1
Introduction

1.1 Quantum computing

How is it possible to build a physical quantum computer? This question is an open one since Feynman and his contemporaries first pondered using a computer whose evolution is quantum mechanical. Quantum computing has in the meantime become a buzz term which grabs headlines. With modern computers developing at a rapid pace and processors becoming ever smaller, a quantum computer is seen as the ultimate goal. But just what is a quantum computer?

A quantum computer is based around the idea of mixing quantum mechanics and modern day computing. The modern day computer at its most fundamental level is a series of simple gates, put a 1 or a 0 in, get a (or many) 1 or a 0 out. If one has N gates, then N operations are performed. Two classical gates which are performed simultaneously do not have an influence on each other.

In contrast to this classical computer, the quantum computer doesn't merely have bits 0 and 1, rather it has quantum bits (qubits), which are superpositions of two orthogonal states. A qubit can be described by

$$\mid \Psi\rangle = \alpha \mid 0\rangle + \beta \mid 1\rangle, \tag{1.1}$$

where $\{\alpha, \beta\} \epsilon \mathbb{C}$ and $\mid \alpha \mid^2 + \mid \beta \mid^2 = 1$.

In addition, the qubits of a quantum computer can be intertwined or entangled, which means that the qubits are no longer purely individual entities. Operations on a qubit of the quantum register influence the evolution of the other entangled qubits. It is this possibility which vastly improves the computing power for the number of operations per switching unit. It is possible to have a unitary operation which effects all of the possible state permutations at once, i.e. the unitary operation performs 2^N operations on N qubits.

This increased power should mean faster operations if one is able to harness it. To this end algorithms have been developed which require a quantum computer to implement them. These algorithms can search a random database of N entries using $\sqrt{N}$ steps rather than the classical N/2 steps [Grover, 1996] or factorize large numbers in polynomial time frames, which is considerably faster than that required for known classical algorithms [Shor, 1997]. These commercially friendly applications are joined by the prospect of being able to solve dynamics of complex quantum mechanical problems [Kaye *et al.*, 2007]. What is required to execute such an algorithm?

In fact it is only necessary to show that individual qubit rotations and conditional logic are possible to execute a quantum algorithm. A rotation is an operation which changes the superposition of the orthogonal states by a prescribed amount. Conditional logic however is somewhat more complicated, but can be brought in analogy to the classical XOR gate [Barenco *et al.*, 1995].

This discussion, which has briefly described the main principles behind a quantum computer, has not answered the first question posed: How to physically implement it? It has been established that the physical system must have two basis states and qubits must interact with each other: What else is required? The DiVincenzo [DiVincenzo, 2000] criterium has five main points which describe the physical requirements for a system which is a quantum computer. There exist a further two points in this criteria, it is often referred to as the 5 (+2) conditions for a quantum computer. These extra two conditions refer to quantum communication techniques and are not discussed here. The five conditions on the the other hand are

1. The **system must be scalable** and the **qubits must be well characterized**. This means that a single qubit must be well understood in terms of its internal Hamiltonian, the couplings to other states, the interactions with other qubits and the coupling to external fields. The coupling with external fields is particularly important as this has a direct influence on the fidelity with which intended operations can be performed. It is also important that it is possible to entangle qubits and that the system can be scaled so that the advantage of the quantum computer can be obtained.

2. It must be possible to **prepare the qubits** to an intended state. Preparation means that an arbitrary qubit state can be obtained with high fidelity.

3. **Decoherence times** must be longer than gate operation times. The qubit is quite often subject to interactions with its environment. Such interactions can effect either the superposition of the orthogonal states of the qubit, or the phase between the occupation of the two states. The requirement of this point is saying that such processes can be dealt with, so long as they aren't likely to happen whilst performing the gates which make the calculation. A guide given in [DiVincenzo, 2000] is that the decoherence time should be $10^4 - 10^5$ times larger than the time needed for an individual gate. This estimation includes using the effect of quantum error correction techniques.

4. **Universal gates** must be possible. In the previously cited [Barenco *et al.*, 1995] it is shown that the set of one-bit quantum gates and the two bit

exclusive-or gate is sufficient to describe all unitary operations on arbitrary many bits. So for a unviersal gate to be possible, single qubit rotations and conditional logic operations must be possible.

5. It must be possible to **measure the different qubits**.

1.1.1 Methods

The DiVincenzo criteria is a handy check list for knowing what is required for the physical manifestation of a quantum computer. In this section the current progress of the scientific community in pursuing this aim is summarized.

The earliest implementations of quantum computing systems were done in Nuclear Magnetic Resonance (NMR) experiments. Here algorithms were shown to work with up to seven qubits [Vandersypen & Chuang, 2005]. These implementations were only intended as proof of principle experiments, as the upper limit of NMR qubits is predicted at 30 [Jones, 2000].

A proposal which has made fast progress in recent years is based on nitrogen vacancy (NV) centres in diamond [Neumann *et al.*, 2008]. Two partite and three partite entanglement for two carbon atoms about the vacancy or for the two carbon atoms and the vacancy itself has been shown. Proposals exist for how to scale this system by entangling distant centers through emitted photons [Barrett & Kok, 2005].

A scheme which has been garnering many headlines in recent times is semiconductor qubits. Semiconductors as qubits is a young technology: it is only recently that two qubit algorithms have been demonstrated [DiCarlo *et al.*, 2009]. If technical problems such as coherence times (currently on the order of μs [Clarke & Wilhelm, 2008]) and gate fidelities You & Nori [2005] are overcome this may well prove to be a viable option for a quantum computer.

Orthogonally polarized photons are used in quantum communication as qubits. It is certainly not difficult to envisage how information can be transported if one is using photons, they travel quite easily! Experiments performed by [Lu *et al.*, 2007] show that it is possible to create six photon graph states where the fidelity with which these states are produced is F = 0.593 $\pm$ 0.025.

In this thesis none of the above technologies are pursued to achieve a quantum computer. This thesis is concerned with individual ions in an electrodynamic trap. Although ion traps have existed for much of the previous century [Paul *et al.*, 1958], using them for quantum information is a relatively new development [Cirac & Zoller, 1995].

1.2 Ion trap quantum computers

The proposal to use ion traps as a quantum computer came from [Cirac & Zoller, 1995]. It was envisioned that the ions would be trapped in a linear trap. The

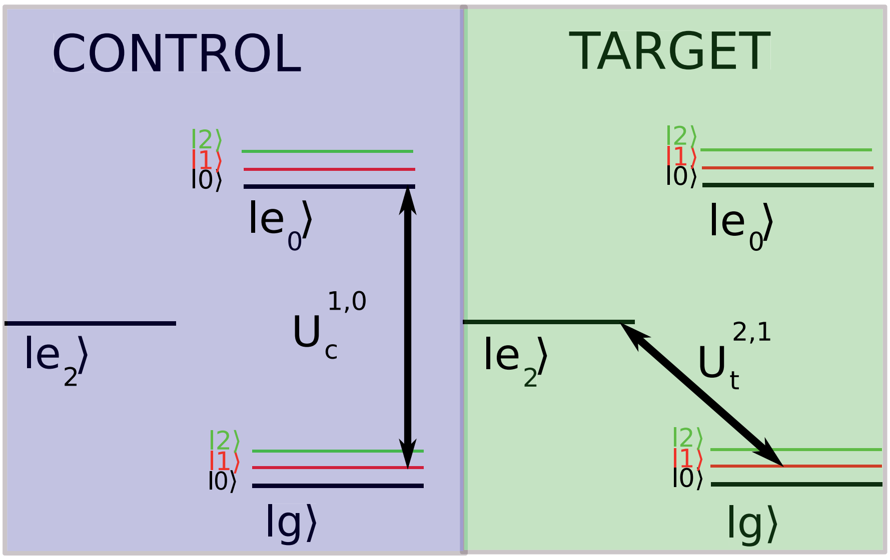

Figure 1.1: In this graphic the two operations which are used to create the Cirac Zoller CNOT scheme are shown. The left ion scheme represents the control qubit, the right the target qubit. The marked unitary operations correspond to those explained in table 1.1

ions would be individually addressable by different lasers being focussed on each individual ion. A universal rotation would be achieved by allowing a laser tuned to the the qubit transition to be switched on for a prescribed amount of time, where the phase and power of the light are controlled. Conditional logic is also possible: this is made possible through use of a bus qubit, which serves to the entangle ions. The bus qubit comes in the form of the collective quantized motion of the ions. The motion of an ion or ions in a trap can be quantized, much like a quantum mechanical harmonic oscillator. A string of N ions in a trap will have 3N common motional modes.

In [Cirac & Zoller, 1995] an example of two ions in a trap is given. Both ions are in the motional ground state of the string. Here the basis of the conditional gate is that if one of the ions is excited to a higher motional state by means of a bus qubit, then both of them will be. Two qubits are shown in figure 1.1, one named the control qubit the other the target. The first pulse, a π-pulse, is tuned such that the the control qubit receives a motional quanta only if it is in the excited state. If the control qubit receives a motional quanta, the target qubit does too. The second operation, a 2π-pulse, is tuned in frequency such that it is only successful if the target qubit has a motional quanta. This 2π-pulse to an external level, adds a phase to the qubit which it would not otherwise have. Finally the sequence is closed by repeating the first π-pulse on the control qubit to remove the motional quanta from the system. The different input and output states are tabulated in table 1.1.

A suitable qubit transition must have a natural linewidth narrow enough so that the transitions involving different motional quanta are cleanly distinguishable. The description of the possible transitions on the various sidebands is given by the

Initial State	$U_c^{1,0}$	$U_t^{2,1}$	Final State $U_c^{1,0}$
$\|g\rangle_c \|g\rangle_t \|0\rangle$	$\|g\rangle_c \|g\rangle_t \|0\rangle$	$\|g\rangle_c \|g\rangle_t \|0\rangle$	$\|g\rangle_c \|g\rangle_t \|0\rangle$
$\|g\rangle_c \|e_0\rangle_t \|0\rangle$	$\|g\rangle_c \|e_0\rangle_t \|0\rangle$	$\|g\rangle_c \|e_0\rangle_t \|0\rangle$	$\|g\rangle_c \|e_0\rangle_t \|0\rangle$
$\|e_0\rangle_c \|g\rangle_t \|0\rangle$	-i$\|g\rangle_c \|g\rangle_t \|1\rangle$	i$\|g\rangle_c \|g\rangle_t \|1\rangle$	-$\|e_0\rangle_c \|g\rangle_t \|0\rangle$
$\|e_0\rangle_c \|e_0\rangle_t \|0\rangle$	-i$\|g\rangle_c \|e_0\rangle_t \|1\rangle$	-i$\|g\rangle_c \|e_0\rangle_t \|1\rangle$	$\|e_0\rangle_c \|e_0\rangle_t \|0\rangle$

Table 1.1: Cirac Zoller conditional logic gate. The unitary operations are illustrated in figure 1.1.

Hamiltonian term

$$H_{int} = \frac{1}{2}\,\hbar\;\Omega_R\;(\sigma_+ + \sigma_-)(e^{i[\eta(a_l+a_l^+)-\omega_M t]} + H.c.), \tag{1.2}$$

where Ω_R is the Rabi frequency of the driving field, ω_M is the rotating frequency of the driving field, t is time, $\sigma_\pm = \sigma_x \pm i\sigma_y$ the Pauli matrices, a_l and a_l^+ are the lowering and raising operators and η is the Lamb Dicke factor. This factor, which will later be explained as the most important factor in this Hamiltonian, is given by

$$\eta = \sqrt{\frac{\frac{(\hbar k)^2}{2m}}{\hbar\omega_1}}, \tag{1.3}$$

where $\vec{k}$ is the wave vector of the incoming light, m is the mass of the ion and ω_1 is the trap frequency. For a microwave transition the Lamb Dicke factor is 10^4 times smaller than that for an optical transition, due to the inverse proportionality of η to the wavelength λ.

So building a quantum computer should be easy right? A five step check list, powerful sounding algorithms and talented scientists it must all be a doddle....

Well not quite, but there have been nonetheless some interesting developments towards building a quantum computer in recent years. Not least, the results from Innsbruck where 8 qubits were shown to be entangled [Häffner *et al.*, 2005]. Entanglement with a higher number of ions has not been attempted due to the large amount of measuring time required and the increasing classical computational time and power required to reconstruct the density matrix. Theoreticians are currently working on more elegant ways that experimentalists can use to prove that qubits are entangled.

Experimentalists in recent years have focussed on how ion traps could be built on much larger scales. To this end it is intended to incorporate chip technology

[Labaziewicz *et al.*, 2008], which would involve several ion traps where ions can be ferried between different trap regions [Kielpinski *et al.*, 2002] whilst maintaining entanglement [Jost *et al.*, 2009]. The future with ion trap quantum computing looks bright indeed!

1.3 Thesis outline

This thesis does not build directly on the great achievements just listed but investigates more humble beginnings. All the experiments shown here are motivated by the hope that the results can be used in potential ion trap schemes or traps involving many ions. The experiments are all performed with a single ion. In particular technical problems foreseen in implementing a variation to the Cirac Zoller scheme based on an “ion spin molecule” which uses microwave radiation, are investigated.

The theory of ion traps, the ions and isotope used, how laser cooling works and finally the alternate “ion spin molecule” scheme are all presented in chapter 2. A problem which will arise in this scheme is an uncertainty in the resonant frequency of individual ions. Pulses which are robust to such uncertainaties are introduced. Decoherence and the main method to overcome it are discussed. Sensitivity to decoherence from ambient magnetic field noise is forseen as a problem in the “ion spin molecule” scheme. An alternative scheme based on the concept of dressed states is introduced which should be robust to such noise. In fact such a scheme could also be used with the “ion spin molecule” to increase interaction strengths between entangled ions. Dressed states themselves are described before examining different possible atomic configurations of implementing such a scheme. Preparation of the states and physical requirements to implement such a system are also considered.

Chapter 3 describes the experimental setup used for the experiments presented in this thesis. This includes the trap (section 3.1), the optics (section 3.2) including the laser systems and the λ-meter, the non optical electromagnetic fields (section 3.3) and the static fields (section 3.3.3). The detection is described (section 3.4) and a brief overview of the experimental control system is given in section 3.5. The data evaluation method used and a possible alternative is described in section 3.6

Chapter 4 describes how the robust pulses introduced in chapter 2 were tested. Two types of pulses are shown: shaped pulses which are based on optimal control theory and simpler composite pulses. These are compared to both expected theoretical values and simple rectangular pulses.

The measurements in chapter 5 are made to investigate the reason that the fidelities in the measurements in chapter 4 were lower than expected. Methods to determine this include an additional laser for state preparation (section 5.1), an additional repumper laser (section 5.2) and changing the photomultiplier (section 5.3). The negative effect of ambient magnetic field noise on the magnetic field sensitive levels is shown in section 5.4 and the improvement after triggering the experiment to the A/C line is also shown. The coherence time of the magnetic field insensitive transition is measured in section 5.5.

Finally chapter 6 describes some investigative measurements for the implementation of a tweaked "ion spin molecule" scheme based on dressed states. It is envisioned that this tweaked scheme could either be used to combat the decoherence caused by ambient magnetic field noise, or to amplify the existing scheme. Preparation methods of the dressed states are measured (section 6.1), the suitability of a two level system (section 6.2) and a three level system (section 6.3) are also measured.