1 Introduction

1.1 Motivation

For the design of bar-shaped timber members, the stability verifications for flexural buckling and lateral torsional buckling are essential. Such members are usually loaded by a combination of axial forces and bending moments. In the design, the longitudinal stresses from axial forces and bending moments, the shear stresses from shear force and torsion, and the deformations are compared with the corresponding resistances.

The equations for stability verification of columns in EN 1995-1-1 [57] and FprEN 1995-1-1 [69] were derived on the basis of extensive experimental, analytical, and numerical investigations on softwood columns and provide reliable and economic results for their design within the safety concept of the Eurocodes. But only a limited number of studies were conducted on columns made of other European wood species or wood products, and no studies were known on the novel material beech laminated veneer lumber GL75.
For lateral torsional buckling, only a few experimental investigations were known, even fewer on full-size softwood glued laminated timber beams, and none on softwood glued laminated timber beam-columns loaded by combined axial compression and bending. The lateral torsional buckling verifications in EN 1995-1-1 [57] and FprEN 1995-1-1 [69] are therefore mainly based on investigations with analytical models, which were not sufficiently validated by full-scale lateral torsional buckling tests yet. Thus, e.g. the background of the nonlinear $N_{x,c}$-$M_{y,1}$-interaction of the k_c-k_m-method was the subject of extensive discussions in the revision of EN 1995-1-1 [57] as there were no experimental results for validation available. Furthermore, the main part of the k_m-method for lateral torsional buckling verification in EN 1995-1-1 [57] is a pure regression model that does not fully consider all relevant parameters. Finally, there were no systematic investigations of geometrical and structural imperfections of timber beams, and the imperfection assumptions of the flexural buckling and lateral torsional buckling verifications in EN 1995-1-1 [57] are inconsistent.

Given these known shortcomings of the lateral-torsional buckling verification method in EN 1995-1-1 [57] and the inconsistencies of the flexural buckling and lateral torsional buckling verification methods, it is likely that the verification methods in EN 1995-1-1 [57] provide partly uneconomical results of varying reliability.

Within three research projects, conducted at the Institute of Structural Design of the University of Stuttgart from 2019 to 2024, systematic measurements of geometrical imperfections of timber beams were carried out in the research project DIBt - ZP 52-5-13.194 [107], flexural buckling tests on beech laminated veneer lumber columns in the research project RP 7-1 of the Cluster of Excellence IntCDC [109], and lateral torsional buckling tests on softwood glued laminated timber beam-columns in the research project IGF No. 21285 N [108]. This thesis combines the results of these three research projects.

1.2 Objectives

The main objective of this thesis was to formulate mechanically sound, consistent verification methods for imperfection-sensitive timber members with combined bending and axial compression, i.e. timber beam-columns. These methods should be capable of considering all relevant effects for both softwood glued laminated timber and novel timber products, including geometrically and materially nonlinear behaviour, geometrical and structural imperfections, size effects, and long-term behaviour.

This resulted in the sub-goals: (i) to experimentally investigate the lateral-torsional buckling behaviour of softwood glued laminated timber beams with combined bending and axial compression; (ii) to experimentally investigate the flexural buckling behaviour of columns made of the novel material beech laminated veneer lumber GL75; (iii) to create a representative database of geometrical imperfections of timber beams; (iv) to develop analytical and numerical models to simulate the buckling behaviour of timber beam-columns and to validate these models with before mentioned experimental results; (v) to develop a model for investigating the scattering structural imperfections of softwood glued laminated timber beams; (vi) to carry out numerical parameter studies on the load-bearing behaviour of imperfection-sensitive timber beam-columns with combined bending and axial compression; and (vii) to evaluate the buckling verifications in FprEN 1995-1-1 [69] and to further develop them for a more reliable, consistent, and economical design of timber beam-columns.

The focus was on lateral torsional buckling. However, as there is a fluent transition between flexural buckling and lateral torsional buckling and a separation is not necessarily mechanically reasonable, the stability phenomena were always studied holistically.

The overarching objective of this thesis was to link the flexural buckling and lateral-torsional buckling of timber members on the basis of extensive experimental, analytical, and numerical calculations and thereby establish the basis for a comprehensive understanding of the buckling phenomena of timber members. To this end, the emphasis was on examining elementary cases, such as single-span beams with a constant rectangular cross-section. This in-depth understanding should provide a sound foundation on which subsequent research can build on and expand.

1.3 Scope

The investigations in this thesis cover: (i) single span bar-shaped members with a constant rectangular cross-section and fork supports; (ii) loading by axial compressive forces and uniaxial bending moments; (iii) softwood glued laminated timber and beech laminated veneer lumber GL75 according to EN 14080 [54] and ETA-14/0354 [63]; (iv) ultimate limit state verifications of stresses in grain direction; and (v) short-term and long-term load-bearing behaviour. The majority of the investigations refer to service class 1, but methods for considering the creep deformation in service classes 2 and 3 are discussed.

Concerning the influence of eigenmodes, eigenvalues, and equivalent member lengths, it is referred to the relevant literature. The verifications in the event of fire or under cyclic loading and the verifications of adjacent members such as fork supports, bracings, and connections are not discussed.

1.4 Methodology

The most important methods utilised in this work were (i) on-site measurements and laboratory experiments, (ii) the finite element method and finite element based design according to FprEN 1993-1-14 [68] and Töpler and Kuhlmann [162], and (iii) the Monte Carlo method.
The on-site measurements were performed to analyse the real geometrical imperfections of timber members in buildings and to create a database for the numerical parameter studies. The laboratory experiments were conducted to investigate which physical phenomena have a relevant influence on the load-bearing capacity of imperfection-sensitive timber beam-columns and to collect data for the validation of the prediction models.
The finite element method was applied to model the lateral torsional buckling and flexural buckling behaviour of timber beam-columns. Finite element based design embraces newly developed methodologies and design concepts that are integrated into the 2^{nd} generation of the Eurocodes, see FprEN 1993-1-14 [68]. These enable the generation of numerical test results by means of a verified and validated finite element model and the application of these results for design.
Furthermore, analytical models can be used to describe stability phenomena, see e.g. Hörsting [90] and Section 4. As significant simplifications have to be made in analytical models, e.g. a constant bilinear plasticising over the member length and beam theory, there are significant differences between these models and the experimentally and numerically observed stability behaviour, e.g. the variable non-linear plasticising over the member length and the cross-sectional warping, see Sections 3.3.3.2, 3.4.3.2, and 6.2.1. While analytical models are useful for building practice, they were not suitable for scientifically investigating the load-bearing behaviour of imperfection-sensitive timber members in this thesis. For this reason, this thesis refrains from discussing the refined analytical model of [90] that was presented in earlier publications, see Köppel et al. [104] and Lukas et al. [120].
The Monte Carlo method was utilised to consider the influence of scattering geometrical imperfections and scattering material parameters on the load-bearing capacity of slender timber beam-columns in finite element analyses.

1.5 Outline and Overview

The structure of this thesis follows the research questions, see Section 1.2, and the methods used, see Section 1.4. The basis of the thesis is the discussion of the state of the art in Section 2, in which the main research questions are developed and the knowledge, data, models, and methods required for this thesis are compiled. The imperfection measurements, the flexural buckling tests on beech laminated veneer lumber columns, and the lateral torsional buckling tests on softwood glued laminated timber beam-columns are described in Section 3. Section 4 discusses analytical investigations on the mechanical background of the stability phenomena. The numerical models, the implementation of material models and scattering geometrical and structural imperfections, the model verification and validation, and the numerical parameter studies are presented in Section 5. An overview of the finite element based design methods in FprEN 1993-1-14 [68] and Töpler and Kuhlmann [162] is provided in the same section. The results of the numerical parameter studies on equivalent imperfections and the $N_{x,c}$-$M_{y,1}$-interaction are given in Section 6, compared with the experimental and analytical investigations, and discussed in the context of the literature. The design model, which was developed based on these results, is presented in Section 7. The thesis closes with the conclusions and outlook in Section 8. The annexes contain the relevant measurement results and background information on the numerical models.

2 State of the art

2.1 General

Whether stability phenomena can occur for straight members depends on whether the unavoidable imperfections and deformations lead to significant additional internal forces. In the following, therefore, the terms *imperfection-sensitive members* and *non-imperfection-sensitive members* are used if all stability phenomena are addressed. The stability phenomena of bar-shaped members can be verbally subdivided into *flexural buckling*, *torsional buckling*, and *Lateral Torsional Buckling* (LTB). However, in reality, the transitions between these phenomena are fluid, and often a clear distinction is hardly possible. Since torsional buckling is not relevant for the closed cross-sections typical in timber construction, it is not discussed.

The characteristic deformation behaviour of imperfection-sensitive timber members with closed cross-section loaded by (combined) bending and axial compression, also referred to as *imperfection-sensitive timber beam-columns*, is illustrated in Figure 2.1. In addition, the corresponding internal forces $M_{x,2}$, $M_{y,2}$, and $M_{z,2}$ are specified. These internal forces are based on the equations given in Sections 2.2.1 and 2.2.2, with the associated simplifying assumptions. A distinction is made between (i) loading by axial compressive forces $N_{x,c}$, bending moments $M_{y,1}$, or $N_{x,c}$-$M_{y,1}$-interaction; (ii) bow imperfections in y- or z-direction, e_y or e_z, or twist imperfections e_θ; and (iii) cross-sections with $H \approx B$ or $H \gg B$. Cases for which no stability phenomenon occurs are greyed out. The slenderness and the load are each sufficiently high for a stability phenomenon to occur.
A basic distinction can be made between the stability behaviour of (i) members with cross-sections of similar height and width, $H \approx B$, and (ii) members with cross-sections of significantly larger height than width, $H \gg B$.
For slender cross-sections with $H \approx B$, stability behaviour can occur for $N_{x,c}$ combined with bow imperfections $e_{y/z}$ or generally for $N_{x,c}$-$M_{y,1}$-interaction. The particularity of the latter case is that the stability behaviour due to $N_{x,c}$ is caused by the deformations w_1 due to bending $M_{y,1}$, and no imperfections are necessary. Stability behaviour for approximately square cross-sections is therefore always caused by $N_{x,c}$ combined with bow imperfections or deformations. Pure $M_{y,1}$ or twist imperfections e_θ do not cause any stability behaviour.
For slender cross-sections with $H \gg B$, stability behaviour can only occur for $N_{x,c}$ combined with bow imperfections in the y-direction e_y and for $M_{y,1}$ combined with bow imperfections in the y-direction e_y or twist imperfections e_θ. Bow imperfections in the z-direction e_z or $N_{x,c}$ combined with twist imperfections e_θ do not cause any stability behaviour.
For approximately square cross-sections with $H \approx B$, there are no significant additional twists due to stability behaviour. Only additional flexural deformations v_2 and w_2 in the same plane as the imperfections and $M_{y,1}$ are caused. This case is therefore known as *flexural buckling*. For slender cross-sections with $H \gg B$ loaded by $M_{y,1}$ (and $N_{x,c}$) there are no significant deformations in the z-direction, the strong member axis, but only twists and lateral deformations in the y-direction due to stability behaviour. This case is therefore

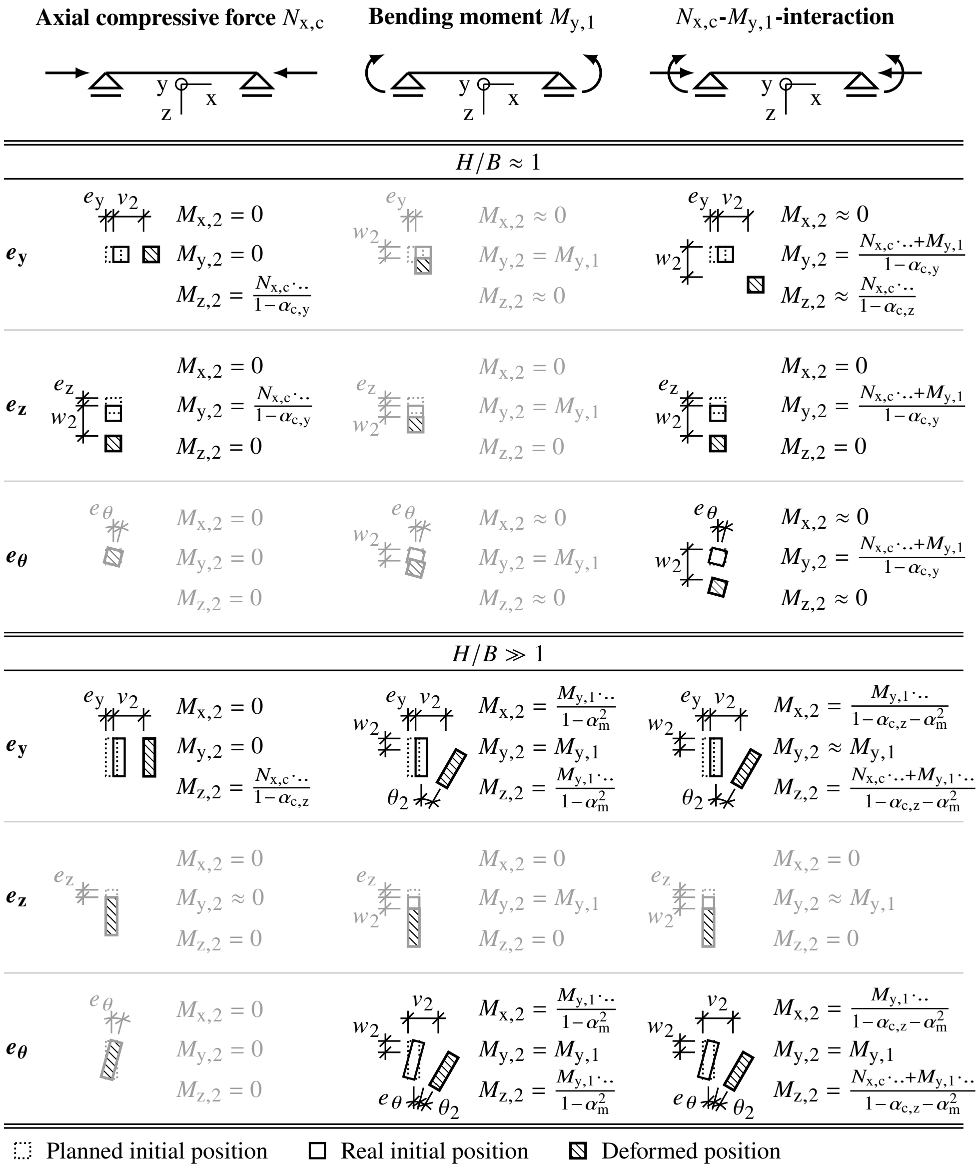

Figure 2.1: Deformation behaviour and corresponding internal forces of imperfection-sensitive timber members with closed cross-section loaded by compression and bending at midspan; top: structural systems; middle: deformation behaviour of cross-sections with $H \approx B$; bottom: deformation behaviour of cross-sections with $H \gg B$; greyed out are cases for which no stability behaviour occurs.

known as *Lateral Torsional Buckling* (LTB). For slender cross-sections with $H \gg B$, loaded only by $N_{x,c}$ and with pure bow imperfections e_y, no twist deformations occur, and the case can be categorised as flexural buckling.
The additional internal forces $M_{x,2}$, $M_{y,2}$, and $M_{z,2}$ due to stability behaviour occur analogously to the additional deformations θ_2, v_2, and w_2.
If the correct stability phenomenon is determined based on the cross-sectional slenderness, loading, and imperfections according to Figure 2.1, this will also reveal which additional deformations and internal forces should be considered in the design.

In building practice, there is practically always a combination of the three basic imperfections e_y, e_z, and e_θ due to growth-related material inhomogeneities as well as production- and assembly-related tolerances. In addition, load-eccentricities that are not planned but which are practically unavoidable, can also lead to significant additional internal forces. Thus, there is practically always a more or less pronounced combination of several of the stability phenomena illustrated in Figure 2.1. For building practice, it is therefore important to define limit criteria for which pure flexural buckling and pure LTB can be assumed and simplified design equations can be applied.

The stability behaviour of imperfection-sensitive timber beam-columns is characterised by a nonlinear increase in deformations and internal forces as the loading increases, see Figure 2.2. The load-bearing capacity of such members is governed either by a member failure due to exceeding the strength (C1), by the maximum of the load-deformation curve (C2), or by a deformation limit criterion (C3). Phenomenologically, this nonlinear stability behaviour can be subdivided into the *geometrically nonlinear load-bearing behaviour* and the *materially nonlinear load-bearing behaviour*.

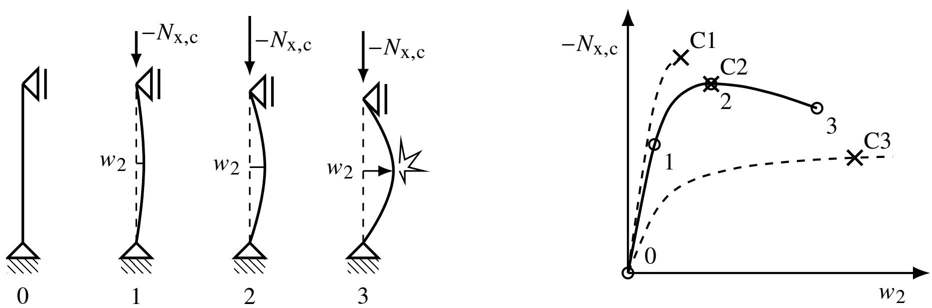

Figure 2.2: Load-deformation behaviour of a timber column with flexural buckling; the flexural buckling behaviour can be caused by geometrical imperfections, load-eccentricities, etc.; left: structural system and three stages of deformations, with 0 being the unloaded column, 1 being the moderately loaded column with a small geometrically nonlinear lateral deformation w_2, 2 being the column at maximum load with a moderate geometrically nonlinear deformation w_2, and 3 being the column at tensile failure due to a significant geometrically nonlinear deformation w_2; right: three possible load-deformation curves with the one of the column on the left as a solid line.

In Sections 2.2 and 2.3, the geometrically nonlinear behaviour and the materially nonlinear behaviour of timber beam-columns are discussed. Within geometrically nonlinear behaviour, flexural buckling and LTB are considered as special cases, and imperfections are discussed as an essential influencing parameter. Concerning the influence of eigenmodes, eigenvalues, and effective lengths, it is referred to the relevant literature. Section 2.3 about materially nonlinear behaviour examines the relevant tensile, compressive, and shear behaviour in grain direction. The failure behaviour of wood in tension, compression, and shear in grain direction is described in Section 2.4. Section 2.5 provides a brief excursus on long-term behaviour. An overview of the stability design methods for timber beam-columns in EN 1995-1-1 [57] and FprEN 1995-1-1 [69] is given in Section 2.6. Section 2.7 discusses methods for simulating scattering material properties of timber. The section closes with the summary and discussions in Section 2.8.
In most cases, the focus is on timber-related aspects, but some topics are also discussed across different materials.

2.2 Geometrically nonlinear behaviour

2.2.1 Flexural buckling

2.2.1.1 General

If the loading of imperfection-sensitive timber (beam-) columns by $N_{\mathrm{x,c}}$ (and $M_{\mathrm{y,1}}$) leads to significant deformations in the xy- and/or xz-plane and bending moments about the y- and/or z-axis but not twisting or torsional moments, this is referred to as *flexural buckling*, see Figures 2.1 and 2.2. Such members are subsequently referred to as *columns*. Their load-bearing behaviour is characterised by a nonlinear increase in deformations and internal forces, see Figures 2.2 and 2.3.
The experimentally determined load-deformation curve of an eccentrically loaded GL 24h column with dimensions $140 \cdot 160 \cdot 2300\ \mathrm{mm}^3$ from Frangi and Theiler [71] is presented in Figure 2.3. The x-axis displays the horizontal deformation at midspan w, and the y-axis exhibits the axial compressive force $N_{\mathrm{x,c}}$. The behaviour was nonlinear from the beginning. After the load-bearing capacity $N_{\mathrm{x,c,R}}$, i.e. the peak, was reached, the horizontal deformations further increased at a moderate load decrease.
The flexural buckling behaviour of timber columns is influenced by the member geometry, the structural system, the actions, the bow imperfections, the sway imperfections, and the material properties, in particular the elastic modulus, the compressive strength, the compressive plasticity, and the tensile strength in grain direction. For timber products with low shear stiffness and strength, e.g. CLT due to its rolling shear behaviour, see Narcy et al. [123], the shear strength and stiffness can influence the flexural buckling behaviour. Rautenstrauch and Becker [133] demonstrated that the influence of the shear stiffness, and therefore presumedly also the influence of the shear strength, is negligible for softwood *Glued Laminated timber* (GL) columns.

Extensive flexural buckling tests on *Structural Lumber* (SL) and GL columns made of softwood were reported in literature, e.g. earlier by Buchanan et al. [25] and Zahn [182], and later by Frangi and Theiler [71], Lam and Oh [110], Steiger and Fontana [150], and Zahn and Rammer [185]. Ehrhart [49] carried out flexural buckling tests on beech GL columns and demonstrated that these behave similarly to softwood columns but observed higher compressive plasticising in grain direction and therefore reduced load-bearing capacities compared to softwood GL columns. No experiments on beech *Laminated Veneer Lumber* (LVL) GL75 columns were known.

In the following, the physical behaviour of flexural buckling is illustrated, and prediction models for describing the observed load-deformation behaviour are discussed.

2.2.1.2 Slenderness dependent load-bearing and failure behaviour

The experimental studies on flexural buckling by Buchanan et al. [25], Frangi and Theiler [71], Lam and Oh [110], Steiger and Fontana [150], Zahn [182], and Zahn and Rammer [185] demonstrated the decreasing load-bearing capacity of timber columns with axial compression with increasing slenderness due to flexural buckling, see Figure 2.4. This curve of the reduction in load-bearing capacity of columns due to flexural buckling plotted over the slenderness is often referred to as *buckling curve*. The slenderness increases with increasing length and decreasing cross-sectional height or width in the direction of flexural buckling, see Equation (2.11).
With increasing slenderness, the member failure mode changes, as described below according to Buchanan [24] and [71]. While the load-bearing capacity of short, stocky columns depends on the compressive strength in grain direction and compressive failure occurs, see also Section 2.4.3, stability failure occurs for long, slender columns. For the latter, the

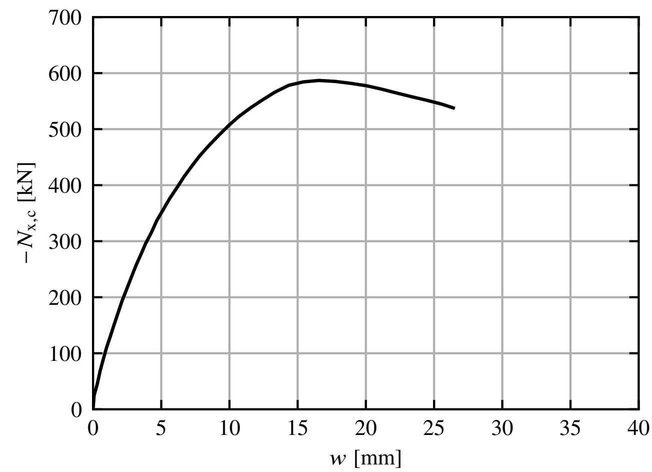

Figure 2.3: Experimentally determined flexural buckling load-deformation curve of an eccentrically loaded GL 24h column with dimensions of $140 \cdot 160 \cdot 2300$ mm^3; axial compressive force $N_{x,c}$ plotted over the horizontal deformation at midspan v; from Frangi and Theiler [71].

load-bearing capacity approaches the critical load and depends on the structural system, the column dimensions, and the elastic modulus in grain direction, or, in summary, the stiffness, see Figure 2.4. With displacement-controlled loading, the load-drop in the load-deformation curve after the peak is exceeded can be mapped, see C2 in Figure 2.2 and Figure 2.3, and columns can be loaded until material failure in tension, which causes member failure. The load drop after the peak is caused by the reduced member stiffness due to compressive plasticising in grain direction. If the load is applied force-controlled, sudden, severe deformations occur when the maximum is reached, which has given the phenomenon the name *stability failure*. This is instantaneously followed by a material failure in tension, causing member failure. For columns with medium slenderness, a transition between compressive failure and stability failure takes place, and the load-bearing capacity is influenced by both compressive strength and stiffness and, additionally, the imperfections, see Figure 2.4. The reduction of the load-bearing capacity compared to the cross-sectional resistance and the critical load, see Figure 2.4, is caused by the geometrically nonlinear behaviour and the materially nonlinear behaviour (compressive plasticising in grain direction) and hence increased internal forces, see Zahn [183] and [185].

As Zahn [183] noted, it is therefore reasonable to utilise a strength criterion for stocky members and a stability criterion for slender members in the design, given the slenderness-dependent failure mode.

In addition to the described member failure behaviour, Ehrhart [49], Frangi and Theiler [71], and Steiger and Fontana [150] reported a local compressive failure in the form of local fibre buckling, which led to a reduction in stiffness and lower load-bearing capacities but not to member failure.

There is no universal definition of where the boundaries between stocky, medium-slender, and slender columns are. In Figure 2.4, a deviation of 10% from the cross-sectional resistance and the critical load was utilised as a criterion, which is reasonable in the

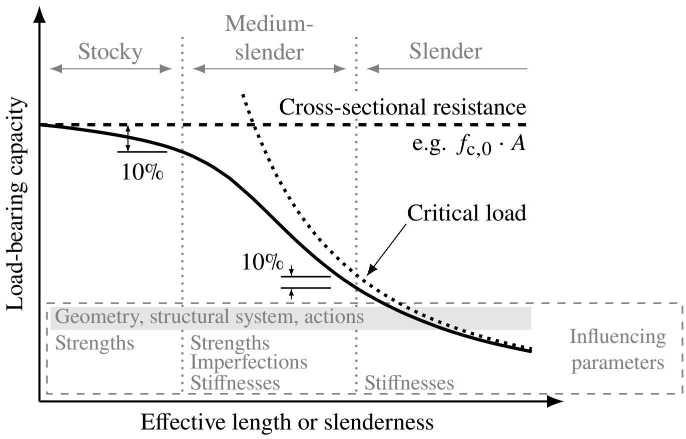

Figure 2.4: Load-bearing capacity of timber beams or columns (solid line) plotted over the effective length or slenderness; with critical load and cross-sectional resistance; illustration of slenderness ranges and associated influencing parameters.