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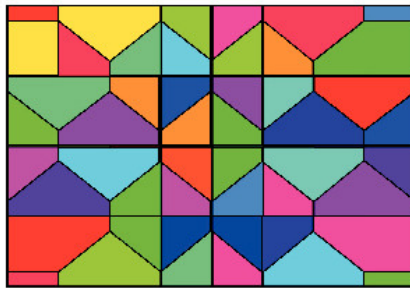
## **Model-Based Predictive Control of Electric Drives**

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 Cuvillier Verlag Göttingen

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## 2 Field-oriented control

In drive applications a DC machine has the great advantage that it has separate field and armature windings. Thus both of its most important states, namely flux and torque, can be controlled independently of each other by the field winding current and by the armature current, respectively. The commutator acting as a mechanical converter makes sure that the current distribution of the rotor is always positioned properly, so that the rotor current and the main flux are always in quadrature to each other. Asynchronous machines, on the other hand, which offer great advantages because of their maintenance free operation, have only one active winding, the stator winding. The rotor winding is either short circuited (squirrel-cage motor) or connected with variable resistors (slip ring rotor). Consequently, both flux and torque in the machine have to be raised via the stator windings. Splitting the control structure in a simple way into flux and torque control is not possible. However, for high drive performance, both states must be controlled independently of each other [57].

A remedy for solving this problem is the use of the so-called *field-oriented control* [79], which can be described with the help of *space vector representation* [70]. The basic idea behind this representation is that the three-phase current system can be represented by a three-axis coordinate system as shown in figure 2.1(a). Unfortunately, the three axes  $a$ ,  $b$  and  $c$  are not linearly independent of each other, a fact that complicates a mathematical description of the actions in a three-phase system. Hence, an alternative two-phase system with two axes linearly independent of each other is constructed. Figure 2.1(b) shows this equivalent system. To obtain an easy representation of the two phase quantities, usually a complex coordinate system is selected. Thus, the corresponding quantities from one coordinate system, e. g. the stator voltage quantities, can be transformed into another coordinate system with the help of the transformation equations (2.1) to (2.4).

$$\mathbf{u}_s = \frac{2}{3} (u_{sa} + \mathbf{a} \cdot u_{sb} + \mathbf{a}^2 \cdot u_{sc}) \quad \text{in which} \quad \mathbf{a} = e^{j \frac{2\pi}{3}} \quad (2.1)$$

$$u_0 = \frac{1}{3} (u_{sa} + u_{sb} + u_{sc}) \quad (2.2)$$

$$u_{s\alpha} = \Re\{\mathbf{u}_s\} = \frac{2}{3} (u_{sa} - \frac{1}{2} u_{sb} - \frac{1}{2} u_{sc}) \quad (2.3)$$

$$u_{s\beta} = \Im\{\mathbf{u}_s\} = \frac{1}{\sqrt{3}}(u_{sb} - u_{sc}) \quad (2.4)$$

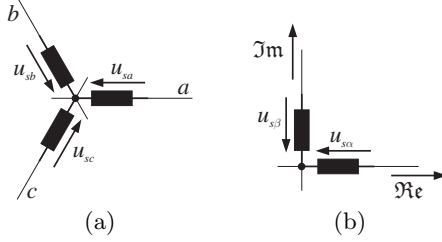


Figure 2.1: Three-phase system and alternative two-phase system

The idea of field-oriented control is based on the fact that the three stator currents  $i_{sa}$ ,  $i_{sb}$  and  $i_{sc}$  in the three stator windings are first transformed into a complex state space vector  $\mathbf{i}_s$ . Therefore, a transformation analog to (2.1) is used for the stator currents. Then, an induction machine can be described in an arbitrary reference coordinate system, rotating with  $\omega_k$ , with the following equations [70, 71]:

$$\mathbf{u}_s = r_s \cdot \dot{\mathbf{i}}_s + \frac{d\boldsymbol{\psi}_s}{d\tau} + j\omega_k \boldsymbol{\psi}_s \quad (2.5)$$

$$0 = r_r \cdot \dot{\mathbf{i}}_r + \frac{d\boldsymbol{\psi}_r}{d\tau} + j(\omega_k - \omega) \boldsymbol{\psi}_r \quad (2.6)$$

$$\boldsymbol{\psi}_s = l_s \cdot \dot{\mathbf{i}}_s + l_h \cdot \dot{\mathbf{i}}_r \quad (2.7)$$

$$\boldsymbol{\psi}_r = l_r \cdot \dot{\mathbf{i}}_r + l_h \cdot \dot{\mathbf{i}}_s \quad (2.8)$$

For current control, as it is implemented in the internal loop of a cascade control structure, it is an adequate solution to choose  $\mathbf{i}_s$  and  $\boldsymbol{\psi}_r$  as state variables. After reformulating according to [56, 57], the following differential equations can be obtained:

$$\dot{\mathbf{i}}_s + \tau_\sigma' \frac{d\dot{\mathbf{i}}_s}{d\tau} = -j\omega_k \tau_\sigma' \dot{\mathbf{i}}_s + \frac{k_r}{r_\sigma} \left( \frac{1}{\tau_r} - j\omega \right) \boldsymbol{\psi}_r + \frac{1}{r_\sigma} \mathbf{u}_s \quad (2.9)$$

$$\dot{\boldsymbol{\psi}}_r + \tau_r \frac{d\boldsymbol{\psi}_r}{d\tau} = -j(\omega_k - \omega) \tau_r \boldsymbol{\psi}_r + l_h \cdot \dot{\mathbf{i}}_s \quad (2.10)$$

in which  $\tau_s = \frac{l_s}{r_s}$ ,  $\tau_r = \frac{l_r}{r_r}$ ,  $\sigma = 1 - \frac{l_h^2}{l_s l_r}$ ,  $l_s' = \sigma l_s$ ,  $l_r' = \sigma l_r$ ,  $k_r = \frac{l_h}{l_r}$ ,  $k_s = \frac{l_h}{l_s}$ ,  $\tau_s' = \frac{\sigma l_s}{r_s}$ ,  $\tau_r' = \frac{\sigma l_r}{r_r}$ ,  $r_\sigma = r_s + r_r \cdot k_r^2$  and  $\tau_\sigma' = \frac{\sigma l_s}{r_\sigma}$ . Figure 2.2 shows the

corresponding signal flow graph. In this work, double lines represent complex values.

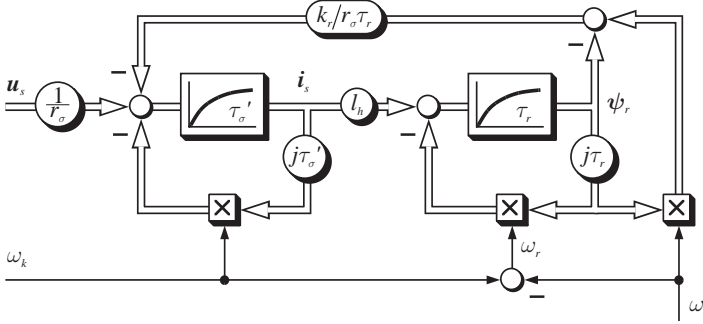


Figure 2.2: Complex machine model

Now a coordinate system rotating with the speed of the rotating field is selected as the base coordinate system with the real axis aligned in the direction of the field. With the help of this new coordinate system the stator current space vector  $\mathbf{i}_s$  can be divided into a torque-producing component  $i_{sq}$  and into a flux-producing component  $i_{sd}$ . As both of these current components can be controlled independently of each other, dynamic control similar to a shunt-wound DC machine can be achieved. If the complex equations (2.9) and (2.10) are divided into their real and imaginary parts, four scalar equations which describe the dynamic behavior of the induction machine can be obtained. Because of field orientation  $\omega_k = \omega_s$  is set; for the same reason  $\psi_{rq} = 0$  is set. The appropriate signal flow graph can be seen in figure 2.3.

$$i_{sd} + \tau_\sigma \frac{di_{sd}}{d\tau} = \omega_s \tau_\sigma' i_{sq} + \frac{k_r}{r_\sigma \tau_r} \psi_{rd} + \frac{1}{r_\sigma} u_{sd} \quad (2.11)$$

$$i_{sq} + \tau_\sigma \frac{di_{sq}}{d\tau} = -\omega_s \tau_\sigma' i_{sd} - \frac{k_r}{r_\sigma} \omega \psi_{rd} + \frac{1}{r_\sigma} u_{sq} \quad (2.12)$$

$$\psi_{rd} + \tau_r \frac{d\psi_{rd}}{d\tau} = l_h i_{sd} \quad (2.13)$$

$$0 = -(\omega_s - \omega) \tau_r \psi_{rd} + l_h i_{sq} \quad (2.14)$$

Equation (2.14) is of no significance for the control and hence, it is not considered for further analysis. However, it describes the condition for field ori-

entation, i. e. the position angle of the field coordinate system with respect to the fixed stator coordinate frame can be determined with it. Therefore equation (2.14) is resolved for the slip or rotor frequency<sup>1</sup>  $\omega_r = \omega_s - \omega$  and after doing this, the following equation can be obtained:

$$\omega_r = (\omega_s - \omega) = \frac{l_h \dot{i}_{sq}}{\tau_r \psi_{rd}} \quad (2.15)$$

The value of  $\psi_{rd}$  required for the calculation of the above relationship can be obtained from the differential equation (2.13). By adding the calculated slip speed  $\omega_r$  to the mechanical rotating speed  $\omega$  of the rotor, the stator speed  $\omega_s$  results. The integration of  $\omega_s$  provides the field angle  $\delta$ . The overall signal flow graph of the induction machine can be seen in figure 2.4.

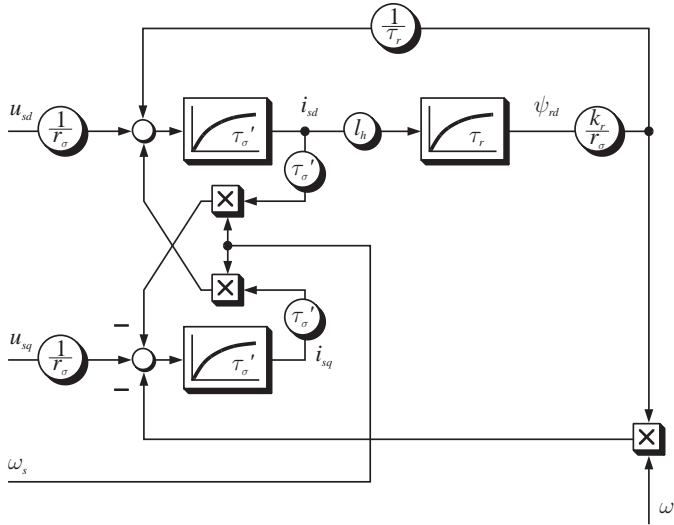


Figure 2.3: Scalar machine model (field coordinates)

<sup>1</sup> The rotor frequency is *not* the mechanical rotating frequency but the frequency of the currents flowing in the rotor.

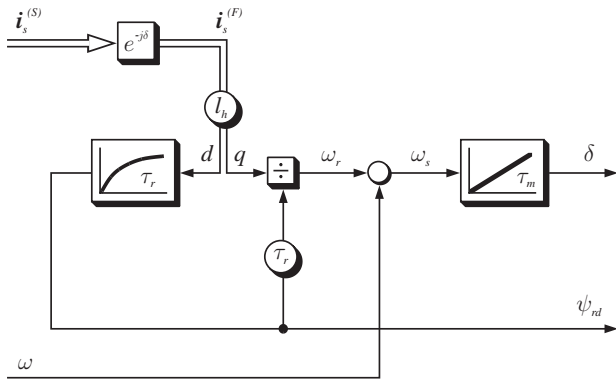


Figure 2.4: Rotor model