

# Table of Contents

<b>Preface</b> .....	<b>V</b>
<b>Abbreviations and Notation</b> .....	<b>IX</b>
<b>List of Figures (line graphics)</b> .....	<b>IX</b>
<b>1. Overview</b> .....	<b>1</b>
1.1 Basic Definitions .....	1
1.2 System and Development Environment .....	3
1.3 Fortran Data Type Definitions (module kinds) .....	4
1.4 Constants and Parameters (module const).....	9
1.5 Composition of Modules (module modules).....	14
1.6 Error Function Tables (subroutine DatError) .....	16
1.7 Test program and Menu.....	23
<b>2. Error Function, <math>y = \text{erf}(x)</math></b> .....	<b>25</b>
2.1 Approximations with high precision .....	29
2.1.1 Intrinsic ERF, Reference GFortran/Silverfrost (2024) .....	29
2.1.2 Coefficients Approximation, Reference Strecok (1968) .....	30
2.1.3 Rational Chebyshev Approximation, Reference Cody (1969).....	32
2.1.4 Chebyshev Expansions, Reference Schonfelder (1978).....	35
2.1.5 Chebyshev Series, Reference Slatec (1993).....	38
2.1.6 Polynomial Approximation, Reference Sun Microsystems (1993).....	43
2.1.7 Rational Approximations, Reference Ooura (1996).....	47
2.1.8 Mill's Ratio Approximation, Reference Dia (2023).....	55
2.2 Approximations with Low Precision.....	58
2.3 Approximations with Series Expansion .....	69
2.4 Approximations with Continued Fraction .....	76
2.5 Summary Error Function .....	83
<b>3. Complementary Error Function, <math>y = \text{erfc}(x)</math></b> .....	<b>85</b>
3.1 Approximations with high precision .....	88
3.1.1 Intrinsic ERFC, Reference GFortran/Silverfrost (2024) .....	88
3.1.2 Rational Chebyshev Approximation, Reference Cody (1969).....	89
3.1.3 Chebyshev Expansion, Reference Schonfelder (1978) .....	92
3.1.4 Chebyshev Expansion, Reference Shepherd/Laframboise (1981) .....	94
3.1.5 Chebyshev Series, Reference Slatec (1993).....	96
3.1.6 Polynomial Approximation, Reference Sun Microsystems (1993).....	101

3.1.7 Rational Approximations, Reference Ooura (1996).....	105
3.1.8 Mill’s Ratio Approximation, Reference Dia (2023).....	112
3.2 Approximations with Low Precision.....	115
3.3 Approximations with Series Expansion .....	115
3.4 Approximations with Continued Fraction .....	122
3.5 Summary Complementary Error Function .....	129
<b>4. Scaled Complementary Error Function, <math>y = \operatorname{erfcx}(x)</math> .....</b>	<b>131</b>
4.1 Approximations with high precision .....	134
4.1.1 Intrinsic ERF_SCALED, Reference GFortran/Silverfrost (2024) .....	134
4.1.2 Rational Chebyshev Approximation, Reference Cody (1969).....	135
4.1.3 Chebyshev Expansion, Reference Schonfelder (1978) .....	138
4.1.4 Chebyshev Approximation, Reference Shepherd/Laframboise (1981).....	141
4.1.5 Chebyshev Series, Reference Slatec (1993).....	143
4.1.6 Rational Approximations, Reference Ooura (1996).....	148
4.1.7 Chebyshev Lookup Approximation, Reference Johnson/Wuttke (2012).....	155
4.1.8 Multiple-Precision Lookup Algorithm, Reference Zaghoul (2024).....	158
4.1.9 Zaghoul Algorithm (optimized), Reference Zaghoul (2024) .....	164
4.1.10 Multi Precision Algorithm w/o Lookup, Reference Höring.....	171
4.2 Approximations with Low Precision.....	179
4.3 Approximations with Series Expansion .....	181
4.4 Approximations with Continued Fraction .....	188
4.5 Summary Scaled Complementary Error Function.....	195
<b>5. Appendix .....</b>	<b>199</b>
5.1 Additional Copyrights, Permissions and/or Obligations .....	200
5.2 Listing of the Test Program .....	204
5.3 Tables for the Error Functions.....	218
5.3.1 Table Error Function $y = \operatorname{erf}(x)$ .....	218
5.3.2 Table Complementary Error Function $y = \operatorname{erfc}(x)$ .....	219
5.3.3 Table Scaled Complementary Error Function $y = \operatorname{erfcx}(x)$ .....	221
5.4 Lookup Function for “ <code>erfcxJohnson.f95</code> ” .....	223
5.5 Include File for “ <code>erfcxZaghoul.f95</code> ” .....	230
<b>6. List of References.....</b>	<b>235</b>