# Introduction

This book[1,2] presents original mathematical models of thermal and phase-transformation stresses in three-component materials, which consist of an isotropic matrix with isotropic cylindrical particles (whiskers) and an isotropic cylindrical envelope on the particle surface. The thermal stresses originate during a cooling process at the temperature $T < T_r$, and are a consequence of different thermal expansion coefficient of the particle and the matrix, where $T_r$ is relaxation temperature of the three-component material. The phase-transformation stresses originate at the phase-transformation temperature $T_{tq} < T_r$, and are a consequence of different dimensions of crystalline lattices, which are mutually transformed during the phase-transformation process in the cylindrical particle ($q = p$), the matrix ($q = m$), the cylindrical envelope ($q = e$).

The mathematical models are determined for a suitable model material system. The model system, which is required to correspond to real three-component materials with cylindrical particles, is characterized by the particle volume fraction $v_p$, the inter-particle distance $d$, the particle length $L_1$, the particle radius $R_1$, the envelope length $L_2$, the envelope radii $R_1$, $R_2$, where $R_1 < R_2$.

The mathematical models include these characteristics of the model material system, which represent structural parameters of real three-component materials with cylindrical particles. Accordingly, the mathematical models are applicable to three-component materials of the particle-matrix type (e.g., martensitic steel: martensitic cylindrical particles with a cylindrical envelope on the particle surface and an austenitic matrix), as well as to those with two types of crystalline grains with different material properties (e.g., dual-phase steel: the cylindrical grains $A$ with a cylindrical envelope on the surface of the grain $A$, and the grains $B$).

The thermal and phase-transformation stresses are derived within a suit-

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[1] This book was reviewed by the following reviewers:
Assoc. Prof. Ing. Robert Bidulský, PhD., visiting professor, Politecnico di Torino, Torino, Italy.

Prof. Ing. Daniel Kottfer, PhD., Alexander Dubček University of Trenčín, Faculty of Special Technology, Department of Mechanical Engineering, Trenčín, Slovak Republic.

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able coordinate system. The coordinate system is required to correspond to the shape of the cylindrical particles and the cylindrical envelope.

The mathematical models results from mechanics of an elastic continuum, and result in different mathematical solutions for the thermal and phase-transformation stresses. The mathematical solutions are required to consider constraints of the elastic continuum, which are defined by mathematical boundary conditions. Due to these different mathematical solutions, the principle of minimum elastic energy is considered.

As usual in mechanics of a solid continuum, stresses in a solid continuum are investigated within an infinitesimal part of the solid continuum at an arbitrary point. The position of this infinitesimal part is determined by a suitable system of coordinates. i.e., by the cylindrical coordinates. The shape of the infinitesimal part results from that of the solid continuum, i.e., a cylindrical infinitesimal part is considered.

The influence of the solid continuum on the cylindrical infinitesimal part is represented by displacements and strains of the cylindrical infinitesimal part, as well as by stresses, which act on surfaces of the cylindrical infinitesimal part [1]. The relationships between the displacement and strains are defined by Cauchy's equations. The relationships between the stresses are defined by the equilibrium equations. The relationships between the strains and stresses are defined by Hooke's law. Cauchy's equations, the equilibrium equations, along with Hooke's law, represent fundamental equations of mechanics of an elastic solid continuum.

If a system of the fundamental equations exhibits different mathematical solutions, then the principle of minimum total potential energy [1] is required to be considered. Strictly speaking, such a mathematical solution is considered, which exhibits minimum total potential energy of the material model system. Finally, such a stress-strain state of the model material system is realized to correspond to this solution.

The mathematical procedures in this book, along with such a mathematical procedure, which is required within the determination of numerical values of the thermal and phase-transformation stresses, are analysed in Appendix (see Section A). The numerical determination is performed by a programming language.

Accordingly, the determination of the mathematical models of the thermal and phase-transformation stresses includes:

- the definition of such a three-component model material system, which corresponds to real three-component materials, and is characterized by the particle volume fraction $v_p$, the inter-particle distance $d$, the particle length $L_1$, the particle radius $R_1$, the envelope length $L_2$, the envelope radii $R_1$, $R_2$, where $R_1 < R_2$ (see Section 1.1),

- the definitions of such a coordinate system and an infinitesimal cylindrical part of the model material system, which correspond to the shape of the cylindrical particle (see Section 1.2),
- the definition of intervals of coordinates (see Section 1.2),
- the analysis of a reason of the thermal and phase-transformation stresses (see Section 1.3),
- the analysis of displacements of the infinitesimal part (see Section 2.1,
- the definition of the fundamental equations for the infinitesimal part (see Sections 2.1–2.3),
- the determination of a system of differential equations, which results from the fundamental equations, and is solved by the method of separation of variables (see Section 2.4),
- the analysis of energy of the model material system with respect to the principle of minimum total potential energy (see Section 2.5) [1].
- the determination of the mathematical boundary conditions, which define the constraints of the model material system (see Section 2.6),
- the determination of the elastic energy of the model material system (see Section 2.7),
- the determination of two mathematical models in Chapters 3, 4 with final formulae for the thermal and phase-transformation stresses in the cylindrical particle and the matrix,
- the analysis of the mathematical procedures, which are considered within the determination of the mathematical models of the thermal and phase-transformation stresses in the three-component materials with the cylindrical particles (whiskers) (see Section A).

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Chapter 1

# Matrix-Whisker Composite

## 1.1 Model Material System

Figure 1.1 shows a model material system, i.e., the multi-particle-envelope-matrix system, which corresponds to real three-component materials [2].

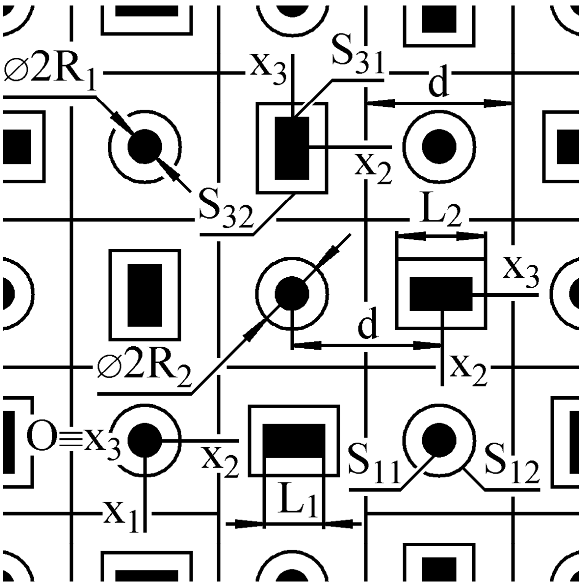


Figure 1.1: The multi-particle-envelope-matrix system is imaginarily divided into identical cubic cells with the dimension $d$ and with a central cylindrical particle. The cylindrical particles with a cylindrical envelope on the particle surface are periodically distributed in the infinite matrix, where $L_1$, $L_2$ and $R_1$, $R_2$ are length and radii of the particle, envelope, respectively. The matrix is infinite along the axes $x_1$, $x_2$, $x_3$ of the Cartesian system $(Ox_1x_2x_3)$, where $O$ is identical with a centre of the cylindrical particles. The surfaces $S_{11}$, $S_{12}$ and $S_{31}$, $S_{32}$ of the particle-envelope, matrix-envelope boundaries with the normal $x_1$ and $x_3$, respectively, and with the surface area $S_{11} = 2\pi R_1 L_1$, $S_{12} = 2\pi R_2 L_2$ $S_{31} = \pi R_1^2$, $S_{32} = \pi R_2^2$, respectively.

The multi-particle-envelope-matrix system consists of isotropic cylindrical particles with an isotropic cylindrical envelope on the particle surface, which are periodically distributed in an isotropic infinite matrix with the inter-particle