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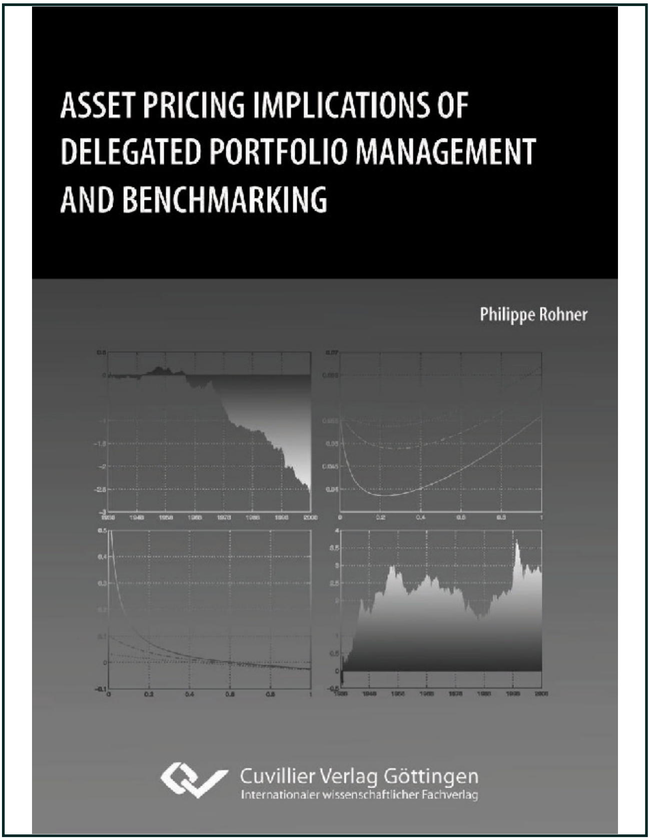
# Asset Pricing Implications of Delegated Portfolio Management and Benchmarking

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# Chapter 2

# Asset Allocation with Benchmark

Relative wealth concerns are important determinants of individual investor behavior. When valuing wealth, investors do not only care about their own wealth, but also about their wealth compared to others. The satisfaction with their own situation depends on how much others have earned. Indeed, a growing stream of literature in financial economics highlights the importance of relative wealth concerns. Frey and Stutzer (2002) establish the concept of happiness research in economics. Abel (1990) and Gali (1994) were the first to model relative wealth concerns in asset pricing. They studied portfolio decisions in the presence of consumption externalities where agents have preferences defining their consumption as well as the average consumption in the economy, and they introduce the notion of "keeping up with the Joneses" (KUJ) preferences. They show that the presence of KUJ preferences lead to a higher equilibrium risk premium than predicted by the CAPM. Garcia and Strobl (2009) formulate the concept of KUJ preferences in a noisy rational expectations equilibrium economy. Gomez, Pristley, and Zapatero (2009) study the cross-sectional implications of relative wealth concerns.

Relative wealth concerns also play an important role in delegated asset management. When investing in mutual funds, individual investors are primarily concerned about the investment performance of their fund relative to a benchmark index. Empirical evidence suggests that money tends to flow into funds that perform well relative to a benchmark, see for example, Gruber (1996), Chevalier and Ellison (1997), Admati and Pfleiderer (1997), or Sirri and Tufano (1998). This gives asset managers an implicit incentive to maximize fund performance relative to a pre-specified benchmark index. The focus on relative fund performance increases the likelihood of attracting new money. Given that the remuneration contracts of most investment funds are based on a fixed percentage of assets under management, it is rational for asset managers to be concerned with relative fund performance. Basak, Shapiro, and Tepla (2006), Basak, Pavlova, and Shapiro (2007), and Basak, Pavlova, and Shapiro (2008) study the effect of implicit incentives in money management. Further, remunerating managers directly, based on relative performance, also gives them an explicit incentive to focus on relative wealth.

The use of a benchmark can also be considered an attempt to align manager behavior with investor incentives. When a manager is compensated on the basis of the portfolio's relative performance vis à vis a benchmark, he is inclined to minimize the risk of deviating from the benchmark. In this situation, the benchmark serves as the risk-free point of reference for the manager. The delegation contract further can be specified by imposing explicit constraints on the risk level of the manager's portfolio. However, constraining a manager's investment universe – either by maintaining a relative performance objective, or by explicitly imposing risk constraints – always causes an efficiency loss in the manager's use of private information. Despite the widespread use of benchmarks, academic literature generally questions the usefulness of benchmark-adjusted compensation. Admati and Pfleiderer (1997) find that benchmark-based delegation contracts are inconsistent with optimal risk-sharing, lead to suboptimal portfolios, and weaken the manager's incentives to expend effort.

This chapter introduces the basic concepts of asset allocation with benchmark orientation and discusses the implications on optimal portfolios. Further, the chapter analyzes different ways to develop a delegation contract by imposing implicit and explicit portfolio constraints.

## 2.1 Mean-Variance Analysis

Before I formulate the portfolio problem with benchmark orientation, I revisit the standard mean-variance theory of Markowitz (1952). Markowitz formulated a theory of optimal asset allocation by jointly managing risk and return. He was the first to offer a mathematical formulation for the concept of portfolio diversification and thereby established modern portfolio theory.

### 2.1.1 Basics of Utility Theory

Investors have utility $U(W_p)$ defined over final wealth $W_p$ of a portfolio of financial assets. $U(W_p)$ is assumed to be a strictly increasing and concave function of $W_p$. That is:

$$\frac{\partial U(W_p)}{\partial W_p} > 0, \tag{2.1}$$

$$\frac{\partial^2 U(W_p)}{\partial W_p^2} < 0. \tag{2.2}$$

The first restriction means that more is preferred to less, which is often referred to as nonsatiation in the economic literature. More wealth always leads to higher utility. The second restriction implies risk averse behavior. The curvature of the utility function determines the intensity of the investor's absolute risk aversion, defined as

$$\rho := \frac{\partial^2 U(W_p)/\partial W_p^2}{\partial U(W_p)/\partial W_p}, \tag{2.3}$$

where $\rho$ is the coefficient of absolute risk aversion. The coefficient of absolute risk aversion is the absolute dollar amount an investor is willing to pay to avoid a gamble of a certain absolute value. Given the choice between a fair gamble with expected payoff $G$ and a risk-free investment with certain payoff $P$, a risk averse investor turns down the gamble if $G \leq P$, since the gamble offers only risk without reward. To bear risks, a risk averse investor always demands a premium $G - P > 0$.

#### Exponential utility

To derive a tractable model of portfolio choice, a functional form for $U(W_p)$ must be imposed. One of the most commonly used utility functions is exponential utility with constant absolute risk aversion (CARA), defined as

$$U(W_p) = -\exp(-\rho W_p), \tag{2.4}$$

where $\rho$ is constant. Exponential utility produces simple results when returns are normally distributed. Further, the normality assumption is very tractable for portfolio analysis, as the weighted sum of normally distributed random variables is also normally distributed. The portfolio problem of a rational investor is how to maximize expected utility with respect to the optimal portfolio strategy. Assuming that portfolio wealth is normally distributed, $W_p \sim \mathcal{N}(\mathbb{E}[W_p], \mathrm{Var}(W_p))$, then, the expected utility under an exponential utility function (2.4) can be expressed as[1]

$$E[U(W_p)] = -\exp\left(-\rho\left(\mathbb{E}[W_p] - \frac{1}{2}\rho\,\mathrm{Var}(W_p)\right)\right). \tag{2.5}$$

Maximizing expected utility under an exponential utility function implies that the optimal investment strategy that maximizes $E[U(W_p)]$ is the one that maximizes the trade-off between expected value and variance of final wealth $W_p$.

The CARA setup has a serious shortcoming, however. The assumption of constant absolute risk aversion implies that absolute risk aversion is constant. However, the assumption of CARA is not in line with actual investor behavior. CARA suggests that a rich investor will be as concerned about a potential loss of \$ 1,000 as a poor person might be. In other words, all investors invest a fixed amount of wealth into risky assets, regardless of their level of wealth. This feature of exponential utility is clearly not in line with reality. It is commonly thought that absolute risk aversion should decrease in wealth. In contrast, power utility implies that absolute risk aversion is decreasing and that relative risk aversion, defined in terms of relative wealth, is constant. From an economic point of view, power utility is preferred to exponential utility. However, exponential utility produces very tractable results in a Gaussian setting and is therefore preferred in many applications and models with exogenous information acquisition. For this reason, I assume for the rest of the thesis that investors have exponential utility. I note that the limitations of CARA discussed above do not apply to the classic mean-variance theory presented in this chapter. It can be shown that power utility and other classic utility functions also give rise to a mean-variance optimization.

### 2.1.2 Notion of a Portfolio

Final wealth $W_p$ is the amount of wealth that results from investing initial wealth $W_0$ into a portfolio $\boldsymbol{q}$ of $N$ risky assets. $\boldsymbol{q} := [q_1, ..., q_N]$ is an $N \times 1$ vector of portfolio fractions. Asset returns are captured by an $N \times 1$ vector $\boldsymbol{r} := [r_1, ..., r_N]$. Hence, final wealth is

$$W_p = (1 + r_p)W_0, \tag{2.6}$$

where $r_p := \boldsymbol{q}'\boldsymbol{r}$ is the weighted return on the risky assets and $\boldsymbol{q}'\mathbf{1} = 1$. $\mathbf{1}$ is the $N \times 1$ vector of ones. In the presence of a risk-free asset with constant rate of return

[1] A detailed derivation of this result can be found, e.g., in Gollier (2001).

$r_f$, $\boldsymbol{q}'\mathbf{1}$ must not equal one. Thus, the investor invests a fraction of wealth $\boldsymbol{q}'\mathbf{1}$ into a combination of risky assets and the remainder $(1-\boldsymbol{q}'\mathbf{1})$ into the risk-free asset. Final wealth, then, is

$$W_p = W_0 + r_f W_0(1-\boldsymbol{q}'\mathbf{1}) + \boldsymbol{q}'\boldsymbol{r}W_0 = (1+r_f)W_0 + \boldsymbol{q}'(\boldsymbol{r} - r_f\mathbf{1})W_0 \tag{2.7}$$

The asset value of a financial portfolio at the end of the period is the sum of future wealth from a risk-free strategy and the excess return of a risky strategy $\boldsymbol{q}$. Risky asset returns are normally distributed,

$$\boldsymbol{r} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{2.8}$$

where $\boldsymbol{\mu} := [\mu_1, ..., \mu_N]$ is an $N \times 1$ vector of expected returns and $\boldsymbol{\Sigma}$ is a symmetric and positive semi-definite $N \times N$ covariance matrix. It follows that the expected return of the risky portfolio is $\mu_p := \boldsymbol{q}'\boldsymbol{\mu}$ and the the variance is $\sigma_p := \boldsymbol{q}'\boldsymbol{\Sigma}\,\boldsymbol{q}$.

### 2.1.3 The Markowitz Paradigm

A rational investor maximizes expected utility over final wealth with respect to the portfolio strategy $\boldsymbol{q}$,

$$\max_{\boldsymbol{q}} \;\; \mathbb{E}[U(W_p)] \tag{2.9}$$

Under exponential utility, expected utility maximization results in the classic mean-variance formulation[2]

$$\max_{\boldsymbol{q}} \;\; \boldsymbol{q}'\boldsymbol{\mu} - \frac{1}{2}\rho\,\boldsymbol{q}'\boldsymbol{\Sigma}\boldsymbol{q} \tag{2.10}$$

subject to

$$\boldsymbol{q}'\mathbf{1} = 1. \tag{2.11}$$

(2.11) is the constraint that the sum of all portfolio positions must equal one. In the traditional Markowitz portfolio problem, no position in the risk-free asset is allowed and the optimal portfolio is fully invested in risky assets. In many practical applications, (2.26) is solved subject to an additional constraint:

$$q_i \;\geq\; 0, \quad \forall\, i = 1, ..., N. \tag{2.12}$$

[2]The mean-variance problem can also be derived under other common utility functions such as quadratic utility $U(W_p) = aW_p - bW_p^2$ or power utility $U(W_p) = (W_p^{1-\rho} - 1)/(1-\rho)$, cf. Campbell and Viceira (2002).