Chapter 1

Introduction

In order to achieve large scale energy production from fusion reactions, plasmas with a mix of deuterium and tritium are heated to temperatures of 100 million degrees. These temperatures are needed for the particle to overcome the Coulomb barrier. Such fusion plasmas are confined by strong magnetic fields, build in toroidal geometry in order to avoid losses parallel to the magnetic field lines. Important for a fusion reactor is the confinement of the energy measured with the confinement time τ_E, which, together with plasma density n and temperature T, forms the triple product. For ignition, where a self-sustained burning fusion plasma is reached, this quantity has to fulfil the Lawson criterion $nT\tau_E > 4 \cdot 10^{21}\,\mathrm{m}^{-3}\,\mathrm{keV\,s}$ [1]. However, achieving ignition has been proven to be a quite challenging task. Increased heating power leads to higher temperatures but it also entails turbulent fluctuations and turbulent transport. Turbulence reduces the confinement time and severely limits the performance of a future reactor.[1]

The search for improved confinement regimes, which will bring the plasma closer to ignition, has long been subject to fusion research [3, 4]. Enhanced performance modes can be achieved due to specific heating or fuelling scenarios and careful wall preparation [5]. With peaked density profiles the density gradient decay length is reduced below a critical value which can stabilise ion temperate gradient instabilities. In 1982 a new type of improved confinement was discovered at the ASDEX tokamak where the transition to a high confinement regime (H-mode) appeared spontaneously [6–8]. Due to a transport barrier in the edge of the confined plasma [9], the turbulent transport was strongly reduced and the energy confinement time doubled. The transport reduction can be explained by a flow shear layer which hinders turbulent outward transport due to the decorrelation of turbulent struc-

[1] The confinement time scales negative with heating power ($P^{-0.5}$) [2] which results in a low confinement regime (L-mode) and leaves the triple product mostly unchanged.

tures (BDT-criterion) [10–13].[2] But the mechanism behind the occurrence of the bifurcation in the confinement time is still not fully understood. Turbulence self-generated zonal flows might play an important role in this H-mode transition [14–18]. These transient shear flows can partially suppress turbulent transport which would in turn increase the ion pressure gradient and thus the background shear flow connected with it. Such behaviour, with a limit cycle oscillation in the intermediate phase between the low and high confinement modes, was indeed confirmed by many experiments [17, 19–25]. However, the physics behind the LH-transition remains a controversial issue as it was found in [26, 27] that the measured turbulent drive was too small to accelerate a zonal flow. And a more resent study suggests that the turbulence zonal flow interaction might not substantially contribute to the LH-transition [28]. Owing to these contrary positions a deeper understanding of the physics related to the drive of zonal flows is highly desirable.

Zonal flows are a phenomenon known before from fluid turbulence [29–31]. The band like structures on Jupiter are probably the most prominent example, but zonal flows also appear in the earth's atmosphere (jet streams) and oceans. On Venus such jets can exhibit velocities faster than the rotation of the planet (super-rotation), and the zonal flows in the interior of the sun are linked to the solar dynamo. Their existence in various physical systems shows that they are a rather universal phenomenon of 2D turbulence.
In toroidal fusion experiments the plasma is confined with an axial (toroidal) magnetic field where the turbulent fluctuations extent far along the field lines and the plasma turbulence is thus quasi two-dimensional. With the additional poloidal magnetic field component, which is needed for stable confinement, the field lines and the elongated turbulent structures are twisted around the torus. The present studies are carried out on a stellarator device where the magnetic field is entirely generated by external field coils and the plasma has a three-dimensional shape. It has been shown that the turbulence in this device resembles that expected in tokamaks, which, in contrast to stellarators, are axisymmetric but need a strong externally induced plasma current to generate part of the field. An illustration of these two confinement concepts is shown in figure 1.1.

Zonal flows exhibit unique properties compared to other turbulent modes. With a homogeneous potential structure along the flux surfaces (called zonal potential) and a finite radial extent zonal flows are intrinsically connected to

[2] Turbulent transport is effectively reduced when the shearing rate $\omega_{E\times B}$ is larger than the maximal linear growth rate $\gamma_{\max}$: $|\omega_{E\times B}| > \gamma_{\max}$.

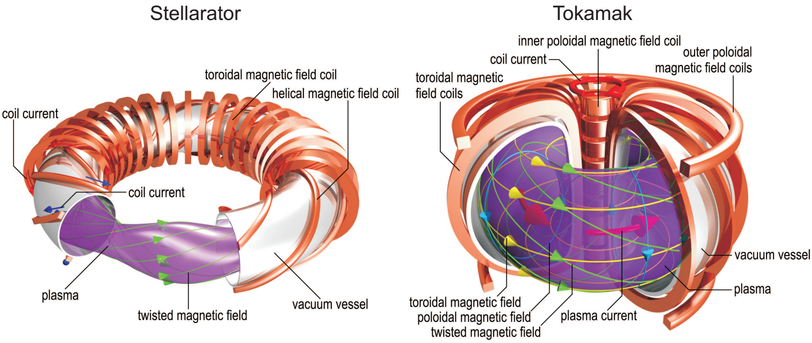

Figure 1.1: Illustration of the two major toroidal magnetic confinement concepts. The combination of toroidal and poloidal magnetic field results in twisted field lines which span the flux surfaces, enclosing volumes of constant magnetic flux. Closed flux surfaces, which do not touch the wall, constitute the confinement region, otherwise the scrape-off layer (SOL). The complex coil geometry of stellarators is reflected in the shape of the plasma. [32]

a zonal shear flow [33]. Because of their symmetry, these mesoscale turbulent structures do not contribute to turbulent cross-field transport and can suppress radial transport by shearing off turbulent eddies. Like in a self-organisation process, the zonal flow is generated by the ambient turbulence itself with a vortex-thinning mechanism [34, 35]. The vortices are tilted and drive the shear flow, which leads to a self-amplification of the zonal flow [36–38]. For tilted vortices the so-called Reynolds stress $\mathcal{R} = \langle \tilde{v}_r \tilde{v}_\theta \rangle$ is non-zero and the radial gradient of this flux surface averaged quantity, as indicated by the brackets, drives the zonal flow. From a theoretical point of view, mostly the physical picture known from fluid mechanics can be transferred to plasma physics. But plasma turbulence, especially in complex geometries, has its own characteristics and phenomena. In contrast to neutral fluids, e.g., the zonal flow drive in plasmas should crucially depend on the cross-coupling of the potential and the density structures. The key parameter in this system is the collisionality C, defined as the normalised electron collision frequency [39]. For the adiabatic case ($C \to 0$) both quantities are closely coupled, while in the hydrodynamic case ($C \to \infty$) density and potential decouple and the zonal flow growth is broken.

The basic concept of large-scale structure formation is clear and elementary processes, like Reynolds stress drive, limit cycle oscillations, and transport suppression, have been demonstrated [40, 41]. However, the experimental verification remains limited as such measurements were mostly restricted to single points in the plasma and do not regard the three-dimensional dynamic of the zonal flow. The complexity of the magnetic field, especially in stellarators, with the consequences for the zonal flow are, up to now, rarely studied in experiment and theory. And a realistic treatment of the geometry in turbulence simulations poses a challenging task which still cannot fully be mastered.

The objective of the present thesis is a detailed analysis of the zonal flows in a toroidally confined plasma with a special focus on the driving mechanism and its collisional dependence. This includes the direct study of the Reynolds stress, and its gradient, together with the connected energy transfer between turbulence and zonal flow. The relevant parameters are measured on the complete poloidal circumference of the confined plasma which allows studying the complex dependency on the magnetic field.

To investigate turbulence, especially the Reynolds stress, multiple measurement points at high time resolution are required which is beyond the limits of the actual diagnostic possibilities in fusion plasmas. Toroidal experiments with low temperature plasmas can fill the gap as their whole confinement region is accessible to probe diagnostics. Probes posses a very high time and a relatively high spatial resolution at the same time. The actual plasma parameters are of course not in the range of those in a fusion plasma, but operation regimes can be chosen such that the normalised parameters relevant for turbulence are comparable to those in the edge region of large fusion experiments. With their flexibility such experiments are predestined for basic research where local magnetic field effects can be very well studied. The experiments for this work have been conducted at the stellarator experiment TJ-K where multi-probe configurations have been exploited to resolve turbulent fluctuations.

This work is organised as follows. The theoretical background of drift waves, the predominant micro instability in the experiment TJ-K, and zonal flows is given in chapters 2 and 3, respectively. This is followed by the description of the techniques of data analysis (Chap. 4), especially the calculation of the energy transfer, and the experimental setup (Chap. 5). All measurements in this work have been performed with newly constructed limiters which result in well-defined boundary conditions. The characterisation of the achieved plasma parameters is presented in chapter 6. This is the foundation for the scaling analysis applied throughout the work. Afterwards the occurrence of the zonal flow is studied in detail where chapter 7 addresses basic properties of the zonal flows. With the conditional averaging technique the temporal evolution can be visualised. In chapter 8 the connection of the Reynolds stress to the magnetic field geometry is studied in detail. Using non-linear analyses techniques, the different energy transfer channels connected with the zonal flow development are studied in chapter 9. Finally, in chapter 10 the results are summarised and discussed with regard to possible consequences of local measurements and the conclusions are presented.

Chapter 2

Plasma turbulence

Turbulence is ubiquitous in nature with a variety of phenomena. This chapter introduces the basic description and characteristics of fluid (Chap. 2.1) and plasma turbulence (Chap. 2.2–2.4). The description is mostly limited to 2D turbulence which is the relevant one for turbulence in fusion plasmas. The fundamental equations introduced here are the basis for the consideration of large structure formation shown in the following chapter.

2.1 Principles of turbulence

In a first part (Sect. 2.1.1) basic formulas, as the Navier-Stokes equation and the vorticity equation, are collected. This is followed by the identification of the conservation laws (Sect. 2.1.2) which entails the turbulent cascades (Sect. 2.1.3) with completely different manifestations in two and three dimensions.

2.1.1 Basic equations

The Navier-Stokes equation [42, 43] is the momentum balance equation for a Newtonian fluid, which, for a complete description, has to be complemented by the continuity equation and an equation for the energy. For an incompressible fluid, with constant mass density and viscosity, it is an extension of the Euler equation by internal friction and describes the evolution of a fluid element in a divergence free velocity field $\mathbf{v}$,

$$\mathrm{D}_t\mathbf{v} \equiv \frac{\partial}{\partial t}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v}\ , \tag{2.1}$$

$$\nabla\cdot\mathbf{v} = 0\ . \tag{2.2}$$

The mass density ρ_m is thereby included in the pressure p and in the (kinematic) viscosity μ. The differential operator D_t is the hydrodynamic derivative, or material derivative, and describes the rate of change in the co-moving frame of reference. This system of second order nonlinear partial differential equations has to be supplemented by appropriate boundary and initial conditions for the velocity and pressure field, which, at least for two dimensions [44], determine a unique solution.[1] If the nonlinear convective term (second term on the left hand side) can be neglected, e.g. if the viscosity is very high, equation (2.1) reduces to a simple diffusion equation (Stokes equation) and a number of special solutions can be obtained (Stokes or creeping flow [45]). But for the majority of more general flows the non-linear term is essential to the dynamics of the flow. The relative strength of this convective term in comparison to the viscous term finally determines the state (laminar or turbulent) of the flow. As only a dimensionless control parameter can be of fundamental significance, the viscosity has to be normalised to a typical length L and velocity V of the system, leading to the Reynolds number

$$Re = \frac{L\,V}{\mu}\ . \tag{2.3}$$

For low Reynolds numbers momentum diffusion by viscosity dominates and the flow is laminar. With increasing Reynolds number the momentum convection gains importance, which leads to the excitation of a few unstable modes with specific flow pattern like, e.g., a Kármán vortex street of alternating vortices. The number of excited modes gets larger with increased control parameter. Eventually, they get nonlinearly unstable and will finally lead to chaotic behaviour and turbulence.[2]

The parameters used in the definition of the Reynolds number (2.3) also define the possible scales of the turbulence. For large structures the typical geometrical size L defines the integral scale where energy is introduced into the system. On the other hand, the viscosity sets a limit for the size of the small structures, i.e. the Kolmogorov dissipation scale. Due to the Laplace operator in the viscous term, viscous diffusion strongly gains influence for smaller structure sizes where the energy is then dissipated into heat.

A characteristic of turbulent flows is that they are rotational. Therefore, the vorticity $\mathbf{\Omega}$, defined as rotation of the velocity field,

[1] For three dimensions, the existence and smoothness of a solution is not yet proven and is one of the 'Millennium Problems' announced by the Clay Mathematics Institute.

[2] Different mechanisms for the onset of turbulence are known but the exact route is yet unclear. In the development of drift-wave turbulence the Ruelle-Takens scenario [46] was confirmed [47–49].

$$\mathbf{\Omega} = \nabla \times \mathbf{v} \;, \tag{2.4}$$

plays an important role in the study of turbulence and describes the rotation of fluid elements about their centroid. An evolution equation for the vorticity can be deduced from the Navier-Stokes equation by taking the curl of (2.1). With the vector identity for the convective term,

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = \nabla \frac{\mathbf{v}^2}{2} - \mathbf{v} \times (\nabla \times \mathbf{v}) = \nabla \frac{\mathbf{v}^2}{2} - \mathbf{v} \times \mathbf{\Omega} \;, \tag{2.5}$$

equation (2.1) leads to

$$\frac{\partial}{\partial t}\mathbf{\Omega} = \nabla \times (\mathbf{v} \times \mathbf{\Omega}) + \mu \nabla \times \Delta \mathbf{v} \;. \tag{2.6}$$

Because of $\nabla \times (\nabla(\mathbf{v}^2/2 + p)) = 0$, the pressure has been eliminated from the equation. The first term on the right hand side can be simplified with $\nabla \cdot (\nabla \times \mathbf{v}) = 0$ and the incompressibility condition (2.2) to

$$\begin{aligned} \nabla \times (\mathbf{v} \times \mathbf{\Omega}) &= \mathbf{v}(\nabla \cdot \mathbf{\Omega}) - \mathbf{\Omega}(\nabla \cdot \mathbf{v}) + (\mathbf{\Omega} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{\Omega} \\ &= (\mathbf{\Omega} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{\Omega} \;. \end{aligned} \tag{2.7}$$

Also the third term of equation (2.6) (with the viscosity μ) can be reformulated, using incompressibility, $\nabla \cdot \mathbf{v} = 0$, to

$$\begin{aligned} \nabla \times \Delta \mathbf{v} &= \nabla \times (\nabla(\nabla \cdot \mathbf{v})) - \nabla \times (\nabla \times (\nabla \times \mathbf{v})) \\ &= -\nabla \times (\nabla \times \mathbf{\Omega}) = \Delta \mathbf{\Omega} \;. \end{aligned} \tag{2.8}$$

With both rearranged terms (Eqs. (2.7) and (2.8)), equation (2.6) results in the vorticity equation in three dimensions

$$\frac{\partial}{\partial t}\mathbf{\Omega} + \mathbf{v} \cdot \nabla \mathbf{\Omega} = (\mathbf{\Omega} \cdot \nabla)\mathbf{v} + \mu \Delta \mathbf{\Omega} \;, \tag{2.9}$$

describing the time evolution of the vector $\mathbf{\Omega}$. Two terms are originating from the nonlinearity of the Navier-Stokes equation, cf. equation (2.5) and (2.7), which exist also in the absence of viscosity (ideal fluid). The second term of equation (2.9) is the convection of vorticity, and the third term describes the stretching of a vortex line[3], leading to an amplification of the vorticity. This

[3]Curves defined as everywhere tangential to the vorticity vector.

vorticity amplification is a consequence of the conservation of circulation Z (Helmholtz's theorem [50]) for ideal fluids,

$$Z = \oint \mathbf{v} \cdot \mathrm{d}\mathbf{l} = \int \nabla \times \mathbf{v} \, \mathrm{d}\mathbf{S} = \text{const} \; . \tag{2.10}$$

Thereby, the integration path of the line integral follows a closed vortex line moving with the fluid. The second part of the equation, after Stokes' law is applied, motivates the use of the vorticity, originally defined in (2.4). From equation (2.10) it is now clear that if the cross section of a vortex $\mathbf{S}$ is reduced through convection of the flow, the vorticity has to increase in order to keep the circulation constant. This mechanism produces intense, fine-scale structures as indeed observed in turbulence [51, 52].[4] But also in the vorticity equation, the viscous diffusion term (last term in Eq. (2.9)) is present, which counteracts the vorticity amplification and sets a limit to the structure size. For sufficiently small scales the viscosity becomes important, leading to a diffusion which smoothes out the vorticity field and stops the amplification.

For a two-dimensional flow, i.e. $\mathbf{v} = (v_x, v_y, 0)$, the vorticity has only a component perpendicular to the plane $\mathbf{\Omega} = \nabla \times \mathbf{v} = \Omega \mathbf{e}_z$. Since the derivative of the flow velocity parallel to the vorticity vector is always zero, the first term on the right hand side of equation (2.9) vanishes, and the vorticity equation reduces to

$$\frac{\partial}{\partial t}\mathbf{\Omega} + \mathbf{v} \cdot \nabla \mathbf{\Omega} = \mu \Delta \mathbf{\Omega} \; . \tag{2.11}$$

In two dimensions, the vorticity equation has reduced to a simple advection-diffusion equation where the vorticity does not act back on the turbulent flow. The missing vorticity stretching is the main difference between two- and three-dimensional turbulence and has, as will be shown later (Sect. 2.1.3), far reaching consequences for the turbulent system.[5]

2.1.2 Conservation laws

Dynamical systems described by the Navier-Stokes equation exhibit deterministic chaos which, in some sense, can be referred to as being sensitive

[4] The funnel of a tornado or the vortex above the outlet of a bathtub, also it is a laminar flow, arises due to the same principle.

[5] Some authors suggest that, because of the missing vortex stretching, flows in two dimensions cannot be seen as turbulent systems, and turbulence is intrinsically three-dimensional.