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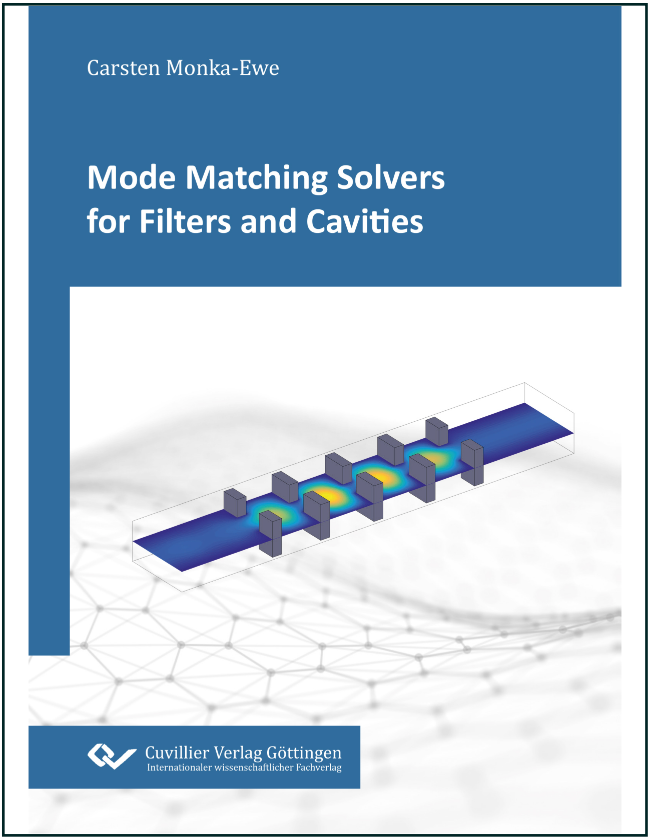
Mode Matching Solvers for Filters and Cavities



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1. Introduction

In 1864 James Clerk Maxwell published his "Dynamical Theory of the Electromagnetic Field" [1] where he postulates that light is an "electromagnetic disturbance" [1], whose velocity of propagation, i.e. the speed of light, can be deduced from purely electric measurements. Maxwell himself expressed this idea as follows:

"The agreement of the results seems to show that light and magnetism are affections of the same substance and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws." [1, p. 499]

Although Maxwell did not comment on the existence of "relatively long" [2] electromagnetic waves [3], today, Maxwell's theoretical work is nevertheless regarded as the foundation of modern electromagnetic theory in the context of microwave engineering. This is because Maxwell's paper introduced the radically new concept of displacement current [3], which is the key to all electromagnetic wave propagation phenomena.

Maxwell had developed his theory entirely from theoretical considerations and it took until 1888 that Heinrich Hertz proved the existence of electromagnetic waves. Regarding Maxwell's postulate on the nature of light, in his paper [4] Hertz concluded:

„Die schon durch viele Wahrscheinlichkeitsgründe gestützte Hypothese, dass die Transversalwellen des Lichtes electrodynamische Wellen seien, gewinnt feste Grundlage durch den Nachweis, dass es wirklich electrodynamische Transversalwellen im Luftraume gibt, und dass diese sich mit einer der Geschwindigkeit des Lichtes verwandten Geschwindigkeit ausbreiten."[1] [4, p. 569]

Soon after Hertz' proof of the existence of electromagnetic waves the rapid development of communication systems began, which should revolutionise the way people live.

Since the earliest days of research on electromagnetic waves engineers have been confronted with the problem of guiding electromagnetic waves by means of transmission lines. While earliest transmission lines were two-wire lines [5], with the shift towards higher frequencies, hollow waveguides and coaxial cables have become standard [6].

Although today's widespread use of microwave systems would have been impossible without the transition to compact, low-cost planar transmission lines [6,7] such as microstrip lines [8] and coplanar waveguides [9], hollow waveguides are still used in a variety of applications [6]:

One typical application are satellite communication systems where waveguides are used both because of their low loss and high power handling capability [10] as well as due to their ability to handle two polarisations, thus enabling polarisation multiplexing [5]. Besides that, waveguides are often found both in millimeter wave test and measurement equipment [6] as well as in material characterisation setups (see chapter 15). Finally, waveguides are widespread in industrial microwave heating systems [11].

[1] "The hypothesis that transverse waves of light are electrodynamic waves, which has already been supported by several reasons of likelihood, gains solid foundation by the experimental proof that transverse electrodynamic waves indeed exist in free space as well as by the fact that they are propagating with a velocity related to the speed of light." [4, p. 569 - translated by the author of this thesis.]

The history of waveguides is a history of multiple rediscoveries [12]. Heaviside [13] considered the idea of electromagnetic waves propagating in a hollow tube, but came to the conclusion that two conductors are required in order to transfer electromagnetic energy.

The propagation of electromagnetic waves in hollow waveguides of rectangular and circular cross-section was predicted by Lord Rayleigh in his paper “On the passage of electric waves through tubes, or the vibrations of dielectric cylinders” [14] as early as 1897. Interestingly enough, Lord Rayleigh was already aware of the waveguides’ infinite discrete spectrum of Eigenmodes. However, the theoretical discovery of waveguides was ahead of its time because high frequency sources were far away from common practice [12] and as a consequence, the idea got forgotten [12].

The “final” rediscovery of hollow waveguides occurred in 1936 when George C. Southworth [15] and Wilmer L. Barrow [16] independently discovered that the propagation of electromagnetic waves inside hollow tubes was indeed possible [12].

In the following, waveguides became common practice and considerable efforts were made to theoretically analyse various waveguide geometries as well as waveguide discontinuities. The possibly largest effort on analysing waveguide structures was carried out in the years 1942 to 1946 at the M.I.T. Radiation Laboratory [17] and based on these efforts, the possibly most complete work on the fine art of the analytical treatment of waveguides and discontinuities, Marcuvitz’ Waveguide Handbook [17], was published.

With the advent of computers in the second half of the last century, the focus regarding the analysis of waveguides changed towards numerical techniques [7]. Today, the application of computational electromagnetics tools can be considered state of the art.

Computational electromagnetics methods can be grouped into four classes as is shown in figure 1.1 for some selected methods. In the following, we will discuss the general concepts as well as advantages and disadvantages of these methods. Because variational methods are of high importance for many computational electromagnetics methods, a concise overview of this subject is provided in appendix A.

Integral Equation methods	**Partial Differential Equation methods**
Method of Moments	Finite Difference Time Domain method
	Finite Integration technique
	Finite Element method
Modal Expansion techniques	**High Frequency methods**
Mode Matching technique	Geometrical Optics method
Spherical Wave Expansion	Physical Optics methods

Figure 1.1.: Some computational electromagnetics methods, with modifications from [18]

We will now discuss the first class of computational electromagnetics methods, **Integral Equation methods** such as the *Method of Moments* (MoM) [19].

In order to apply the Method of Moments, a suitable *integral equation* has to be found by careful analysis of the electromagnetic problem [20]. In the context of electromagnetic waves, such integral equations typically exhibit an unknown current distribution $\mathfrak{u}$ as well as a known forcing field distribution $\mathfrak{f}$ [21].

The "physical environment surrounding the radiator" [21, p. 132] or scatterer is included in the integral equation formulation by means of *Green's functions* [21], which provide the "essential link" [20, p. 290] between the partial differential equation and the integral formulation [20]. Green's functions can be interpreted as a spatial impulse response to a source or as field propagation function [21].

In order to determine the unknown current distribution, the field problem under form of the integral equation has to be inverted by converting it into a linear system of equations [20]. This is done by applying an *indirect variational method*, e.g. the *collocation* or *Galerkin methods*, which yields a matrix equation under form of (A.12) [20]. In order to populate the problem's matrix, the inner products in (A.12) have to be determined by integrating over the meshed surfaces of radiators or scatterers included in the problem [20].

The Method of Moments is well-suited for solving open-domain problems because it inherently implements radiation and only requires surfaces rather than the entire volume to be meshed [21]. However, for closed-domain problems, the method of moments is inferior to the partial differential equation methods discussed in the following because the method of moments yields a dense matrix [21], which is unfavourable both in terms of memory requirements and the time required for inverting the matrix.

In contrast, **Partial Differential Equation methods** such as the *Finite Element method* (FEM) [20], the *Finite Difference Time Domain* (FDTD) method [22, 23] and the *Finite Integration* (FI) technique [24] are the most common choice for solving closed-domain problems [21]. For these methods, a space-segmentation of the domain is required.

Similar to the method of moments, the finite element method is based on variational methods. While both direct and indirect methods can be used as an approach to the Finite Element method [21], here we will consider the *direct Rayleigh-Ritz method* only.

As discussed in appendix A, the Rayleigh-Ritz method strives to extremise the *functional* corresponding to the partial differential equation to be solved [20].

In order to obtain a solution to the partial differential equation for the domain of interest, the domain is firstly separated into subdomains [18], e.g. triangles or tetrahedra [20]. Inside these elements the fields are approximated by suitable polynomials [20, 25]. The corresponding coefficients of these polynomials can be expressed in terms of the values of the solution at the elements' vertices [20].

Careful analysis of the element governing equations [20] and subsequent combination of all elements' equations [20] yields an expression for the overall domain's value of the functional [20].

By forcing the expression's partial derivatives with respect to the values at the elements' vertices to be zero, a system of equations is obtained, which may be solved by standard means in order to determine the solution's values at the vertices of all elements, thus solving the field problem [20].

In contrast to the aforementioned techniques, the two other partial differential methods, namely the FDTD method and the FI technique both operate “directly” on Maxwell’s equations rather than using a functional or an integral equation.

The FDTD technique uses finite difference equations to approximate the partial derivatives included in Maxwell’s equations both in time and space [18]. The well-known *Yee grid* is used for the discretisation of the domain under consideration [22]. This *staggered grid* leads to finite difference equations for the spatial derivatives, which “are central-difference in nature and second-order accurate” [23, p. 60]. Moreover, the Yee grid is divergence free [23, 26] as discussed in greater detail e.g. in [26, p. 500f].

In contrast, the FI technique discretises an integral representation of Maxwell’s equations [27]. When put under integral form, the curl and divergence operators become contour or surface integrals [27, 28], which may easily be evaluated for the grid cells, thus providing the so-called Maxwell grid equations [27, 29].

Since both methods operate on staggered grids [28], the field problem, which is either given under form of the FDTD method’s finite difference equations or the FI technique’s Maxwell grid equations, may be solved in a leapfrog manner [29, 30].

A backdraft of all three aforementioned techniques is the fact that if complicated or large geometries need to be segmented, the obtained meshes become very large and thus lead to both high memory requirements and long computation times [21].

The *Mode Matching technique*, which belongs to the third class of computational electromagnetics methods, **Modal Expansion techniques**, aims to avoid this issue for the price of reduced general applicability.

The idea of the Mode Matching technique is to decompose the structure to be analysed into sub-domains whose spectrum of Eigenmodes is known analytically. By performing orthogonal expansion of the yet unknown tangential field distributions at the interfaces between these sub-domains, a system of equations is obtained which can be solved for the Eigenmodes’ amplitudes.

As the Eigenmodes of the geometries involved must be known, there is a certain limitation regarding the general applicability of the method. If, however, these limitations do not represent a problem for the structure under investigation, the Mode Matching technique outperforms partial differential equation methods by far as we will see later in this thesis.

Since Mode Matching generally treats individual discontinuities only, the Mode Matching technique is commonly used in combination with techniques such as the *generalised scattering matrix method* [31] in order to cascade multiple waveguide discontinuities.

However, in this thesis, a different approach is used, that is, all matrices obtained either by Mode Matching at the discontinuities or by analysing the Eigenmodes’ propagation on waveguide sections of uniform cross-section are assembled into one large system matrix. This system matrix is a sparse matrix with a very narrow bandwidth, which only depends on the number of Eigenmodes included at the interfaces.

Another Modal Expansion technique, which can be used to apply the Mode Matching concept to certain open-domain problems, is *Spherical Wave Expansion*. [18].

For the sake of completeness, the fourth class, **High Frequency methods** shall be mentioned. This class includes ray-based methods such as *Geometrical Optics* (GO) and wave physical methods such as *Physical Optics* (PO), which are relevant for solving electrically large problems, for example calculating the radar cross-section of ships [18].

1.1. State of the Art of the Mode Matching Technique

The development of the Mode Matching technique was started in the 1960s by papers published by Wexler [32] and Clarricoats [33] and it was Wexler [32] who coined the term Modal analysis.

Apart from its limitation to a certain subset of geometries with known Eigenmodes, the most severe backdraft of the Mode Matching technique is the problem of *relative convergence* [34–39]:

As we will see later in this thesis, the series corresponding to the orthogonal expansions need to be truncated for the purpose of implementation. However, if this truncation is carried out in an improper manner, convergence occurs more slowly [38, 40, 41] and for some unfavourable geometries such as a bifurcation of a parallel plate waveguide containing an infinitely thin septum the solution may converge against an incorrect solution [34, 39].

The problem of relative convergence has widely been studied [34–39] and it was found that this problem can be circumvented if the ratio of the numbers of Eigenmodes included on two waveguides adjacent to a discontinuity is chosen corresponding to the waveguides' dimensions.

The successful application of the Mode Matching technique has been reported for a variety of structures such as waveguide filters [42–44], waveguide transformers [43], antennas such as corrugated conical horns [45, 46] and stepped rectangular horns [47]. While we will investigate the analysis of rectangular and tubular filters in great detail, antennas are out of the scope of this thesis. Still, some remarks on the general concept appear imperative:

As discussed in [42, 43], one possible approach for solving antennas using the Mode Matching technique is to terminate the horn antenna in a very large waveguide, which approximates free space conditions. Because for $f \gg f_c$ the waveguide's wave impedance approaches the free-space wave impedance (see figure 10.3 on page 82), this is indeed a valid assumption. The far field is then calculated by means of equivalent Huygens sources for the aperture fields. After numerically solving the source integrals for the vector potentials describing the radiated fields, the electric and magnetic fields are obtained as derivatives of the vector potentials (see e.g. [48]).[2]

While the Mode Matching technique was originally developed for the analysis of discontinuities in hollow waveguides, the analysis of planar microstrip circuits has received attention as well. In order to derive a suitable set of orthogonal Eigenmodes, Wolff et. al. [49,50] modelled microstrip lines as dielectrically filled parallel-plate waveguides with perfectly magnetically conducting sidewalls [38]. Based on this model, for example in [51] planar filters were analysed.

Finally, it should be noted that the Mode Matching technique has also been applied for the investigation of accelerator structures (see [29] and references therein).

In order to alleviate the limitations regarding the geometries which can be treated using the Mode Matching technique, the method has been combined with various other computational electromagnetic techniques. Among those *Hybrid methods* are combinations of Mode Matching and Finite Element methods [52,53] the *Boundary Contour Mode Matching technique* extensively studied by Arndt and Reiter [54, 55] and other methods [10].

[2] In [42, 43] it is said that the far field is obtained by integration of the aperture fields. This most likely refers to the aforementioned approach.

1.2. Organisation of this Thesis

The scope of the current thesis spans a wide range of topics from waveguide theory to the final applications, waveguide filters and electromagnetic cavities for dielectric material characterisation. The material presented in this thesis is grouped into three parts, which are organised as follows:

Part I: Introduction

After a brief introduction given in **chapter 1** with additional remarks provided in **appendix A**, the first part of this thesis covers the theory of cylindrical waveguides.

Cylindrical waveguides have a uniform cross-section in the wave's direction of propagation. We will investigate the boundary value problem imposed by such a waveguide and show that its solution is a discrete spectrum of orthogonal Eigenmodes in **chapter 2**.

In **chapter 3** we will then exploit the orthogonality property of the Eigenmodes in order to perform orthogonal expansion of transverse field components.

Part II: The Mode Matching Technique

In the second part of this thesis, the Mode Matching formalism is developed based on our insights gained in the two introductory chapters 2 and 3.

Firstly, in **chapter 4** we will study the treatment of an individual waveguide discontinuity. In this context, the edge condition and the problem of relative convergence will be discussed. Next, in **chapter 5** additional means for describing waveguide segments of uniform cross-section will be established.

Finally, in **chapter 6** we will discuss a method for cascading the building blocks of waveguide structures introduced in the two preceding chapters and investigate the structure's overall system of equations.

For the development of Mode Matching solvers a sound understanding of the coupling mechanisms at waveguide discontinuities is mandatory. This knowledge is not only important for the validation of the obtained results, but also for determining the smallest set of Eigenmodes which allows a complete treatment of the structure for a given excitation. This allows to drastically reduce the required computation time. In **chapter 7** we will exemplarily investigate coupling at a symmetric H-plane discontinuity. While the outcome of this investigation is well known, the obtained results are far from easy to establish. This exemplary discussion of a waveguide discontinuity should enable the reader to carry out appropriate analyses for other types of discontinuities.

In **appendix B** the Eigenmodes of all waveguides considered in this thesis are given.

Chapters 8 and 10 focus on the post-processing of the Mode Matching solution:

In **chapter 8** we will study the calculation of scattering parameters. As a matter of fact, the correct calculation of scattering parameters is far from easy because problems treated by the Mode Matching technique often include ports with different characteristic impedances. In order to correctly calculate scattering parameters, an in-depth understanding of the definition of voltage and current waves and of the waveguide's characteristic impedance is required. Supplementary information on this subject will be given in **appendix C**. The development of equivalent circuits for waveguide discontinuities will be covered in **chapter 10**. These equivalent circuits will prove highly valuable in the context of filter design later in this thesis.

Numerical aspects of the Mode Matching technique will be covered in chapters 9 and 11. In **chapter 9** means to assess the convergence of the Mode Matching solution are addressed.

Then, in **chapter 11** we will see that the Mode Matching representation of a structure leads to a band-limited sparse matrix, which can be exploited to reduce both the methods' memory requirements as well as the computation time for solving the structure's system of equations. One possible approach to minimising the computation time required to solve the structure's inhomogeneous system of equations is to apply an LU decomposition algorithm which is optimised for working on banded matrices. This will be investigated in **appendix E** in greater detail.

Although solving the structure's system of equations may become time-consuming for larger structures, the computational effort required for populating the structure's system matrix is considerably larger. Thus, in chapter 11 we will also investigate various means to speed up matrix population. We will then conclude chapter 11 by pointing out some possible approaches for accelerating Mode Matching solvers using parallelisation.

The final chapter of part II, **chapter 12**, will cover the extension of the Mode Matching technique for electromagnetic cavities, which allows to accurately determine the resonant frequency and unloaded quality factor of perturbed cavities.

Part III: Applications and results

The third part of this thesis will cover applications for the newly developed Mode Matching solvers and discuss the obtained results. Two main fields of application are considered, waveguide filters and electromagnetic cavities used for material characterisation.

In **chapter 13** we will compare the performance of the Mode Matching solver developed during the preparation of this thesis with the performance of a commercial FEM code, namely Ansys HFSS. We will consider three filter structures, a tubular stepped-impedance filter, a tubular direct-coupled bandpass filter with foreshortened transmission line resonators and a direct-coupled rectangular waveguide shunt-iris filter.

For all three filter structures we will find that the Mode Matching solver developed in the scope of this thesis outperforms the commercial FEM solver by far.

Although the above-mentioned filter structures were either kindly provided by SF Microwave GmbH or have been obtained from literature, it nevertheless appears appropriate to embed the obtained results in a concise discussion of waveguide filter theory. In doing so, the close interplay between the electromagnetic analysis of filter structures and the fine art of microwave filter design is nicely illustrated.

In tubular filter designs it might be desirable to insert dielectric tubes in order to both center the inner conductor as well as to increase the filter's power handling capability. In order to treat such a filter using the Mode Matching technique the Eigenmodes of such a partially filled coaxial waveguide must be known. **Chapter 14** provides a complete derivation and discussion of the Eigenmodes relevant in tubular filter designs.

Finally, in **chapter 15** we will investigate the impact of "secondary effects" on cavity perturbation material measurements, which are induced by approximations made during the method's derivation. In order to do so, approximated quantities provided by the analysis equations of the cavity perturbation material measurement technique are compared to exact ones obtained by performing full-wave solution of the perturbed cavity using a Mode Matching solver for resonant problems.

Chapter 16 concludes this thesis and provides an outlook on future work.

1.3. Contributions

This thesis offers the following contributions to the current state of knowledge:

- **A complete treatise of the Mode Matching Technique**

  Starting from graduate-level electromagnetics as discussed in [48], this thesis provides a detailed thorough development of a Mode Matching formulation. Important theoretical background topics such as the orthogonality relation are reviewed and critical pitfalls such as the problem of relative convergence are outlined.

  In later chapters this thesis focuses on topics related to the efficient implementation of the Mode Matching technique, e.g. the efficient population of a structure's system matrix. While implementation aspects are usually left out in standard literature on the subject [38], this thesis summarises the insights gained by the author in recent years in a comprehensible fashion. An outlook on the future parallelisation of the method concludes this topic.

- **A performance comparison between the Mode Matching Technique and a commercial FEM tool**

  In the scope of this thesis, three filter topologies, a stepped-impedance lowpass filter, a tubular direct-coupled bandpass filter with foreshortened transmission line resonators and a direct-coupled rectangular wave shunt-iris filter will be analysed using the Mode Matching solver developed in the scope of this thesis and a performance comparison with Ansys HFSS is carried out.

  While the Mode Matching solver discussed here typically solves the aforementioned structures in a few minutes at maximum, Ansys HFSS may easily require between half an hour and more than two hours of computation time depending on the complexity of the structure and on whether parallelisation is used.

  It will also be shown that at least for the given structures, the Mode Matching Technique is more robust in terms of convergence while the FEM code requires additional settings regarding the generation of the mesh in order to converge in a satisfying manner.

- **A demonstration of the close interplay between filter design and full-wave analysis of waveguide filters**

  In order to demonstrate the close interrelation between the electromagnetic analysis of waveguide filters and the art of filter design, the results on solving such filters using the Mode Matching technique are embedded in a concise review of filter theory.

  The interplay between these fields cannot be stressed enough because in the process of designing filters, aspects from both fields strongly influence the design process.

  One aspect of this close interplay is that computationally efficient full-wave solutions of filter structures are crucial for the filter design process because filter structures derived from prototype filters usually require manual optimisation in order to mitigate the effect of parasitics.

  A second aspect is the fact that filter polynomials often lead to symmetries in the filters' geometries, which may be exploited to accelerate the solution process of such filters, e.g. during the population of the filter's system matrix.

- **A complete rigorous discussion of the Eigenmodes of partially filled coaxial waveguides**

  In tubular filter designs, dielectric tubes might be desirable to increase the filter's power handling capability as well as to center the inner conductor. While such waveguides can be treated using the generalised theory of multi-layered cylindrical waveguides [48, 56, 57], a concise presentation of the waveguide's Eigenmodes relevant in tubular filter designs is not available in the body of literature.

  Moreover, applying the Mode Matching technique to such waveguides will be demonstrated.

- **A rigorous analysis of "secondary effects" in cavity perturbation material measurements**

  During the derivation of the equations required for cavity perturbation material measurements several approximations have to be made. These approximations are shown to introduce "secondary effects", which may skew measurements of the complex permittivity.

  We will highlight that for materials with a low loss tangent of $2 \cdot 10^{-4}$ the error[3] of the imaginary part may easily be in the region of 10 % even for dielectrics with a low permittivity if a cavity with a moderate unloaded quality factor is used.

  Moreover, it will be shown that at least the deviation of the real and imaginary part of the complex permittivity caused by the field approximation can be corrected by employing the concept of permittivity-dependent geometry factors introduced in this thesis.

  In order to do so, a Mode Matching solver for electromagnetic cavities, which was developed in the scope of this thesis, is used which is capable of calculating the resonant frequency, the unloaded quality factor and the field distribution inside a perturbed cavity in a quasi-analytical fashion.

Terminology, Nomenclature and List of Symbols

The current thesis has a strong theoretical focus and as a consequence, a rigorous nomenclature is mandatory. A detailed overview of the terminology and nomenclature used in this thesis is given on pages 235ff.

[3] See remark on terminology on p. 235.