Introduction

This book[1,2] presents original mathematical models of phase-transformation stresses in composite materials, along with mathematical models of phase-transformation-stress induced micro-/macro-strengthening and phase-transformation-stress induced intercrystalline or transcrystalline crack formation. The materials consist of an isotropic matrix with isotropic ellipsoidal inclusions. These stresses originate during a cooling process at the phase- transformation temperature T_{tq}, and are a consequence of the difference between dimensions of crystalline lattices, which are mutually transformed during the phase-transformation process in the inclusions ($q = IN$) or the matrix ($q = M$).

The mathematical models are determined for a suitable model system. The model system is required to correspond to real isotropic matrix-inclusion composites. The phase-transformation stresses are derived within a suitable coordinate system. The coordinate system is required to correspond to a shape of the ellipsoidal inclusions.

The mathematical determination results from mechanics of an isotropic elastic continuum, and result in different mathematical solutions for the phase-transformation stresses. Due to these different mathematical solutions, the principle of minimum elastic energy is considered.

The mathematical models of the phase-transformation stresses, which result from the superposition method, along with the mathematical models of the phase-transformation-stress induced micro-/macro-strengthening and crack formation, include microstructural parameters of a real matrix-inclusion composite, i.e., the inclusion dimensions a_1, a_2, a_3, the inclusion volume fraction v_{IN}, as well as the inter-inclusion distance $d = d\left(a_1, a_2, a_3, v_{IN}\right)$.

Consequently, the mathematical models are applicable to composites with ellipsoidal inclusions of different morphology, i.e., $a_1 \approx a_2 \approx a_3$ (dual-phase

[1] This book was reviewed by the following reviewers:
Assoc. Prof. Ing. Robert Bidulský, PhD., visiting professor, Politecnico di Torino, Torino, Italy.

Prof. Ing. Daniel Kottfer, PhD., Alexander Dubček University of Trenčín, Faculty of Special Technology, Department of Mechanical Engineering, Trenčín, Slovak Republic.

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steel), $a_1 \gg a_2 \approx a_3$ (martensitic steel).

In case of a real matrix-inclusion composite, such numerical values of the microstructural parameters can be determined, which result in maximum values of the micro- and macro-strengthening, and which define limit states with respect to the intercrystalline or transcrystalline crack formation in the matrix and the ellipsoidal inclusion. This numerical determination is performed by a programming language. The mathematical procedures in this book are analysed in Appendix (see Section 10.4).

Chapter 1

Matrix-Inclusion Composite

1.1 Model System

Figure 1.1 shows a model system, corresponding to real matrix-inclusion composites, which is considered within the mathematical models of the phase-transformation stresses. This model system consists of an infinite isotropic matrix and isotropic ellipsoidal inclusions with the dimensions a_1, a_2, a_3 and the inter-inclusion distance d along the axes x_1, x_2, x_3 of the Cartesian system $(Ox_1x_2x_3)$, respectively, where O represents a centre of the ellipsoidal inclusion.

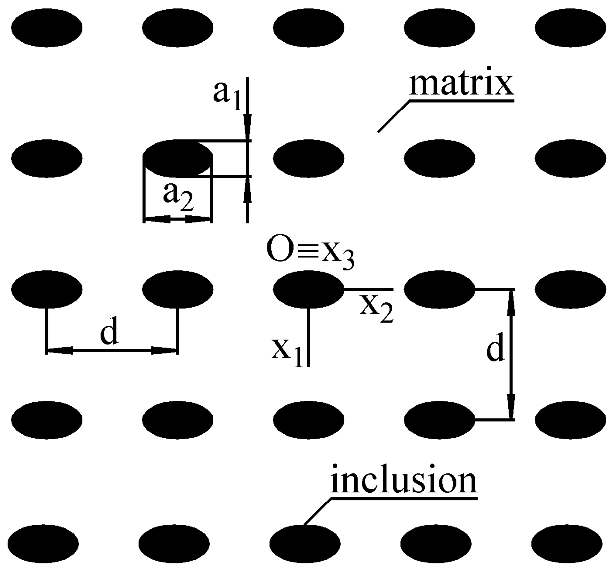

Figure 1.1: The matrix-inclusion system with an infinite isotropic matrix and isotropic ellipsoidal inclusions with the dimensions a_1, a_2, a_3 and the inter-inclusion distance d along the axes x_1, x_2, x_3 of the Cartesian system $(Ox_1x_2x_3)$, respectively, where O represents a centre of the ellipsoidal inclusion.

As presented in [1]–[22], the phase-transformation stresses are determined in the cubic cells with the dimension d along the axes x_1, x_2, x_3 and with central ellipsoidal inclusions (see Figure 1.2). Due to the infinite matrix, the phase-transformation stresses, which are determined for one of the cubic cells, are identical with those, which are determined for any of the cubic cells [1]–[22]. With regard to the volume $V_{IN} = 4\pi a_1, a_2 a_3$ [23] and $V_C = d^3$ of the ellipsoidal inclusion and the cubic cell, the inter-inclusion distance d as a function of the inclusion volume fraction v_{IN} is derived as

$$v_{IN} = \frac{V_{IN}}{V_C} = \frac{4\pi a_1 a_2 a_3}{3d^3} \in \left(0, \frac{\pi}{6}\right), \quad d = \left(\frac{4\pi a_1 a_2 a_3}{3v_{IN}}\right)^{1/3}, \tag{1.1}$$

where the value $v_{INmax} = \pi/6$ results from the condition $a_i \to d/2$ $(i = 1,2,3)$. Accordingly, the phase-transformation stresses are functions of the material parameters a_1, a_2, a_3, v_{IN}, d.

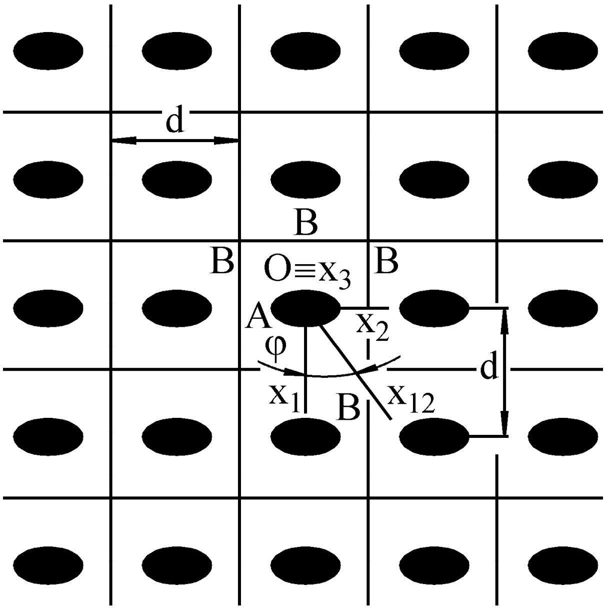

Figure 1.2: The cubic cells with the dimension d along the axes x_1, x_2, x_3 of the Cartesian system $(Ox_1x_2x_3)$ and with the plane $x_{12}x_3$, where O represents a centre of the ellipsoidal inclusion, and ($x_{12} \subset x_1x_2$, $x_{12}x_3 \perp x_1x_2$. The phase-transformation stresses in the cell A and the neighbouring cells B are mutually affected.

Additionally, the phase-transformation stresses in the cell A and the neighbouring cells B are mutually affected. In contrast to [1]–[13], [15]–[22], this effect is explicitly determnined [14].

1.2 Coordinate System

Figure 1.3 shows the ellipse E with the dimensiions a, b along the axes x, y, respectively. The ellipse E is described by the function

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1. \tag{1.2}$$

Any point P of the ellipse E is described by the coordinates [23]

$$x = a\cos\alpha, \quad y = b\sin\alpha, \quad \alpha \in \langle 0, 2\pi\rangle, \tag{1.3}$$

where the normal n of the ellipse E at the point P is derived [23]

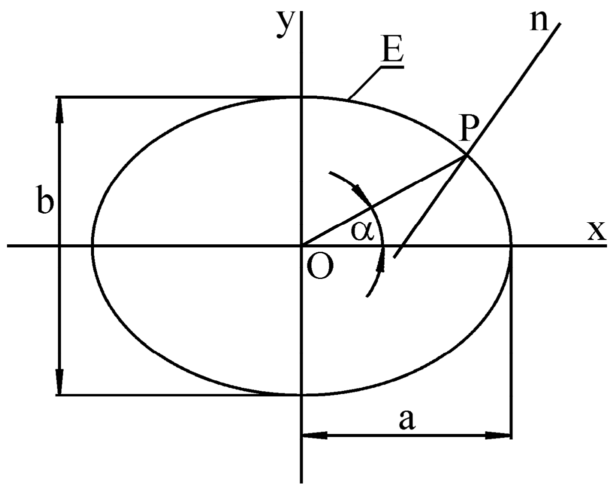

Figure 1.3: The ellipse E with the dimensions a, b along the axes x, y of the Cartesian system (Oxy), respectively, and the point P related to the angle α.

$$y = \frac{x\,a\tan\alpha}{b} - \frac{\left(a^2 - b^2\right)\sin\alpha}{b}. \tag{1.4}$$

The phase-transformation stresses are determined by the spherical coordinates $(r, \varphi\nu)$ (see Figure 1.4). The model system in Figures (1.1), (1.2) is symmetric, and then the phase-transformation stresses are determined within the intervals $\varphi \in \langle 0, \pi/2\rangle$, $\nu \in \langle 0, \pi/2\rangle$ [1]–[22].

Figure 1.4 shows the ellipsoidal inclusion for $\varphi, \nu \in \langle 0, \pi/2\rangle$ with the centre O and with the dimensions $a_1 = O1$, $a_2 = O2$, $a_3 = O3$ along the axes x_1, x_2, x_3 of the Cartesian system (O, x_1, x_2, x_3) (see Figures (1.1), (1.2)), respectively. Finally, $\left(P, x_n, x_\varphi, x_\nu\right)$ is a Cartesian system at the point P, where the axes x_n and x_ν represents a normal and a tangent of the ellipse E_{123} at the point P, respectively, $x_{12}x_3 \perp x_1x_2$, $x_{12} \subset x_1x_2$, $x_\varphi \perp x_{12}$. Figure 1.5 shows the cross section $O567$ of the cubic cell in the plane $x_{12}x_3$ (see Figures 1.2, 1.4). The angle $\nu \in \langle 0, \pi/2\rangle$ defines a position of the point P with the Cartesian system $\left(P, x_n, x_\varphi, x_\nu\right)$ (see Figure 1.4) for $\nu = \nu_0$ (see Figure 1.5a), $\nu \in \langle 0, \nu_0)$ (see Figure 1.5b), $\nu \in (\nu_0, \pi/2\rangle$ (see Figure 1.5c). The points P_1, P_2 represent intersections of the normal x_n with $O567$.

With regard to Equations (1.2)–(1.4), the angle ν_0 represents a root of the following equation [24]

$$\frac{\cos\nu_0}{a_3}\left[\frac{d\sqrt{a_1^2\cos^2\varphi+a_2^2\sin^2\varphi}}{2\,f(\varphi)\,\sin\nu_0}+a_3^2-\left(a_1^2\cos^2\varphi+a_2^2\sin^2\varphi\right)\right]-\frac{d}{2}=0,$$

$$f(\varphi)=\cos\varphi,\quad \varphi\in\left\langle 0,\frac{\pi}{4}\right\rangle;\quad f(\varphi)=\sin\varphi,\quad \varphi\in\left\langle\frac{\pi}{4},\frac{\pi}{2}\right\rangle, \tag{1.5}$$

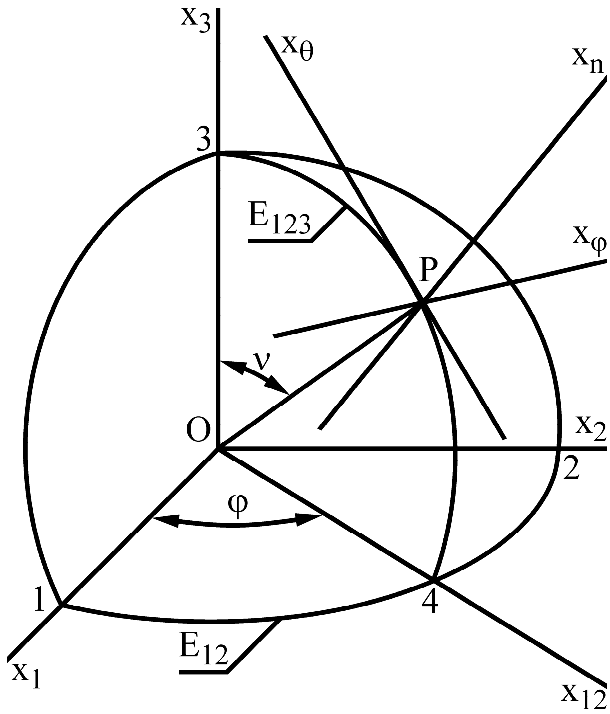

Figure 1.4: The inclusion with the centre O and with the dimensions $a_1 = O1$, $a_2 = O2$, $a_3 = O3$ along the axes x_1, x_2, x_3 of the Cartesian system (O,x_1,x_2,x_3), respectively, where E_{12}, E_{123} represent ellipses in the planes x_1x_2, $x_{12}x_3$, respectively, and $x_{12}x_3 \perp x_1x_2$, ($x_{12} \subset x_1x_2$, $x_\varphi \perp x_{12}$. The point P on the inclusion surface is defined by $\varphi, \nu \in \langle 0, \pi/2\rangle$, $\nu \in \langle 0, \pi/2\rangle$, and $\left(P, x_n, x_\varphi, x_\nu\right)$ is a Cartesian system at the point P, where $P \subset E_{123}$. The axes x_n and x_ν represents a normal and a tangent of the ellipse E_{123} at the point P, respectively.

and this root is determined by a numerical method. The angle $\theta = \angle\left(x_n, x_3\right)$ is derived as [24]

$$\cos\theta=\frac{\sqrt{a_1^2\cos^2\varphi+a_2^2\sin^2\varphi}}{\sqrt{a_1^2\cos^2\varphi+a_2^2\sin^2\varphi+(a_3\tan\nu)^2}},$$