



Hartmut Hafermann (Autor)
**Numerical Approaches to Spatial Correlations in
Strongly Interacting Fermion Systems**



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