2 Thruster and System Model

One of the most important tasks when designing a spacecraft is deciding on the size and properties of the actuators since it is crucial to have enough control authority to overcome the disturbances acting on the satellite, as well as getting the spacecraft into its desired state. To command the spacecraft to a determined attitude or rates, different methods can be used. The most common active actuation methods use magnetic torques (magnetorquers), angular momentum storage and exchange devices (control moment gyros, momentum wheels and reaction wheels), and thrusters [94]. A thruster system provides a direct actuation for each direction without dependencies, therefore it is especially recommended for active control of a satellite of six-degrees of freedom (or attitude stabilized satellite). The design of the thruster system for the attitude and translation control of the spacecraft involves many design factors such as the number of thrusters that should there be, the kind of controller, and the physical distribution in terms of position and orientation (thruster configuration) [37]. Such parameters also have to be known as accurately as possible if one wants to make a precise control of the spacecraft, because they basically describe the system. Therefore in this chapter, the system model is presented and also the generic thruster model that will be used in this work. The nature and origin of the thruster parameters are explained in detail.

2.1 System Model

In a high precision spacecraft system, the forces and torques necessary to compensate the external influences or to command the vehicle to a certain position are produced by the thruster system. The thruster systems are designed to provide the sufficient amount of forces and torques to deal with the expected disturbances and to command the spacecraft to carry out specific tasks. They are arranged in a complete system whose parameters, the location of the thrusters, direction of the thrusts, and the thrust itself, must be identified. The location and the direction are parameters that should be known from the design of the spacecraft propulsion system itself. The thrust beam path depends on the type of thrusters and on the precision of the models used to characterize their behaviour. Normally a set of equations are established, which translates the commands into real forces and torques, and the accurate determination of the parameters of the system will grant their efficiency. However, the characterization of the system is not very precise because there are many parameters that can not be determined on ground, and there are many factors that change the response, behaviour and the characteristics (such as the location and direction of the thrusters) of the whole system when the spacecraft is already on its orbit [98][100].

The translation of commands into forces and torques depends basically on the thruster configuration matrix, which is specified by the direction of the thrusts, and implicitly the location of the thrusters on the spacecraft. The force-thrust relation can be expressed as

$$\mathbf{F} = \mathbf{A}\mathbf{T} \tag{1}$$

where **F** is the force vector, which consists of forces and torques; **A** is the thruster configuration matrix which defines the position and orientation of the thrusters, in general the direction of the thrust (force) exerted by the thrusters; and **T** is the command vector, which specifies the activation of each individual thruster as well as the magnitude of the thrust. In its proper form, Eq. 1 can be rewritten as

$$\left[\frac{\mathbf{F}_f}{\mathbf{F}_m}\right] = \left[\frac{\mathbf{A}_f}{\mathbf{A}_m}\right][\mathbf{T}]$$

where $\mathbf{F}_f$ and $\mathbf{F}_m$ are vectors of forces and torques (moments) respectively; $\mathbf{A}_f$ is a matrix which specifies the direction of the thrust force, and $\mathbf{A}_m$ is a matrix with the radial locations of the thrusters, which generates moments. In other words, the elements of **A** are the influence coefficients defining how each thrust affects each component of the vector **F**.

There are an infinite number of combinations of thrusts **T** that can generate the desired force **F**, because the matrix **A** has more columns than rows due to the fact that there must be at least one more thruster than the degrees of freedom of the system; in general **A** is a $n \times m$ matrix, where n is the number of degrees of the object and m is the total number of thrusters [37]. The problem can be solved and a unique solution can be found if the norm of **T** is minimized. It is known as a thruster controller, and the type of controller is the the result of the type of norm used in the minimization [97].

As previously stated, the relation from Eq. 1 lies on the thruster configuration matrix **A**, and this equation is the basic relation on which the study is based. So, the proper operation of the propulsion system will depend on the accurate knowledge of such a matrix. That is why some methods to determine the actual parameters (values) of the matrix **A** are needed.

2.2 Thruster Model

2.2.1 The Thruster Principle

A propulsion system accelerates matter to provide a force of thrust that moves a vehicle or rotates it about its center of mass. In the same way, a thruster is based on the principle of the conservation of momentum, it accelerates and exhausts a propellant to generate a force on the spacecraft. If the line of action of this force does not go through the satellite's center of mass, it can also be used to generate a torque about the center of mass, allowing both active position and attitude control. Note that to generate a pure torque, a pair of thrusters on opposite sides of the spacecraft must be fired to create a couple. The thrust of a thruster is potentially the largest source of force and torque that can be generated on a spacecraft, and does not depend on its altitude.[2]

The force is generated then by expelling mass through a nozzle. The simplest and most common valve is strictly on-off and single level, but variable and dual-level thrusters are available. The expelled mass multiplied by its velocity results in a product called linear momentum denoted as [32]

$$\rho = m_p \, v_p \tag{2}$$

where ρ is the linear momentum, m_p the expelled mass of the propellant and v_p the velocity of the expelled mass. The force is equal to the time of change of the momentum, and this change takes place in the direction in which the force acts

$$\begin{aligned} F &= \dot{\rho} \\ &= m_p \, \dot{v}_p = \dot{m}_p \, v_p \end{aligned}$$

where m_p or v_p can be variable. Therefore with a simple conservation of momentum, the force acting upon a body will cause a reaction force of the same intensity in the opposite direction, producing the force acting upon the spacecraft. The force acting on the spacecraft is then

$$F_{thr} = -\dot{\rho} \tag{3}$$

The force is the first parameter that characterizes a thruster. It is measured in Newtons (N) and depends highly on the thruster design, its functional principle and the used propellant. For typical small satellite attitude control applications this force is in the range of μ-mN. From Eq. 3 one can see that higher exhaust velocities v_p creates the same flow at lower mass flow rate $\dot{m}_p$. This gives the information on how effective the proppelant is used, and this is also the reason why the use of heavy gases or liquid metals propellants is preferable because they can generate

[2]The generation of forces using external disturbances is dependant upon many factors such as altitude, latitude, attitude, total mass and surface of the spacecraft between others. However the altitude determines mostly the magnitudes of the external disturbances except for the solar pressure disturbances (see Section 5.6).

bigger forces. This leads to another characteristic of the thrusters, the impulse. The impulse of the force acting on a particle is equal to the change in momentum of a particle. The impulse is measured for the thrusters as a specific impulse I_{sp} and it is used for the comparison of propulsion systems [84]

$$I_{sp} = \frac{F}{\dot{m}_p \, g_0} = \frac{v_p}{g_0} \tag{4}$$

where g_0 is the gravity on the Earth's surface. Equation 4 compares basically the thrust derived from a system as a function of the propellant mass flow rate. Therefore, the higher the exaust propellant velocity, the less the propellant to be carried on board the spacecraft.

The main advantage of using thrusters is that they can produce an accurate and well defined force/torque on demand, and supply a reliable force/torque in any direction. The accuracy of the control depends on the minimum impulse bit allowed by the thrusters used. The main disadvantage is that a spacecraft can only carry a limited amount of propellant.

In the family of propulsion systems for the spacecrafts, there are a wide variety of thrusters, which vary depending on the force impulse used and the methods used to generate this impulse. The impulse is one of the most important characteristics of thrusters and defines how they are classified. So, the way the propellant is accelerated determines the family in which the thrusters are classified. Electric propulsion is nowadays one of the most promising family of thrusters; some of these electric propulsion techniques are an already proven technology and their characteristics expand the future of this technology, because of their high quality generating very small thrust forces with very high specific impulses. Between the vast family of electric thrusters, those who produce specific impulses in the range of mili-Newtons are the devices used for the satellite attitude control. Proportional thrusters, which can vary the thrust levels used, are the general solution for new satellite precise scientific missions, like drag-free satellites, because they are capable of supplying the accuracy of thrust needed to maintain a free-fall of the test-masses inside the satellites. These kinds of thrusters allow the use of explicit thrust-force relation of Eq. 1, which is the basis of the research. There are many technologies and classifications in the electric propulsion, here a generic electric propulsion is taken into account and a model is presented in the next section. To go into further depth on the topic see [32], [84], and [75], a report produced at the ZARM institute.

2.2.2 Generic Thruster Model

The simplest model of a thruster is a linear one, and most of the estimation methods adapted for the thruster parameter estimation (see Chapter 3) take this model as a default. However the thrusters are not linear, and they have some dynamics.

Therefore a simple model of the thruster is used and during the simulations it is possible to switch between the linear and the non-linear one.

The linear model of a thruster can be represented as a simple gain as shown in Figure 1.

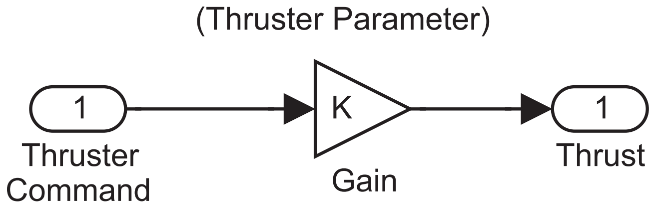

Figure 1: Linear model of a thruster

The gain K determines how the thrust-command is converted into a force-thrust, in other words, it is the ratio between the real and commanded force. In a simple situation, where the thruster is aligned with a principal body axis of the satellite, the gain represents the gain-parameter of the thruster in this axis. If the thruster has an arbitrary position, the total magnitude of K will be divided in portions proportional to the angles between the unitary vector of the direction of the thruster-force and the principal body axes. In general, the gain K will be

$$K = \sqrt{K_x^2 + K_y^2 + K_z^2} \tag{5}$$

where

$$\begin{aligned} K_x &= K \cos\alpha \\ K_y &= K \cos\beta \\ K_z &= K \cos\gamma \end{aligned} \tag{6}$$

and α, β, and γ are the coordinate direction angles. The real thruster direction can then be represented by the unitary vector

$$\begin{aligned} \hat{u}_{thr} &= \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k} \\ &= \frac{K_x}{K}\,\mathbf{i} + \frac{K_y}{K}\,\mathbf{j} + \frac{K_z}{K}\,\mathbf{k} \end{aligned} \tag{7}$$

in thruster coordinates. If the thrust force does not pass through the center of mass of the satellite, it will also generate a torque resulting in three more parameters for the thruster. The parameters are dependent on the total gain K, or to be more specific on the unitary gains $\{K_x, K_y, K_z\}$, and the lever arm of the position of the thruster, of the distance between the position of the application point of the force and the center of mass of the body. Mathematically, it can be represented as

$$\left\{r_{thr_x}\mathbf{i} + r_{thr_y}\mathbf{j} + r_{thr_z}\mathbf{k}\right\} \times \{K_x\mathbf{i} + K_y\mathbf{j} + K_z\mathbf{k}\} = K_M$$

where r_{thr} is the lever arm, and K_M is the vector of the three new parameters $\{K_{M_x}, K_{M_y}, K_{M_z}\}$. Each individual parameter will then be

$$\begin{aligned} K_{M_x} &= r_{thr_y}K_z - r_{thr_z}K_y \\ K_{M_y} &= r_{thr_z}K_x - r_{thr_x}K_z \\ K_{M_z} &= r_{thr_x}K_y - r_{thr_y}K_x \end{aligned} \tag{8}$$

Note that the gains now have a subscript M to differentiate them from the force parameters. For simplifications the force parameters of Eq. 6 will be described as $\{K_{F_x}, K_{F_y}, K_{F_z}\}$. Therefore the total vector of gain-parameters of an individual thruster will have six elements (or less if less degrees of freedom are taken into consideration). It can be represented as:

$$\mathbf{K}_{thr} = \begin{bmatrix} K_{F_x} \\ K_{F_y} \\ K_{F_z} \\ K_{M_x} \\ K_{M_y} \\ K_{M_z} \end{bmatrix} \tag{9}$$

and each thruster of a spacecraft can be totally described by this parameter-vector (or column). However thrusters, like all real systems, are not linear. They have their own dynamics depending on the thruster technology used. Nevertheless, a generic thruster model is proposed for this work. The model is based on the work already created for the ZARM by Bindel.[3] Therefore, there is no intention to rewrite all the work, just to put in detail what a generic thruster model should be.

In general a simple thruster with dynamics can be represented by a first-order transfer function with a time delay (FOLPD) plus some noise, as shown in Figure 2.

The model has a first-order transfer function depending on the time-constant T, and the gain K of the thruster; a time-delay τ indicating the time difference between the time of activation and execution of a command; and a random noise n

[3] The generic thruster model is based on the work by Daniel Bindel for the ZARM institute. Two models were generated, for the FEEP and Colloid technology in order to test a LISA-pathfinder simulator. See [13] and [12].

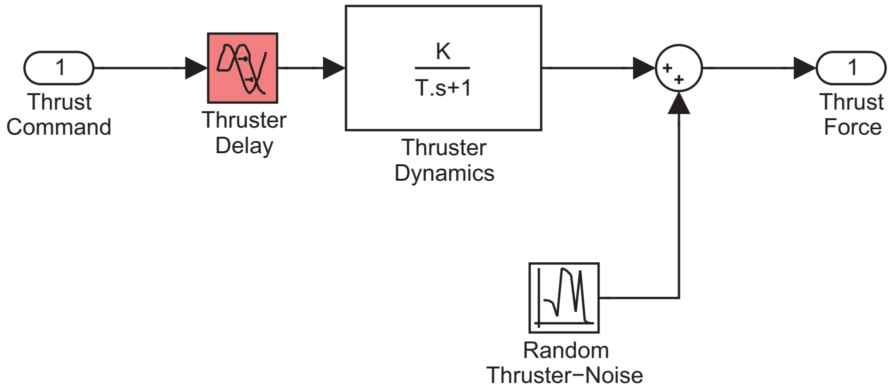

Figure 2: FOLPD Thruster Model

in the direction of the thrust-force. However, the model does not take into account the nonlinearity nature of a thruster, its digital control and the fluctuations in the direction of the generated force. So that, a more sophisticated model of a thruster should have at least five submodules as suggested in [12]. A generic model then has a pre-processing, non-linearity, dynamic, noise, and beam flutter model (see Figure 3).

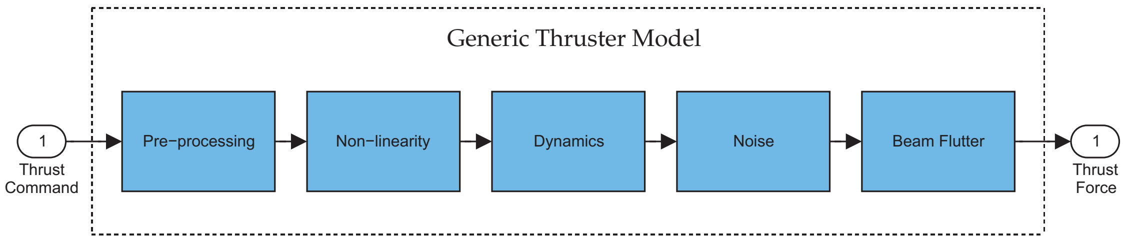

Figure 3: A Generic Thruster Model

In the pre-processing submodel of the generic thruster model, the saturation of the input signal is checked out and it limits the command to the maximum and minimum magnitude level allowed by the thruster (due to physical constraints), and the command is also discretized. The non-linearity submodel tries to represent the non-linear behaviour of the thruster. The response may vary by several factors such as age, temperature, and magnitude of the commanded force. Therefore the non-linear model can vary a lot according to the different technologies used. To simplify it, a look-up table can be used to model such non-linearities. The dynamics simulate not only the time-delay response but the dynamical behaviour of the thruster to the thrust-command. Here for simplifications, the FOLPD model can be used. Note that for the FOLPD model there will be no gain K, it will always be one because the variations of K and its magnitude are taken into account in the non-linear subsystem. In the noise submodel a random noise with a know variance σ_n is added to the response of the dynamics. Finally, in the beam-flutter submodel, the variations of the thrust force are taken into account. A nozzle is normally used to direct the propellant in one direction. However, the exhausted thrust is not directed 100% toward the thruster direction axis, it may vary in all directions (in the coordinate system of the thruster) and it is simulated here. A detailed model is shown in Figure 4, and every part of the model is explained in detail in [12].

Note that the thrust force generated by the engine is in the direction of the thruster and the deviations of the direction are in terms of the thruster axis system. To solve it one has only to rotate the generated direction to the body axis using a simple rotation matrix. A more complicated supposition can be taken into account if the direction of the thrust force is not aligned with the thruster, as was explained for the linear model. So that, six different angles determine the final direction of the thrust force and therefore the gains of the thruster in each direction. The group of equations 6 and 8 will then be rewritten as

$$
\begin{aligned}
K_{F_x} &= K\cos\alpha\,[\cos\theta\cos\psi] + K\cos\beta\,[\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi] \\
&\quad + K\cos\gamma\,[\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi] \\
K_{F_y} &= K\cos\alpha\,[\cos\theta\sin\psi] + K\cos\beta\,[\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi] \\
&\quad + K\cos\gamma\,[\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi] \\
K_{F_z} &= -K\cos\alpha\,[\sin\theta] + K\cos\beta\,[\sin\phi\cos\theta] \\
&\quad + K\cos\gamma\,[\cos\phi\cos\theta]
\end{aligned}
\tag{10}
$$

and

$$
\begin{aligned}
K_{M_x} &= r_{thr_y}K_{F_z} - r_{thr_z}K_{F_y} \\
K_{M_y} &= r_{thr_z}K_{F_x} - r_{thr_x}K_{F_z} \\
K_{M_x} &= r_{thr_x}K_{F_y} - r_{thr_y}K_{F_x}
\end{aligned}
\tag{11}
$$

where α, β, and γ are the true direction of the thrust-force, and ϕ, θ, and ψ are the misalignment of the thruster. The problem gives a set of six non-linear equations with nine variables, therefore the attempt to distinguish the true direction of the thrust to the direction of the thruster is a non-solvable procedure.

In the calibration, one is able to get the six parameters of a thruster, the vector $\mathbf{K}_{thr}$. This vector will contain all information related with thrust direction, thruster misalignment, thrust variations, thrust impingement, etc. However it is not possible

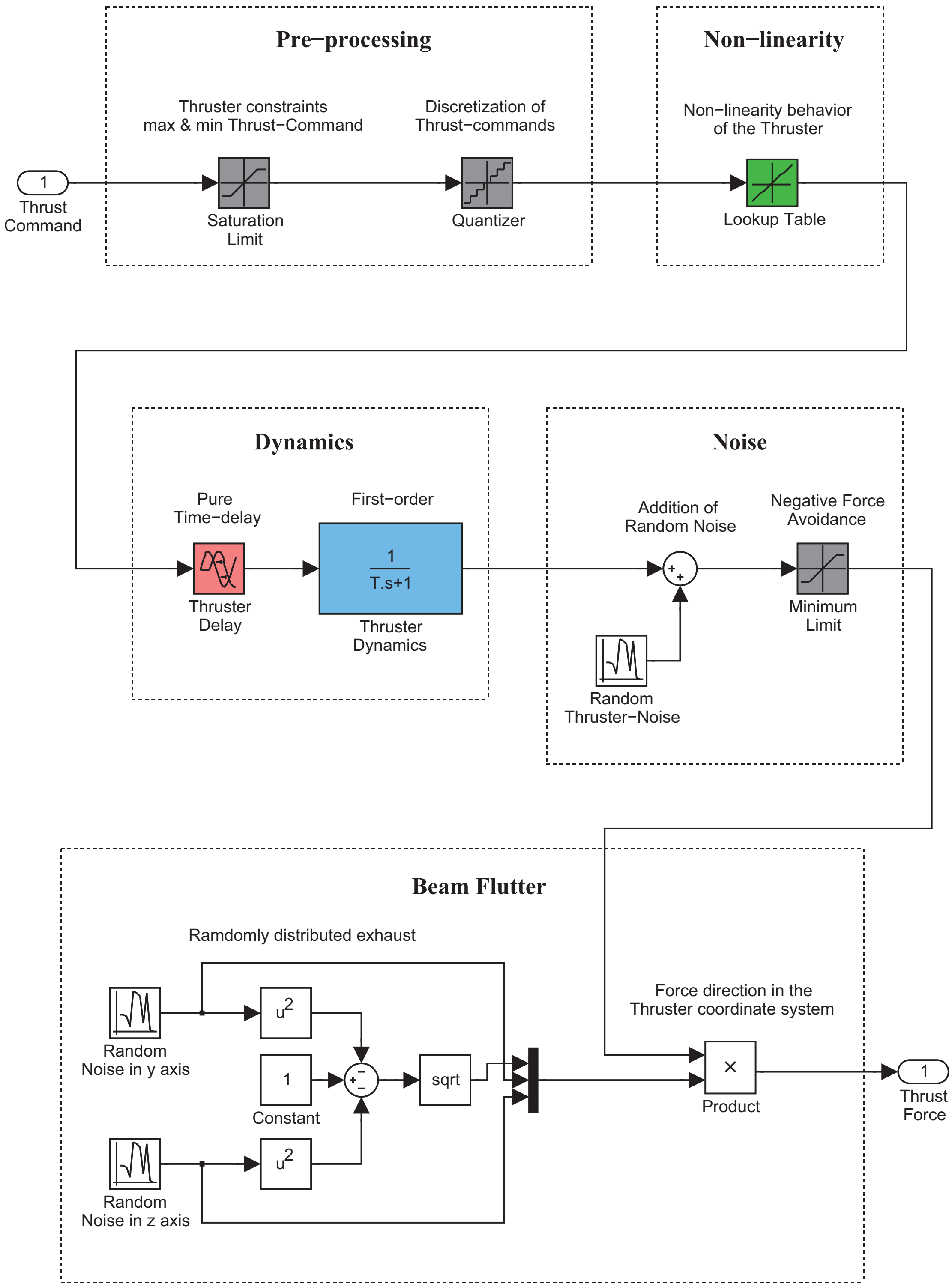

Figure 4: A Generic Thruster Model