2 Metamaterials: Fundamental Revolution and Potential Future

Materials' properties have troubled scientists since old ages [1]. From an electromagnetic outlook, researchers have had different concerns about materials' parameters and have looked at the problem from different viewpoints. While at microwave frequencies the relative dielectric permittivity is of interest, in optics the refractive index (n) is the important parameter. Usual optical materials have a positive dielectric permittivity (ϵ) and magnetic permeability (μ), and n could easily be taken as $\sqrt{\epsilon\mu}$ without any problems. Although it was realized that the refractive index would have to be a complex quantity to account for absorption and even a tensor to describe anisotropic materials, the question of the sign of the refractive index did not arise until the late sixties of the last century [2].

In 1968, Veselago studied analytically a medium that has both negative ϵ and negative μ [3]. He deduced that the medium possesses a negative refractive index. This means the negative square root $n = -\sqrt{\epsilon\mu}$, should be chosen. His result remained an academic curiosity for a long time, as neither real nor artificial materials with simultaneously negative ϵ and μ were available.

However, in the last few years, theoretical studies [4, 5] for specific engineered media whose ϵ and μ could become negative in certain frequency ranges were developed experimentally [6, 7], and this has brought Veselago's result into the limelight. These materials have been called metamaterials (MTMs). It means the materials which are not available in nature. Moreover, negative index media (NIM), double negative media (DNG), backward media, and left-handed media (LHM) have been named for the media when both ϵ and μ are negative. However, nowadays metamaterials are considered to be any engineered structures with unusual properties not readily available in nature [1]. Hence, negative permittivity, negative permeability, less-than-one refractive index, ϵ and/or μ near zero, graded index and bi-anisotropy materials are just some representative examples of metamaterials [8]. This field has become a hot topic of scientific research and debate over the past nine years. This chapter will review the early steps of the subject, and then give an overview of the potential applications.

2.1 Metamaterials - The Story so Far

2.1.1 How the Subject Started?

For ordinary materials both the relative permittivity and permeability are positive values and larger than one. These materials have been identified as right-handed media (RHM) or dou-

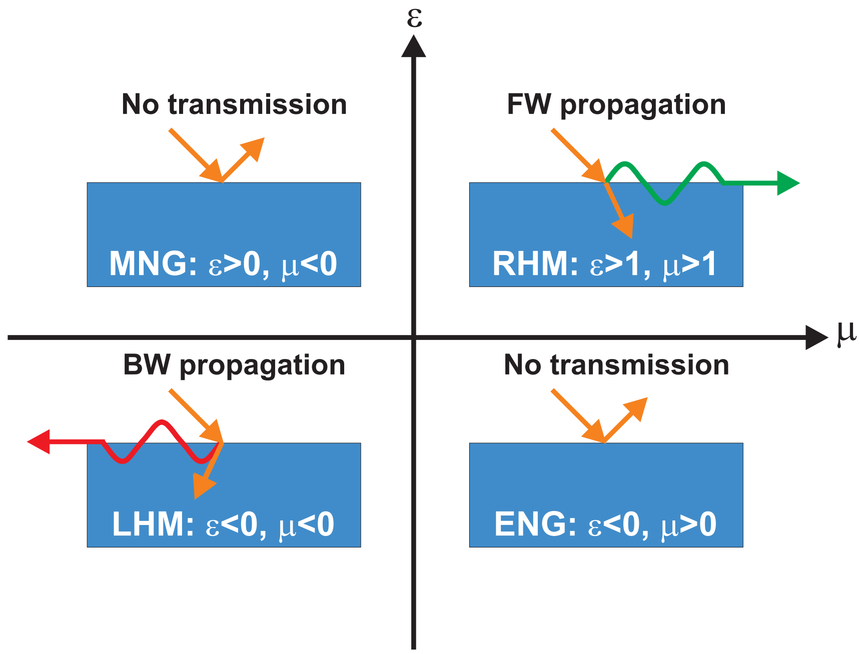

Figure 2.1: Material classification.

ble positive media (DPS). They are represented by the right-top quadrant of Fig. 2.1. In the following, some of the historical milestones for the other quadrants will be discussed.

As in any scientific field, metamaterials have required simultaneous efforts over the last century. It had a long gestation period and many contributors. The first study of general properties of wave propagation in negative index medium, represented by the left-bottom quadrant of Fig. 2.1, has usually been attributed to the work of Russian physicist V. G. Veselago. However, the subject has been studied since at least 1904 [9]. Some historical milestones regarding backward waves and metamaterials have been made over the last century. Fig. 2.2 shows the names versus years for some known researchers who contributed effectively to make that. H. Lamb in 1904 may have been the first person to propose the existence of backward waves. The phase of these waves moves in the direction opposite from that of the energy flow [10]. Lamb´s examples involved mechanical systems rather than electromagnetic waves. Apparently, the first person who discussed the backward waves in electromagnetism was Schuster in 1904 [11]. Schuster briefly annotates Lamb´s work and gives a speculative discussion of its implications for optical refraction. He cited the fact that within the absorption band of, for example, sodium vapor a backward wave will propagate. He was pessimistic about the applications of negative refraction because of the high absorption region in which the dispersion is reversed. Meanwhile, H.C. Pocklington showed in 1905 that in a specific backward-wave medium, the output wave from a suddenly activated source has group velocity which is directed away from the source, while its velocity moves toward the source [12].

After about half a century, Mandelshtam made the earliest known speculation on negative refraction [13]. He noticed that given two media, for a given incidence angle $\theta 1$ of the wave at the interface, Snell's law admits mathematically two solutions: not only the conventional solution $\theta 2$ but also the "unusual" solution π-$\theta 2$. Although we note that he made no reference

Figure 2.2: Main historical milestones of metamaterials.

to negative refraction, it is obvious that his argument refers to the same phenomenon. In 1945, with reference to Lamb, Mandelshtam presented physical examples of structures supporting waves with "negative group velocity". Such structures exhibit periodically varying effective permittivity [14]. From the above reference with Mandelshtam, it is concluded that Veselago was not the first to postulate the existence of LH media in [3]. However, it is an established fact that he was the first to conduct a systematic study of these media and to predict their most fundamental properties. Moreover, Pendry's contributions have awakened scientists to the aforementioned physical phenomenon [4, 5, 15].

2.1.2 Wave Propagation in Left-Handed Media

Let us start with the wave equation to show wave propagation in left-handed media [3]:

$$\left(\nabla^2 - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = 0 \tag{2.1}$$

where n is the refractive index, c is the velocity of light in vacuum, and $n^2/c^2 = \epsilon\mu$. It is anticipated then that lossless left-handed media with $n = -1$ must be transparent [16]. Considering the above equation, we can interestingly conclude that solutions to equation (2.1) will remain unchanged after a simultaneous change of the signs of ϵ and μ. However, when Maxwell's first-order differential equations are explicitly considered,

$$\nabla \times \mathbf{E} = -jw\mu\mathbf{H} \tag{2.2}$$

$$\nabla \times \mathbf{H} = jw\epsilon\mathbf{E} \tag{2.3}$$

where $\mathbf{E}$ is the electric field and $\mathbf{H}$ is the magnetic field, it becomes obvious that these solutions are quite different. For plane-wave fields, the above equations reduced to:

$$\mathbf{k} \times \mathbf{E} = w\mu\mathbf{H} \tag{2.4}$$

$$\mathbf{k} \times \mathbf{H} = -w\epsilon\mathbf{E} \tag{2.5}$$

where $\mathbf{k}$ is the wave vector. Therefore, for positive ϵ and μ, $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{k}$ form a right-handed system of vectors as shown in Fig. 2.3(a). However, if ϵ<0 and μ< 0, then equations (2.4) and (2.5) can be rewritten as:

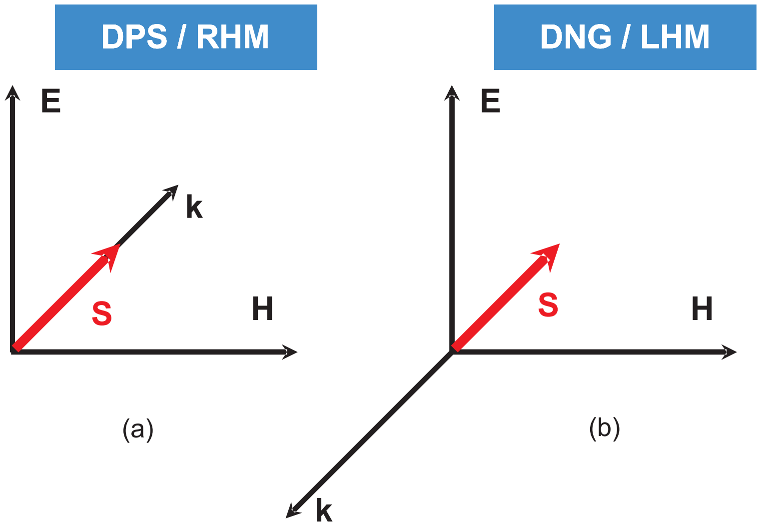

Figure 2.3: Representation of the three vectors **E**, **H** and **k** for a right-handed medium (RHM) (a) and left-handed medium (LHM) (b).

$$\mathbf{k} \times \mathbf{E} = -w|\mu|\mathbf{H} \tag{2.6}$$

$$\mathbf{k} \times \mathbf{H} = w|\epsilon|\mathbf{E} \tag{2.7}$$

showing that **E**, **H**, and **k** now form a left-handed triplet, as illustrated in Fig. 2.3(b). In fact, this result is the original reason for the denomination of negative ϵ and μ, media as "left-handed" media [3].

The main physical implication of the aforementioned analysis is backward-wave propagation. For this reason, the term backward media has been also proposed for media with negative ϵ and μ [17]. In fact, the direction of the time-averaged flux of energy is determined by the real part of the Poynting vector,

$$\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^{*} \tag{2.8}$$

where * denotes the complex conjugate, which is not influenced by a simultaneous change of sign of ϵ and μ. Thus, **E**, **H**, and **S** still form a right-handed triplet in a left-handed medium. Hence, energy and wavefronts travel in opposite directions in such media. Backward-wave propagation in homogeneous isotropic media seems to be a unique property of left-handed media. The phenomena of forward- and backward-wave propagation are demonstrated in Fig. 2.4(a) and Fig. 2.4(b) for a slab of media with refractive index of 1 and -1, respectively. There is free space before and after the media. The backward wave is evident from the electric field distribution inside the media in Fig. 2.4b. So, the wave vector is opposite of the pointing vector.

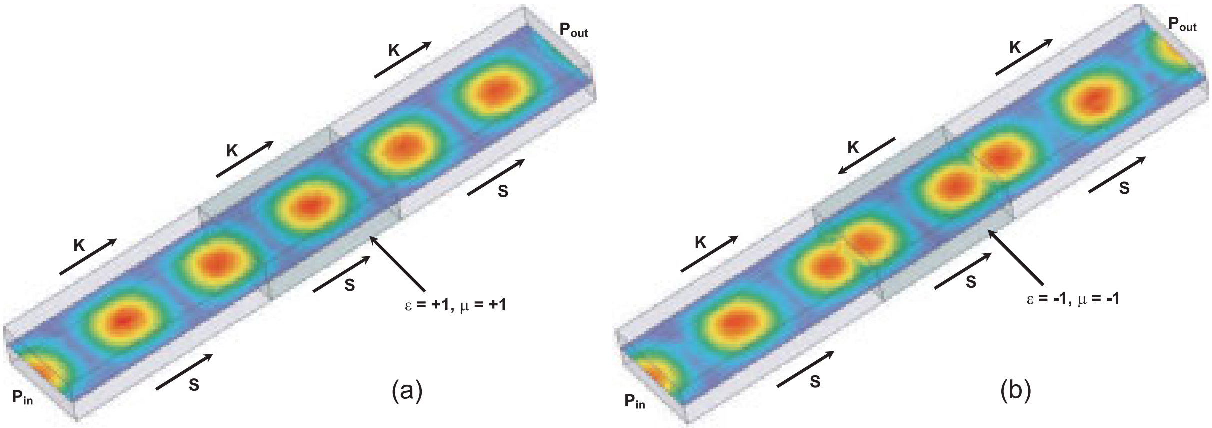

Figure 2.4: Forward- (a) and backward- (b) wave demonstration using a slab of material with ϵ and μ equal 1 and -1, respectively.

2.1.3 Negative Permittivity

It is not secret that higher permittivity low loss materials were needed during the Second World War for radar technology. Therefore, there were great efforts made to develop artificial dielectrics. One of the structures studied was an array of thin wires, which were shown to have an effective plasma frequency [18]. Later, Rotman was motivated to simulate plasmas in order to have more insight into problems such as the effect of rocket exhaust upon the radiation of re-entry vehicle antennas and his paper [19] was a result of that investigation. About quarter of a century later, Pendry et al. obtained similar conclusions [4]. The idea is based on having an array of these wires as shown in Fig. 2.5(a) with an applied electric field along the axes of the wires.

It has been shown that the behavior of an array of thin metallic wires can be explained by the plasma resonance in a metal [4, 20]. The frequency response of a metal to incident electromagnetic radiation is due to the plasma resonance of the electron gas. The following expression describes this behavior in an ideal form:

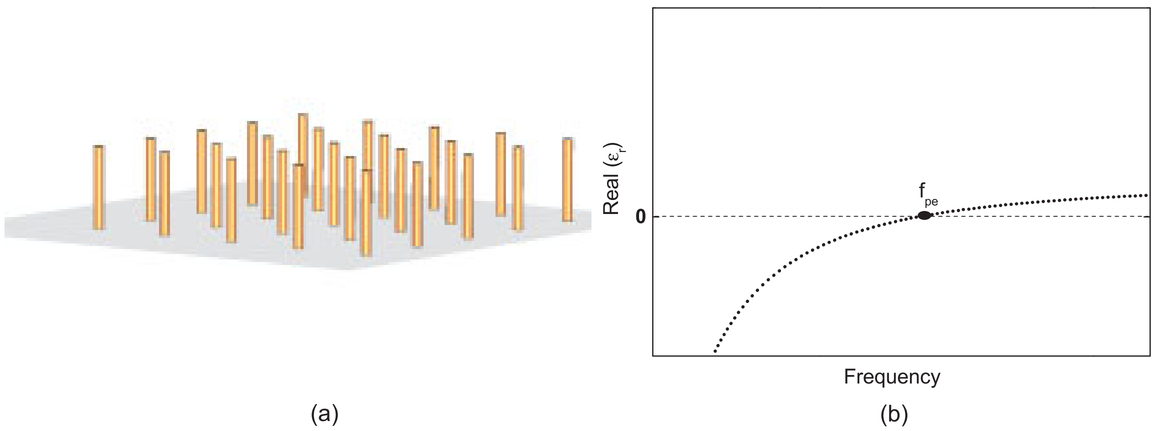

Figure 2.5: (a) Wire medium with an applied electric field along the axes of the wires (b) real part of ϵ_r versus frequency.

$$\epsilon_{metal} = 1 - \frac{w_{pe}^2}{w^2} \tag{2.9}$$

where w_{pe} is the plasma frequency, which is given by:

$$w_{pe}^2 = \frac{ne^2}{\epsilon_0 m_e} \tag{2.10}$$

where, n: electron density of the gas, m_e: electron mass, e: electron charge. An analytical approximation of the thin wire medium shows that:

$$n_{eff} = n\frac{\pi r^2}{a^2} \tag{2.11}$$

$$m_{eff} = \frac{\mu_o \pi r^2 e^2 n}{2\pi} ln\left(\frac{a}{r}\right) \tag{2.12}$$

where n_{eff} and m_{eff} are the average electron density and the effective electron mass, respectively. r and a are the wire radius and the lattice period, respectively. Simple substitutions give the plasma frequency:

$$w_{pe}^2 = \frac{2\pi c_0^2}{a^2 ln\left(\frac{a}{r}\right)} \tag{2.13}$$

By applying equation 2.13, a plot of epsilon for a thin wire medium can be calculated, as is shown in Fig. 2.5(b). The graph explains how epsilon goes from negative values in the lower frequency range (and hence, only evanescent modes are allowed to propagate), up to positive values in the higher frequency range, through the plasma frequency f_{pe}. It is also interesting to note that the plasma frequency can be lowered by increasing the lattice constant (a) of the medium. Fig. 2.6 shows a sweep for four different ratios for a/r. As the ratio increases, the plasma frequency decreases. This is an interesting property of this medium, because the operating frequency range of the final metamaterial configuration is limited by the size of this metallic array. Researchers have been attracted by this kind of thin wire arrays. They have envisaged these types of structures with plasmonic response for the realization of sub-wavelength antennas with enhanced radiation properties [21].

2.1.4 Negative Permeability

Interestingly, unknown to Veselago, the existence of negative permeability had already been shown in such material by Thompson [22] a decade earlier. However, Pendry proposed a novel type of particle called the Split Ring Resonator (SRR) in the late 90´s [5]. This was a major step in the implementation of an LHM. The resonator consists of a pair of concentric rings, with slits etched in opposite sides. By adequately exciting the SRR with a time varying magnetic field oriented in the axial direction of the particle, a strong magnetic behavior can be observed. A highly resonant response with frequency can be observed by studying the equivalent magnetic permeability value. Moreover, a frequency range will exist in which μ will exhibit a very high

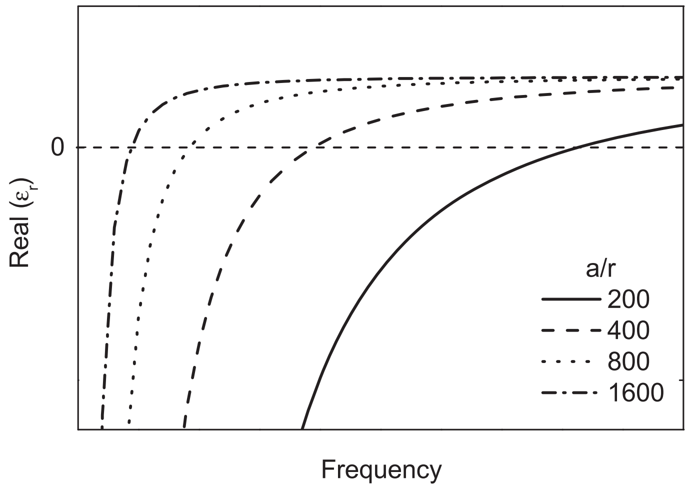

Figure 2.6: Analytical calculation of epsilon for a thin wire medium when a/r ratio is varied.

negative value (close to the quasi-static frequency) up to where it reaches a value higher than 0 (magnetic plasma frequency).

Pendry started his study by taken two concentric cylinders with a slit in each one, in opposition one to the other. A schematic of the resulting structure is shown in Fig. 2.7. In this case, currents are forbidden to move in any of the cylinders. A very high capacitive value is a consequence of that, which enables displacement current to flow. The value of the effective magnetic permeability is given by:

$$\mu_{eff} = 1 - \frac{F}{1 + i\frac{2\sigma}{wr\mu_0} - \frac{3}{\pi^2\mu_0 w^2 C r^3}} \tag{2.14}$$

where F is the fractional volume, given by $\pi r^2/a^2$ and C is the capacitance per unit area between both metallic sheets of the cylinders, given by $C = \epsilon_0/d = 1/dc_0^2\mu_0$. Fig. 2.8 presents the SRRs medium and the analytical calculation of Eq. (2.14) for the effective magnetic permeability of a medium composed by SRR cylinders. It shows the μ_{eff} versus frequency.

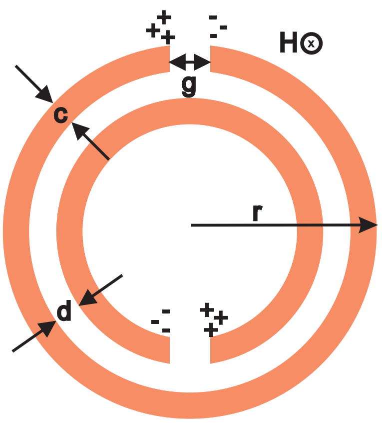

Figure 2.7: Top-view of split ring resonator cylinder structure.