Chapter 1

Introduction

Packing a set of objects into a container or into a set of containers has many applications in everyday life. Some of them are immediately evident, such as packing clothes into a suitcase before going on vacation or loading containers onto a ship; see Fig. 1.1. Some of them appear more hidden, *e.g.,* in *scheduling problems* where the task is to assign a set of jobs to a number of machines. If we model the machines as containers and the jobs as the objects which have to be packed, then minimizing the number of used containers, implies a schedule that uses a minimum number of machines.

Over the years, a huge variety of packing problems has been studied. They differ in the shape of the objects or in the shape of the container(s). Moreover, there are many additional constraints such as constraints on the order in which the objects have to be packed or the placement of the objects inside the container, *e.g.,* rotating objects might be allowed or not.

Popular packing problems are, *e.g.,* the *bin packing problem* and the *strip packing problem.* In the bin packing problem a set of one-dimensional objects with size less than one have to be packed into unit-sized containers. The task is to use as few containers as possible. The strip packing problem asks for a placement of rectangles inside a semi-infinite strip of width one, minimizing the height used. These problems have been studied in one, two, and three dimensions.

In the strip packing problem the container and the objects usually have a rectangular shape. If the objects are not rectangles but some (arbitrary) polygons, the challenge is even harder. The same holds if the objects have a regular shape but not the container. This is often the case in packing problems that arise in real world applications. A packing problem that appears in industrial applications is the computation of the volume of a trunk. Accord-

Figure 1.1: A box with volume one liter, that is used to measure the volume of a trunk (image source: `www.autobild.de`). A ship packed with containers (image source: `www.wikipedia.org`).

ing to the *Deutsches Institut für Normung*[1] *(DIN 70020-1),* the trunk volume is not the continuous volume (*e.g.,* the amount of water that can be filled into it) but rather the number of boxes with a volume of 1 liter that can be packed into it; see Fig. 1.1. A trunk is a three-dimensional—not necessarily convex—polygon. Thus, the task is to pack as many rectangular boxes as possible into a polygon. This problem has, *e.g.,* been studied in [Rei06].

Most of the packing problems are computationally *hard*, meaning that—roughly speaking—an optimal solution can only be found with enormous computational effort. However, for small instances exact solutions can be found in reasonable time. There are three main branches in the study of packing problems: exact algorithms, heuristics, and approximation algorithms.

Exact algorithms for packing problems are often based on methods that enumerate the solution space completely, *e.g.,* the *branch-and-bound method*. These approaches can be accelerated by providing good lower bounds on the solution value. For example, for the bin packing problem, *dual-feasible functions* [LMM02] often quickly provide near-optimal lower bounds. For a survey on exact methods see [FS98]. This survey also contains heuristics that are applied to packing problems. They often yield good (although not provably good) solutions, within a small amount of time.

The focus in this thesis is on approximation algorithms. These algorithms provide near-optimal solutions, and there is a proven bound on the solution quality. For example, for the strip packing problem, Baker *et al.* [BCR80] proved that the algorithm that always chooses the bottommost and leftmost position is a 3-approximation. This means that the height of the packing produced by the algorithm is at most three times higher than an optimal

[1] www.din.de

packing. Moreover, they provide an example in which the factor of three is actually achieved. Hence, there is an upper bound and a lower bound of 3 on the solution quality. A survey on approximation algorithms for packing problems can be found in [Ste08].

Outline of this Thesis In Chapter 2 we present basic definitions used throughout this work.

Chapter 3 studies the problem of dynamically inserting and deleting blocks from an array. The blocks can be moved inside the array and our goals are to minimize the time until the last block is removed and the costs for the block moves. This problem differs from other *storage allocation problems* in particular in the way the blocks can be moved. We present complexity results, different algorithms with provably good behavior, and provide computational experiments.

A variant of the strip packing problem with additional constraints—*Tetris constraint* and *gravity constraint*—is studied in Chapter 4. We present two algorithms achieving *asymptotic competitive factors* of 3.5 and 2.6154, respectively. These algorithms improve the best previously known algorithm, which achieves a factor of 4.

In Chapter 5 we present two closely related problems. They both have in common that point sets with small interior distances have to be selected. In particular, the first problem asks for the selection of grid points from the two-dimensional integer grid such that the average pairwise L_1 distances are minimized. We present the first optimal algorithm for this problem. In the second problem, we have to pack shapes with fixed area into a unit square, minimizing, again, the distances inside the shapes. We present a 5.3827-approximation algorithm.

Every chapter starts with a problem statement and definitions needed in the rest of the chapter. Moreover, we present work related to the problem, at the beginning of every chapter. Additionally, Chapter 3 contains related work at the beginning of the Sections 3.3 and 3.4.

Three papers form the basis of this thesis. All of them were prepared in collaboration with other people. Chapter 3 is based on the paper [BFKS09] and Chapter 4 on the paper [FKS09]. Both were prepared together with **Sándor P. Fekete** and **Tom Kamphans**.

The paper [DFR$^+$09] forms the basis for Section 5.2. It was prepared in collaboration with **Erik D. Demaine**, **Sándor P. Fekete**, **Günther Rote**, **Daria Schymura**, and **Mariano Zelke**.