Chapter 1

Introduction

The most important endeavour of high energy physics in the coming years is the unravelling of the mechanisms by which masses are generated and the electroweak symmetry is broken. The Standard Model of particle physics postulates a new scalar particle, the Higgs boson, which solves both issues simultaneously. Through self-interaction the Higgs field acquires a vacuum expectation value v, $v = (\sqrt{2}G_F)^{1/2} \approx 246\,\text{GeV}$, where G_F is the Fermi constant. In this framework only the mass of the Higgs boson is a free parameter[1]

Once the mass of the Higgs is known, all parameters entering the Standard Model of particle physics are completely determined since the couplings to the other particles are predicted by the Standard Model. The mass of a fermion m_f is, for instance, connected to its coupling to the Higgs boson, the Yukawa coupling λ_f, through the relation $m_f = \lambda_f\, v$. As v and the masses of the fermions are known, measuring the Higgs Yukawa couplings is a stringent test of the validity of the Standard Model.

There are also a large number of models going beyond the Standard Model, and many of them also feature a particle very similar to the Standard Model Higgs boson. The most prominent example is the lightest Higgs particle of the minimal supersymmetric extension of the Standard Model. Once such a particle is discovered, it is therefore very important to measure not only its mass but also its couplings since these discriminate between the different models.

[1] The Higgs mass is expected to lie between the current exclusion bound of 114.4 GeV [3] and 1 TeV. The analysis of electroweak precision observables yields a 95% confidence level exclusion bound of 182 GeV [4].

While the mass of a Standard Model Higgs boson is expected to be measurable at the LHC with a precision well below the percent level for most of the Higgs mass range [5], it is much more difficult to measure its couplings. The couplings to the W and Z bosons can be determined with a precision of approximately $10\% - 30\%$ for higher Higgs masses around 200 GeV but this deteriorates to $\sim 30\%$ for lower Higgs masses (see e.g. [6]). For the Yukawa couplings the situation is worse. One can hope to make a measurement only for the top quark, the τ and the bottom quark, and of these the top Yukawa coupling is the most promising, with a relative uncertainty of $25 - 40\%$ [6] (where a few model-dependent assumptions are made).

The ILC, however, allows measurements of the Higgs properties such as the Higgs width and the Yukawa couplings with a much higher precision without having to introduce model-dependent assumptions.

In general, since the top quark is so heavy, the Higgs is not expected to decay into tops. The top-Yukawa coupling can therefore not be measured through the branching ratio but through top-associated Higgs production both at the ILC and the LHC. At an e^+e^- linear collider the process $e^+e^- \to t\bar{t}H$ proceeds via the Feynman diagrams shown in Fig. 2.1 at leading order and is dominated by those where the Higgs is radiated off one of the quarks. They give a contribution proportional to λ_t^2 in the cross section, leading to a large sensitivity to the value of the top Yukawa coupling λ_t. Since it is a $2 \to 3$ process where all three final state particles are very heavy, the total cross section is however tiny, never exceeding a few fb.

In its first phase, the ILC will run at 500 GeV, i.e. near the $t\bar{t}H$ threshold. This gives rise to two challenges.

The first is more on the experimental side: Near threshold the total production rate is of course especially small due to phase space suppression and the statistical uncertainties grow. In Section 2.2 we will present experimental studies at 800 GeV and at 500 GeV based on the tree-level results illustrating this issue. While the Born cross section suggest a relative uncertainty of $\Delta\lambda_t/\lambda_t \sim 10\%$ at 800 GeV this deteriorates to 25% at 500 GeV for the relatively favourable case of a light Higgs with mass $m_H = 120$ GeV. The errors increase with smaller $\sqrt{s}$ since they are dominated by statistical uncertainties which grow as the rate decreases.

One way of extending the sensitivity of the ILC is to polarize the initial state beams. We included the effect of arbitrary e^+e^- polarization in our calculation and found that the rate can be increased by a factor of up to 2. This would already reduce the statistical errors significantly.

The other challenge comes from the theoretical side: Fixed-order perturbation theory breaks down when for instance the $t\bar{t}$ system becomes non-relativistic. This is especially pressing for center-of-mass energies near the three-particle threshold since the complete phase space is then within the non-relativistic regime where usual perturbation theory is not applicable. This issue can be controlled by switching to an effective theory correctly describing the non-relativistic regime.

In this thesis the process $e^+e^- \to t\bar{t}H$ is described using the effective theory velocity Non-Relativistic QCD (vNRQCD) including all QCD and electroweak next-to-leading logarithmic (NLL) effects. This leads to a consistent description of the cross section near threshold and allows for a meaningful comparison with experimental data.

Our NLL results have been implemented in an experimental feasibility study at 500 GeV [7]. When they are included, the anticipated accuracy of the ILC at 500 GeV doubles, bringing $\Delta\lambda_t/\lambda_t$ down to $10 - 15\%$ even in the first phase of the linear collider.

1.1 Outline

The Born level results for the process $e^+e^- \to t\bar{t}H$ will be presented in Chapter 2 where we will also describe the advantages of beam polarization and give the results for the case of polarized electron-positron beams. The formulas for the cross section expanded in the endpoint of large Higgs energies can be found there as well as the matching coefficients for the current operators at LL and NNLL order.

When the top-quark pair becomes non-relativistic, usual fixed-order perturbation theory breaks down due to the appearance of Coulomb singularities and of large logarithms as is described in Chapter 3.

This problem can be cured by using the effective theory vNRQCD which resums the Coulomb singularities and the large logarithms to all orders. By definition the relative velocity v between the top quarks is small near threshold and so v is introduced as an additional expansion parameter. Chapter 4 gives an overview over the effective theory vNRQCD that was originally developed for top pair production $e^+e^- \to t\bar{t}$ near threshold.

We then apply vNRQCD to the process $e^+e^- \to t\bar{t}H$ at NLL order.

In Chapter 5 the results are presented including the QCD corrections. In particular the process-dependent hard QCD Wilson coefficients are calculated there both for polarized and unpolarized beams.

Due to the instability of the top quark there are electroweak mixing effects at NLL order. We computed the absorptive matching coefficients at NNLL order which lead to a sensitivity on the real part of the Green function, producing a renormalization group running of the Wilson coefficients at NLL order. Such an effect is also caused by the Green function describing the $t\bar{t}$ production in a P wave and the v^2 correction to the S wave production. The difference between the full-theory phase-space and the effective-theory phase-space is taken into account by the procedure of phase-space matching [8]. These electroweak effects are included in Chapter 6.

Finally, we analyze the complete NLL order result in Chapter 7, showing that a meaningful comparison with experimental results is only possible when the non relativistic effects are included. In addition, they greatly improve the prospects of the top Yukawa coupling measurement at the International Linear Collider, especially in the first phase of the ILC at 500 GeV. The experimental study [7] based on our results indicates that the top Yukawa coupling λ_t can be measured with $\Delta\lambda_t/\lambda_t \sim 10 - 15\%$ at 500 GeV. This is twice as accurate as anticipated by earlier studies based on the Born cross section.