Rosengren and Rice [10] used  $J$  as a single parameter to characterize crack tip stress fields under SSY condition in nonlinear materials (HRR solution). Shih [11] demonstrated a correlation between  $J$ -integral and CTOD, implying that both parameters are equally valid for characterizing fracture.

The  $J$ -integral and the HRR solution provide the basis to consider the crack-tip field of the nonlinear fracture problems, however, the limitation arises due to high constraint crack geometries.

### 1.1.2 Constraint effect in ductile fracture

It has been known that the resistance against initiation of ductile crack growth and crack propagation depends on the geometry and the load configuration of the respective specimen of structural component [12–20]. The geometry dependence of the crack growth resistance in ductile materials arises due to the competition between two dissipations, i.e. the plastic deformation around the crack tip and the energy spent in the actual creation of new surface due to microseparation processes [13, 21]. The  $J$ -integral is insufficient to characterize a crack, even in a two-dimensional specimen.

In order to account for different triaxial stress-states in influencing crack tip fields, O’Dowd and Shih [22, 23] proposed the two-parameter  $J$ - $Q$  theory. In this case,  $Q$  represents the constraint at the crack tip.

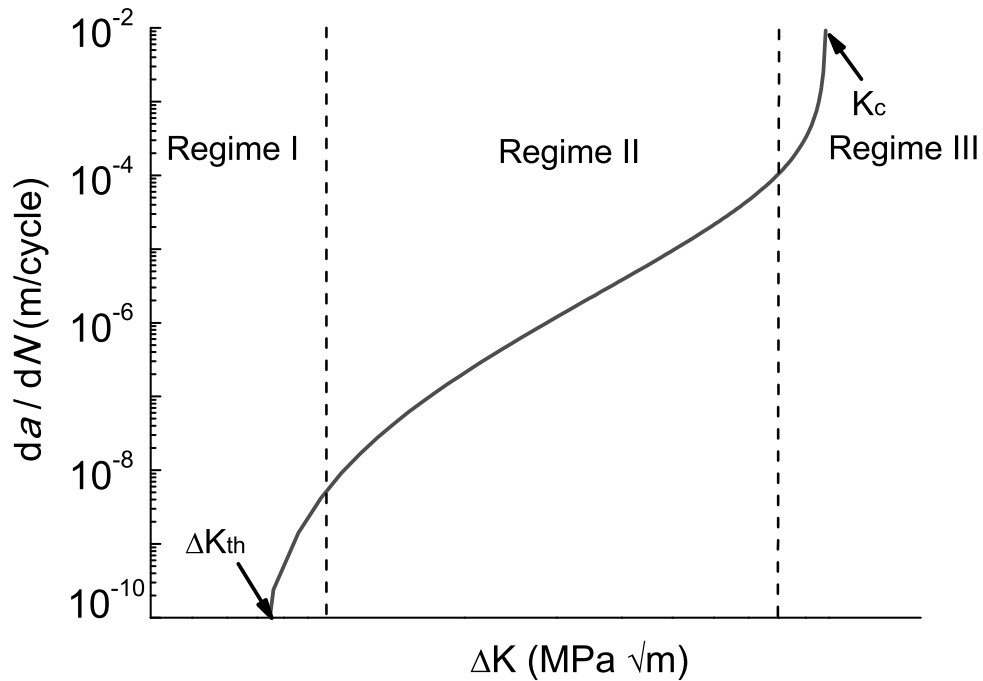
Yuan and Brocks [18] considered the plastic zone size in three-dimensional cracked specimens. The computations confirm that the out-of-plane constraint is not directly related with the in-plane constraint, which is mainly a manifest of the loading configuration to the crack tip field. This knowledge implies that the computational crack model has to include more details about the crack field besides the stress intensity, such as  $K$  and  $J$  et al. The constraint around the crack tip has to be represented in a more suitable way.

In characterization of three-dimensional cracks Brocks and Künecke [24] found the stress triaxiaity is a meaningful crack parameter besides the  $J$ -integral, defined as the ratio of hydrostatic stress over the effective stress,  $\eta = \sigma_h/\sigma_e$ . In fact  $\eta$  is directly related to crack field, while  $Q$  evaluate the difference between the real crack stresses and the plane strain reference crack field.

The constraint effects make the crack assessment methodology based on the conventional crack parameters too complex to apply for engineering. The transferability problem, which means transferring the crack growth resistance data as measured on laboratory specimens to assessing the defects in components, is a real practical concern.

### 1.1.3 Fatigue crack growth

The strength of a component or structure can be significantly reduced by the presence of a crack or discontinuity. However, in most engineering cases, the initial crack or discontinuity size is not critical to cause catastrophic failure. More commonly, subcritical crack growth occurs from the existing crack or discontinuity until a critical crack size is reached, causing fracture. Then, the major question would be how fast the crack grows from an initial length to the critical crack size.



**Figure 1.1:** Schematic demonstration of fatigue crack growth rate versus  $\Delta K$ .

Applying the concepts LEFM or EPFM under SSY condition, the stress intensity factor  $K$  can characterize the stress field around the crack tip involving the applied stress, crack size and the material geometry. Paris and Erdogan [25] firstly suggested that for a cyclic variation of the imposed stress field, the rate of fatigue crack growth ( $da/dN$ ) should be based on the stress intensity factor range ( $\Delta K$ ). The well-known Paris equation can be denoted as

$$\frac{da}{dN} = C(\Delta K)^m, \quad (1.1)$$

where  $C$  and  $m$  are material specific parameters. Many fatigue crack growth data can be obtained under constant load amplitude test condition using sharp cracked specimens. The typical log-log plot of  $da/dN$  vs.  $\Delta K$  shown schematically in Fig. 1.1 has a sigmoidal shape that can be divided into three major regions. Region I is the near threshold ( $\Delta K_{th}$ ) region which indicates the early development of a fatigue crack and the crack growth rate. The threshold value is on the order  $10^{-10} \sim 10^{-9}$  m/cycle. Fatigue crack should not propagate below the threshold value. Region II represents a stable crack propagation zone under small-scale yielding condition. The data follows a linear relation between  $\log(da/dN)$  and

$\log(\Delta K)$  which corresponds to Paris equation. Region III shows a very high crack growth rate which is near the final failure. Unstable crack growth would occur when approaches the fracture toughness  $K_c$ , which in turn depends on the microstructure, the mean stress and the environment.

Forman et al. [26] improved Paris' law by suggesting a new model which is capable of describing Regime III of the fatigue crack growth curve and including the stress ratio effect. Further modifications of the Forman's expression by Erdogan and Ratwani [27] to represent region I, II and III have been accomplished by including the threshold stress intensity parameter  $\Delta K_{th}$ .

Based on the experimental observations, a fatigue crack can close at a remotely applied tensile stress due to a zone of compressive residual stresses left in the crack tip. Therefore, many closure models were proposed based on the effective stress intensity factor range  $\Delta K_{eff}$ . Elber [28, 29] introduced the crack closure concept to analyze crack propagation under variable amplitude loading. Newman [30, 31] used a crack closure approach for predicting the fatigue crack growth life. The crack opening stress,  $\sigma_{op}$ , as a function of crack length and load history was calculated by an iterative solution procedure for a cycle-by-cycle closure calculation using detailed FE programs. Although the estimation of plastic zone size suggested by Irwin is very elementary, the engineering application confirms a high accuracy of this method.

Despite the use of  $K$ -based models have been an enormously successful engineering theory, it is important to recall that most engineering estimates of fatigue crack growth rely on  $K$  under the assumption that the plastic behavior of the material around the crack tip will not affect the crack growth process. However, for low cycle fatigue problems, the material is locally plastic, especially around cracks. Due to a large amount of plastic zone, fatigue crack propagation is very complex and conventional  $K$ -based models cannot describe the crack propagation behavior appropriately. Predicting elastoplastic fatigue crack growth life remains one of the most challenging problems in fracture mechanics.

## 1.2 Cohesive zone model

### 1.2.1 Numerical damage models

The advent of numerical damage models is providing an approach to structural assessment in that these models deal with the damage events in the near-crack tip process zone which are embedded in the global FE model of the component. This way the global FE model prescribes the loading conditions and the component is under onto the damage zone [32].

Analytical and macroscopic approaches used in fracture mechanics have some limitations with respect to the amount of plasticity allowed at the crack tip, constraint and geometry dependency. As Siegmund and Brocks [33] pointed out, to the present, "local approach" is the only really successful methods for prediction of crack growth resistance. In this kind of approach, one can simulate ductile fracture either by employing a micromechanical model of damage, which represents the micromechanism of void initiation, growth and coalescence, or by using a phenomenological model for material separation and coupling it to the surrounding undamaged elastoplastic material [34].

In a micromechanical model, a representative volume element (RVE) or "unit cell" is considered to study the respective mechanism. Such a micromechanical porous plastic-

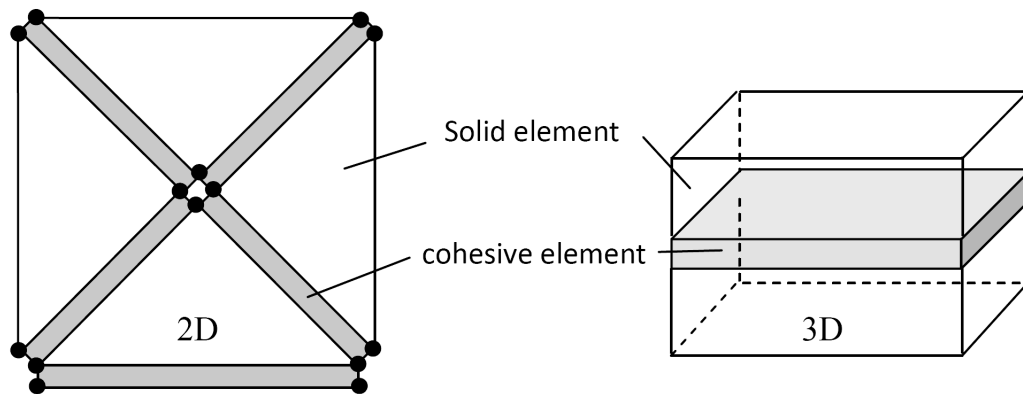
ity model was originally proposed by Gurson [35] and later extended by Tvergaard and Needleman [36] which is so called GTN model.

In a phenomenological model, ductile crack growth in metals can be modeled by introducing a process zone ahead of the crack tip where material degradation and separation occur. This zone is embedded in an elastoplastic surrounding. Within the concept of the energy release rate, a "cohesive zone model" is introduced.

### 1.2.2 Cohesive zone model formulation

The cohesive zone model (CZM) removes the crack tip singularity and represents physics of the fracture process at the atomic scale. It regards fracture as a gradual phenomenon in which material separation takes place across an extended crack tip (cohesive zone) and is resisted by cohesive forces. A cohesive law (traction vs. separation) governs the constitutive behavior of crack opening in addition to the bulk stress-strain relation of surrounding material. No additional criterion is needed for fracture to occur. New crack surfaces are created as a natural result of constitutive evolution, thus maintaining continuity conditions mathematically despite the physical separation [37].

Generally, in crack propagation simulation, cohesive zone can be treated as the prescribed crack extension path, as shown in Fig. 1.2. The locations where material damage may occur have to be provided either from experimental observations, experienced data or numerical analyses. Within the framework of extended finite element methods (XFEM), the potential of CZM for arbitrary crack propagation has been exploited [38–42]. However, by applying XFEM in elastic-plastic problems, numerical difficulties arise significantly which leads to inevitable limitation.



**Figure 1.2:** Schematic representation of the concept of cohesive/volumetric finite elements model [37].

Stress components within the cohesive zone no longer follow the constitutive law of the bulk material. Considering the contribution of the cohesive traction, the mechanical equilibrium statement of the cohesive zone in the form of the principle of virtual work can be written as

$$\int_V (\boldsymbol{\sigma} : d\boldsymbol{\varepsilon})dV + \int_{S_{int}} (\mathbf{T} \cdot d\boldsymbol{\delta})dS = \int_{S_{ext}} (\mathbf{t} \cdot d\mathbf{u})dS, \quad (1.2)$$

where  $\boldsymbol{\sigma}$  and  $\boldsymbol{\varepsilon}$  are the Cauchy stress and strain tensor.  $V$  and  $S_{\text{ext}}$  represent the specimen volume and the external surface.  $\mathbf{t}$  means the traction vector at the external boundary and  $\mathbf{u}$  is the displacement vector. The integral term,  $\int_{S_{\text{int}}} (\mathbf{T} \cdot d\boldsymbol{\delta}) dS$ , over the crack surface and its prolongation in the process zone,  $S_{\text{int}}$ , represents the cohesive surface contribution of the cohesive zone ahead of the crack tip.  $\mathbf{T}$  denotes the cohesive traction components and  $\boldsymbol{\delta}$  is the separation of the two adjacent cohesive surfaces.

Comparing with the conventional stress-strain relations of bulk material, the evolution of local traction vs. separation behavior within the cohesive zone can be defined as a constitutive law for the special field between the virtual crack surfaces.

### 1.2.3 Cohesive laws

The cohesive law is responsible for material degradation under different loading configuration. In general cases, both traction and separation are vectors, so that the cohesive law is defined in a vector equation. For a mode I crack, only normal traction vs. normal separation play a role in material failure, that is, the cohesive law expression reduces to a scalar traction-separation function,  $T(\delta)$ .

Within the cohesive zone, the damage increases when the traction is reduced. Complete failure is assumed when the separation is greater than an ultimate value. Common to all cohesive law shapes, the cohesive strength  $T_0$  and the ultimate separation  $\delta_u$  where the final failure occurs, are sufficient for modeling the separation process.

Fracture mechanics is built on energy balance around the crack tip and during crack propagation. The energy release rate should calibrate crack initiation and propagation in fracture mechanics methodology. In cohesive zone modeling, the crack should initiate and propagate based on the energy dissipation to separate the cohesive zone, which is defined as the cohesive energy  $\Gamma_0$ , as

$$\Gamma_0 = \int_0^{\delta_u} T(\delta) d\delta. \quad (1.3)$$

### 1.2.4 Overview of the application of CZM

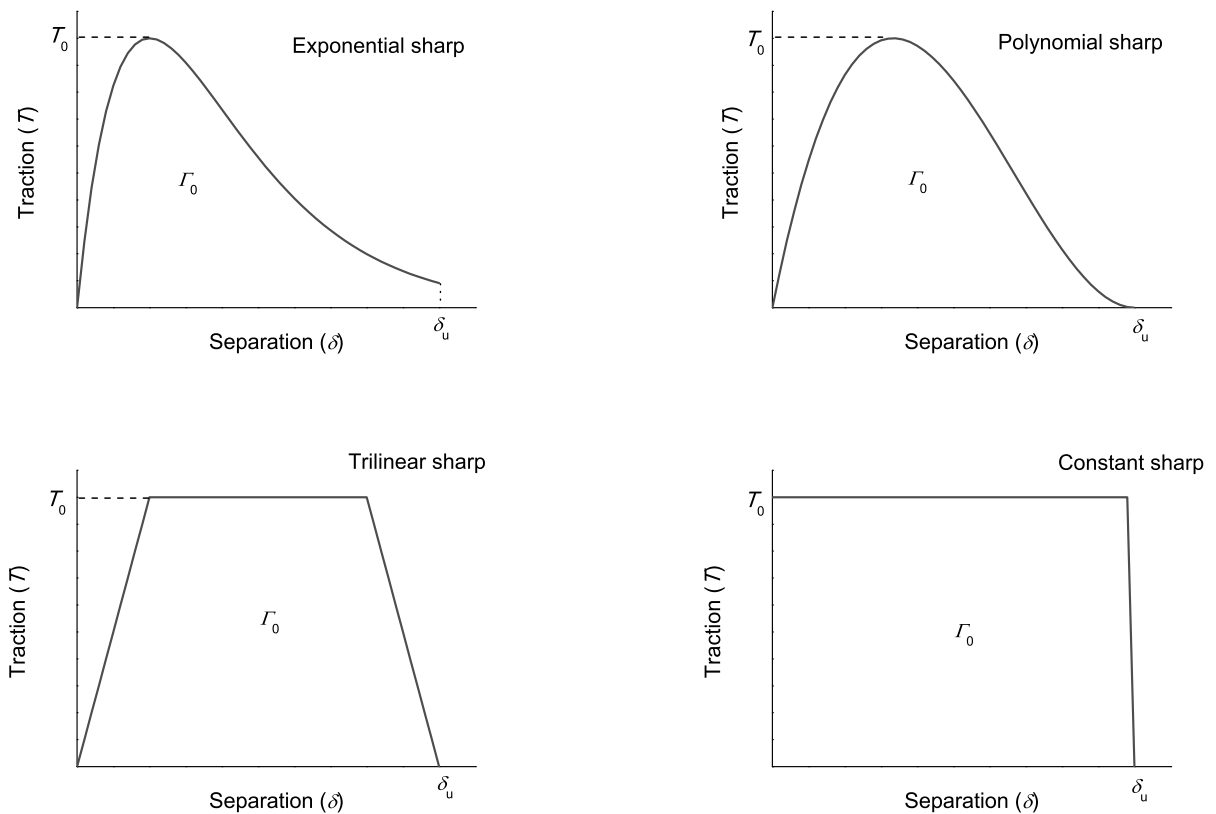
The early pioneering work on CZM can be traced back to 1960s. Dugdale [43] assumed an elastic-perfect plastic material behavior to avoid the unrealistic stress singularity around the crack tip. A narrow strip plastic zone ahead of the crack tip can be considered as cohesive zone. Barenblatt [44] later assumed cohesive force distribution in a sufficient large process zone along the crack ligament and the plastic strength is replaced by a cohesive law. The strip yield models of Dugdale and Barenblatt form the fundamental idea for CZM.

The first application of CZM to simulate the fracture behavior of a material was performed by Hillerborg et al. [45], as early as in 1976. The authors indicated that the cohesive stress at the crack tip has a linear descending relationship with the crack opening to describe the damage behavior of concrete. Needleman [46, 47] firstly induced CZM to investigate non-linear failure. The inclusion debonding in micro and macro scales were simulated by applying an exponential cohesive law.

In 1990, Yuan and Cornec [48] firstly employed CZM to investigate the crack extension in ductile materials. The ductile crack growth in an aluminium alloy was simulated by

using a constant cohesive law. Tvergaard and Hutchinson [13] computed the crack growth and subsequent resistance for an elastic-plastic solid through introducing a multilinear form cohesive law. Later, Scheider and Brocks [49] probed with most different function forms and studied influences on fracture process simulation. Goyal et al. [50] formulated an exponential form cohesive law by including a "brittleness" parameter, so that the flexibility of the cohesive law was enhanced.

In order to find more hints about the CZM, Krull and Yuan [51] investigated the atomistic separation with help of molecular dynamics and concluded the similarity between the exponential cohesive law of the normal failure process in the nano-scale.



**Figure 1.3:** Schematic demonstration of different sharp cohesive law.

Various sharps of cohesive laws are shown in Fig. 1.3. Scheider and Brocks [52] showed that the shape of cohesive law has a significant influence on the results of crack extension of ductile materials. Different cohesive law sharps can make the same results in one specimen with different cohesive parameters. But the results might be significant different in another fracture specimen. Vossen et al. [53] denoted that cohesive zone elements generally do not satisfy rotational equilibrium for arbitrary cohesive laws. The error in the nodal forces can be neglected if the ratio of the ultimate separation to the length of the process zone is small. Yuan and Li [54] revealed that the initial stiffness of the cohesive law can induce significant errors into simulation if it is assumed improperly. Furthermore, the cohesive energy is generally smaller than the  $J$ -integral from FEM computation, caused by elastic

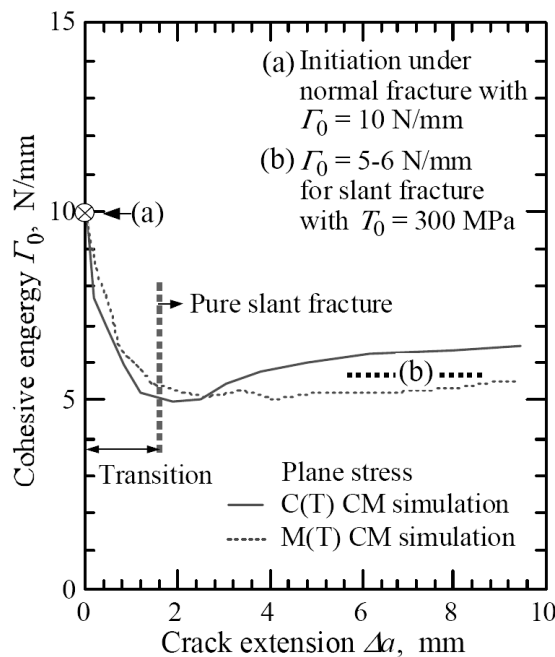
unloading around the crack tip. Based on such observations determining parameters of the CZM needs detailed special experiments. More detailed classification of different kinds of cohesive laws can refer to the literatures [55–60].

### 1.2.5 Constraint dependence of the cohesive parameters

Investigation has confirmed that the stress-state plays a crucial role in the damage growth till failure. Only a model which can account for the variations in stress-state in prediction of initiation and propagation of macroscopic crack would facilitate the transfer of fracture model parameters from a simple laboratory test to real complex structures [61].

However, up to now almost all studies using CZM to describe ductile crack growth assume the cohesive parameters to be material constants. Therefore, the effect of stress-state on the cohesive law has not been accounted for.

An early evidence for constraint dependence of the cohesive parameters can be found in the work of Yuan et al. [62]. The crack extension of thin-walled fracture specimens failing in a slanted manner was controlled by adjusting the cohesive energy in each element by experimental  $v_{LL} - \Delta a$  curves. By reproducing the experimental data it turned out that the crack initiates with a  $\Gamma_0$  value, which is equal to the crack initiation  $J_i$  for normal fracture, and then reduces to significantly lower values during the transition to the slanted fracture mode. After reaching the fully slanted region, the values remain almost constant again [32]. The result is shown in Fig. 1.4.



**Figure 1.4:** Development of the cohesive energy in the transition region from flat to slant fracture [62].

Ivankovic [63] measured a cohesive law in polyethylene using novel experimental method by notched tensile specimens. These results showed that both fracture energy and cohesive strength must be functions of the loading intensity and specimen geometry. It implies different cohesive laws for plane strain and plane stress.

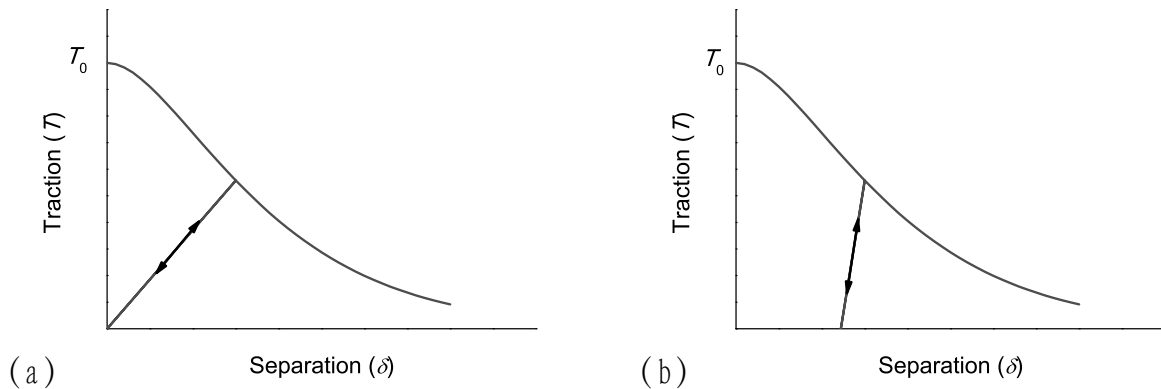


For aforementioned reasons, recently, several papers on relationships between constraint effects and CZM were published. Unfortunately, most published works were mainly on in-plane constraint effects. Detailed discussions on existing models and formulation of a stress-triaxiality-dependent cohesive zone model (TCZM) will be presented systematically in Chapter 4.

## 1.3 Cyclic cohesive zone model

Cohesive zone modeling provides an alternative way to predict crack growth in ductile materials under elastoplastic loading conditions. Application of the cohesive zone model for fatigue crack propagation has to consider damage accumulation which is significant different from monotonic fracture. In order to simulate crack growth under cyclic loading, two aspects have to be taken into account additionally: the cyclic loading path and the damage evolution.

Within the cohesive zone, the constitutive behavior is exactly specified by correlation of the relative opening displacement and the corresponding traction. For the monotonic loading, the cohesive law is uniquely defined and is independent of the loading history, without un/reloading behaviors being considered. Under cyclic loading, in order to account for the irreversibility of the damage process, one has to introduce the un/reloading behaviors. Thus, the damage accumulation in fatigue is determined by the loading path and the material responding in the cohesive zone behaves loading history dependent. The correlation of traction and separation has to be reformulated to entail amount of the damage accumulation.



**Figure 1.5:** Cohesive law with un/reloading path for (a) elastic damage. (b) elastic-plastic damage.

Two types of unloading behaviors have been presumed for brittle and ductile failure respectively, as shown in Fig. 1.5. For an elastic damaged solid, Camacho and Ortiz [64] employed a linear unloading behavior towards the origin. For an elastic-plastic material under the consideration of damage, Chaboche et al. [65] suggested that unloading occurs with the initial stiffness at the origin, and after a completed loading cycle there exists the

possibility for the presence of a residual displacement. In this case, the irreversibility of the inelastic deformation could be taken into account.

Various approaches have been proposed regarding the application of cyclic cohesive zone model (CCZM). The state of art of proposed models will be reviewed in Chapter 5. To the knowledge of the author, most proposed CCZMs can only describe the Paris' law like behavior. The capabilities of CCZM to predict low cycle fatigue crack growth with severe plastification need further development.

### 1.4 Outline of the dissertation

Present work concerns the stress-state effect on the cohesive zone model and the application of the cohesive zone model for three-dimensional low cycle fatigue crack growth predictions. The contents of present thesis are performed by the following chapters:

1. In chapter 2, a special cohesive element is developed to study effects of the cohesive law. The influence of the initial stiffness of the cohesive law and the energy balance in crack simulation are investigated by using cohesive zone models. Interdependence among the threshold value of the cohesive laws, the cohesive stiffness, the cohesive strength and the fracture energy of the cohesive zone is considered systematically.
2. In chapter 3, to validate the efficiency and the predictability of the cohesive zone model in three-dimensional computations, the LCF crack growth behavior of S460 specimens is investigated experimentally under mode I loading condition. Fatigue tests are carried out on the cracked rod bars to examine the surface crack evolution. Furthermore, to quantify the geometry effect on fatigue cracks, the fatigue tests are performed on C(T) specimens with different thicknesses. The failure mechanisms are analysed by comparing the local crack profiles and the fatigue crack growth rates.
3. In chapter 4, a new stress-triaxiality-dependent cohesive zone model (TCZM) is proposed to allow to transfer the cohesive parameters between specimen geometries. The stress-state effect is significant on ductile crack growth, which must influence the cohesive parameters as well. The correlations of the stress triaxiality and the cohesive parameters are obtained based on the experiments, which are performed for quantifying the in-plane and the out-of-plane constraint in three-dimensional cracks. Furthermore, the proposed TCZM is applied for three-dimensional crack propagation simulation.
4. In chapter 5, a new cyclic cohesive zone model (CCZM) is constructed which can characterize the fatigue damage with severe plastic deformation and can predict the fatigue crack growth in Regime III. The experimental fatigue crack growth data of the ductile materials are used for calibrating the new model.
5. In chapter 6, the proposed CCZM is extended to consider the stress-state effect, which can be named as the stress-triaxiality-dependent cyclic cohesive zone model (TCCZM). The influence of the stress-state effect are considered on the cohesive law as well as on the damage evolution equation. The predictions are performed in different specimens for three-dimensional low cycle fatigue crack growth. Numerical