Introduction

The development of novel computational methods that are capable of harnessing the increase in available computing power allow today the tackling of complex scientific and engineering problems. At the same time we observe a trend in applying advanced computational concepts that have been developed for areas such as fluid dynamics to emerging fields such as biology and medicine where advanced computing is a promising tool for quantitative prediction and understanding. This thesis exemplifies this approach in the problem of virtual surgery.

One of the core challenges in virtual surgery is the modeling and simulation of soft biological tissue. This requires not only reliable biomechanical models but also a robust and highly flexible simulation tool as the underlying physical laws and geometries are complex. In biomechanical models we often have to consider a nonlinear elasticity law to describe the large deformations that can appear during a medical intervention. Complex physical phenomena, such as interaction with medical devices, need to be taken into account. As the human body consist mostly of water and many vitals are filled with fluid, biological tissue is often in contact with body fluids. The consideration of such fluid-solid interactions, however, is numerically demanding and requires special numerical techniques. Large-scale simulations are required to resolve the complex geometries of biological structure. The cutting of tissue can cause large topological changes leading to a numerical challenge especially for grid-based methods where this requires the creation of new cells and an update of the connectivity information. We investigate the use of particle methods as a simulation tool for virtual surgery and consider the performance in key requirements

- 1. Robustness and flexibility with respect to physical laws and geometries
- 2. Consideration of fluid-solid interactions
- 3. Behavior with respect of large deformation and topological changes
- 4. Scalability of large scale simulation on parallel computer architecture.

This thesis reviews and extends computational methods to simulate biological systems for the use in virtual surgery.

The thesis is structured as follows:

Chapter 1. Motivation and Objectives

The motivation of this thesis is the need to improve the efficiency and accuracy of virtual surgery simulations. The focus of virtual surgery is on the development, integration and validation of enabling technologies towards advanced computer aided and image guided systems for medical interventions. It supports the complete treatment process from diagnosis, therapeutic planning and simulation via intra-operative action to postoperative care, monitoring and documentation. In this framework, virtual reality based surgical simulation is an area of special interest. Surgical simulators can only fulfill their mission if they provide a realistic and configurable training environment. *How realistic are virtual surgery nowadays?*

A training environment requires that the simulated organs behave authentically in a biomechanically and physiologically perspective and the environment is represented in a realistic manner. However, due to the high complexity of the simulated system, none of the simulators proposed up to now can even approximately achieve the necessary level of realism in simulation and visualization. Chapter 1 illustrates the state of the art of surgical simulators and shows benefits and risks of the virtual reality training concept. In Section

1.5, we presents the key components of a surgical simulator and their influence on the training effect.

Chapter 2. Particle Methods

Particle methods have been successfully applied in a wide range of problems, from astrophysics [84, 107], to computational fluid dynamics and [41, 109] and molecular dynamics [74], but they are hardly used for the simulation of soft biological tissue. *Why do we consider particle methods for the use in virtual surgery?*

The advantages of particle methods include adaptivity and multi-resolution capabilities of the computational elements, good stability properties of the discretization, and, similar to discrete systems, an inherent link of the computational elements to the physics that they represent. The flexible handling of geometries and physics makes particle methods an appealing technique for the numerical challenges in virtual surgery simulations. Chapter 2 includes the fundamental background of continuum particle methods as they are used in this study. It illustrates basic numerical techniques, such as derivative approximations (Section 2.3) and the redistribution of particles (Section 2.4).

Chapter 3. Particle Level Set Method

To describe the evolution of interfaces/surface, we distinguish two classes of computational methods: interface capturing and interface tracking methods. Particle methods are often associated with tracking methods, because interfaces evolution in tracking methods is solved in a Lagrangian fashion using markers. In level set methods, however the interface is captured using an implicit function that are traditionally evolved in an Eulerian way.

How can we combine the level set method with Lagrangian particle methods?

We can naturally solve the underlying level set equation in a Lagrangian frame using particles that carry the level set information as also shown in Section 3.3. The consistent

remeshing procedure enforces the regularization of the particle locations when the particle map gets distorted by the advection field. The Lagrangian description of the level set method is inherently adaptive and exact in the case of solid body motions. The efficiency and accuracy of the method is demonstrated in several benchmark problems in two and three dimensions involving pure advection and curvature induced motion of the interface (Section 3.5). Section 3.6 and 3.7 show that the simplicity of the particle description is well suited for real time simulations of surfaces involving cutting and reconnection as in virtual surgery environments.

Chapter 4. Particle Simulations of Fluids

The particle method used in this study bases on remeshed Smoothed Particle Hydrodynamics (rSPH), a continuous particle method to simulate compressible viscous fluids described by the Navier-Stokes equations.

How can continuous particle methods solve the Navier-Stokes equations?

This chapter shows the governing equations of a compressible viscous fluid and its particle discretization. Moreover, it includes results of a compressible vortex ring simulation and a general discussion on Smooth Particle Hydrodynamics.

Chapter 5. Particle Simulations of Elastic Solids

The correct reproduction of the deformation behavior is a crucial part of any surgical simulator. A correct reproduction requires first of all a reliable model for the stress-strain relationship with suitable parameters. Linear elastic models are suitable up to a stretch of $\lambda = 10\%$. When the deformation exceeds a stretch of 10%, nonlinear effects need to be taken into account. Finally, an accurate numerical solver of the model should deliver the desired simulation results. An established numerical method to solve elasticity problems is the grid-based finite element method that solves the nonlinear elasticity models in the reference coordinate system.