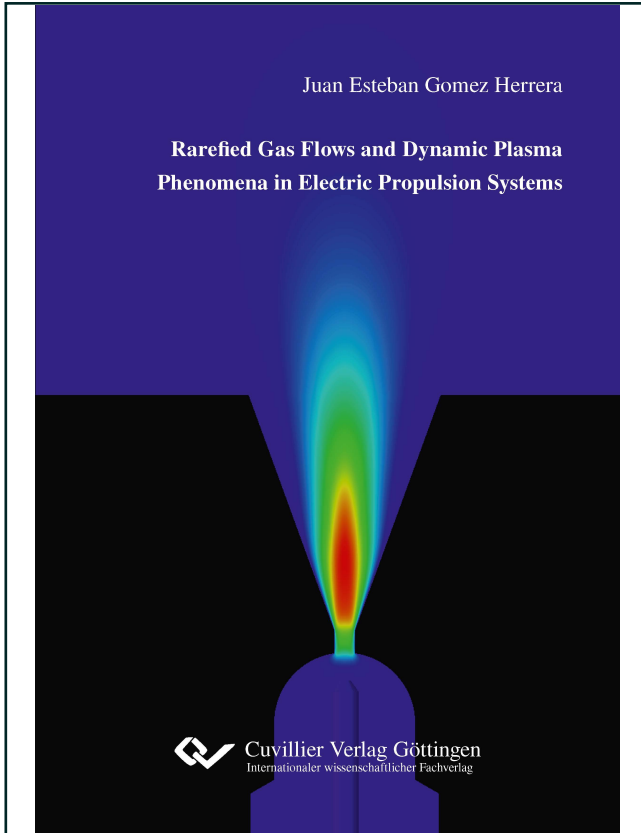




Juan Esteban Gomez Herrera (Autor)  
**Rarefied Gas Flows and Dynamic Plasma Phenomena in  
Electric Propulsion Systems**



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Telefon: +49 (0)551 54724-0, E-Mail: [info@cuvillier.de](mailto:info@cuvillier.de), Website: <https://cuvillier.de>

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