

Contents

Introduction	1
1 Solid Continuum Mechanics	7
1.1 Model Material System	7
1.2 Coordinate System	10
1.3 Reason of Residual Stresses	13
1.4 Radial Stresses p_1, p_2	18
1.4.1 Multi-Particle-Envelope-Matrix System	18
1.4.2 Multi-Particle-Matrix System	20
1.5 Fundamental Equations	20
1.5.1 Cauchy's Equations	22
1.5.2 Equilibrium Equations	23
1.5.3 Hooke's Law	25
1.6 System of Differential Equations	26
1.7 Total Potential Energy	27
1.8 Mathematical Boundary Conditions	30
1.8.1 Multi-Particle-Envelope-Matrix System	31
1.8.2 Multi-Particle-Matrix System	34
1.8.3 Superposition Method	34
2 Mathematical Model 1	37
2.1 Mathematical Procedure 1	37
2.2 Multi-Particle-Envelope-Matrix System.	
Condition $\beta_p \neq \beta_e \neq \beta_m$	39
2.2.1 Cell Matrix	39
2.2.2 Spherical Envelope	42

2.2.3	Spherical Particle	45
2.3	Multi-Particle-Envelope-Matrix System.	
	Condition $\beta_p = \beta_e \neq \beta_m$	46
2.3.1	Spherical Envelope	47
2.3.2	Spherical Particle	48
2.4	Multi-Particle-Envelope-Matrix System.	
	Condition $\beta_p \neq \beta_e = \beta_m$	49
2.4.1	Spherical Particle and Envelope	49
2.4.2	Cell Matrix	51
2.5	Multi-Particle-Matrix System.	
	Condition $\beta_p \neq \beta_m$	53
3	Mathematical Model 2	55
3.1	Mathematical Procedure 2	57
3.2	Multi-Particle-Envelope-Matrix System.	
	Condition $\beta_p \neq \beta_e \neq \beta_m$	60
3.2.1	Cell Matrix	60
3.2.2	Spherical Envelope	68
3.3	Multi-Particle-Matrix System.	
	Condition $\beta_p \neq \beta_m$	75
4	Mathematical Model 3	77
4.1	Mathematical Procedure 3	79
4.2	Multi-Particle-Envelope-Matrix System.	
	Condition $\beta_p \neq \beta_e \neq \beta_m$	83
4.2.1	Cell Matrix	83
4.2.2	Spherical Envelope	92
4.3	Multi-Particle-Matrix System.	
	Condition $\beta_p \neq \beta_m$	99
5	Crack Formation	101
5.1	Mathematical Procedure	101
5.2	Multi-Particle-Envelope-Matrix System	106
5.2.1	Cell Matrix	106
5.2.2	Spherical Envelope	110
5.2.3	Spherical Particle	118
5.3	Multi-Particle-Matrix System	122
5.3.1	Cell Matrix	122
5.3.2	Spherical Particle	126

6	Strengthening	131
6.1	Multi-Particle-Envelope-Matrix System	132
6.2	Multi-Particle-Matrix System	138
A	Selected Topics in Mathematics	141
A.1	Cramer's Rule	141
A.2	Quadratic Algebraic Equation	143
A.3	Determination of Integrals	143
A.4	Wronskian's Method	145
A.5	Numerical Determination	147
	Index	150