PART I: Low-Energy Electron Scattering on ^{12}C and the Structure of the Hoyle State

1 Introduction

The production of the element carbon is a key reaction of stellar nucleosynthesis. The creation of its most abundant isotope ^{12}C is the result of the triple-alpha process [1] which takes place in stellar interiors at temperatures above 10^8 K and represents two successive resonance fusion reactions. In the first step the ^{8}Be nuclei are produced by fusion of two ^{4}He nuclei

$$^4\mathrm{He} \; + \; ^4\mathrm{He} \;\Leftrightarrow\; ^8\mathrm{Be}. \tag{1.1}$$

This reaction populates the short-lived ground state of ^{8}Be which decays back into two helium nuclei. However, at high temperatures a small amount of ^{8}Be in thermal equilibrium is formed. Capture of another alpha particle by ^{8}Be leads to formation of ^{12}C in an excited 0^+ state at an excitation energy $E_x = 7.654$ MeV,

$$^8\mathrm{Be} \; + \; ^4\mathrm{H}e \;\Leftrightarrow\; ^{12}\mathrm{C}^*(7.654\ \mathrm{MeV}). \tag{1.2}$$

The existence of a 0^+ resonance in ^{12}C close to the 3α threshold ($E_{3\alpha} = 7.275$ MeV) was postulated [2] by the astrophysicist Fred Hoyle in 1954 to explain the observed abundance of carbon in the Universe. Indeed, such a state (named Hoyle state hitherto) in ^{12}C was experimentally confirmed by Cook *et al.* [3] in 1957. Figure 1.1 shows a scheme of low-lying states in ^{12}C. Hoyle state produced in the reaction given by Eq. (1.2) preferably decays back into three alpha particles, but there is a small probability of a transition to the ground state by emission of a gamma cascade (γ) or an electron-positron pair ($e^{\pm}$).

Despite its astrophysical relevance, the carbon production rate through the above mentioned reactions is known with insufficient precision only [5,6]. For temperatures above 10^8 K the reaction rate $r_{3\alpha}$ can be written as

$$r_{3\alpha} \propto \Gamma_{rad}\, \exp\Big(-\frac{Q_{3\alpha}}{kT}\Big) \tag{1.3}$$

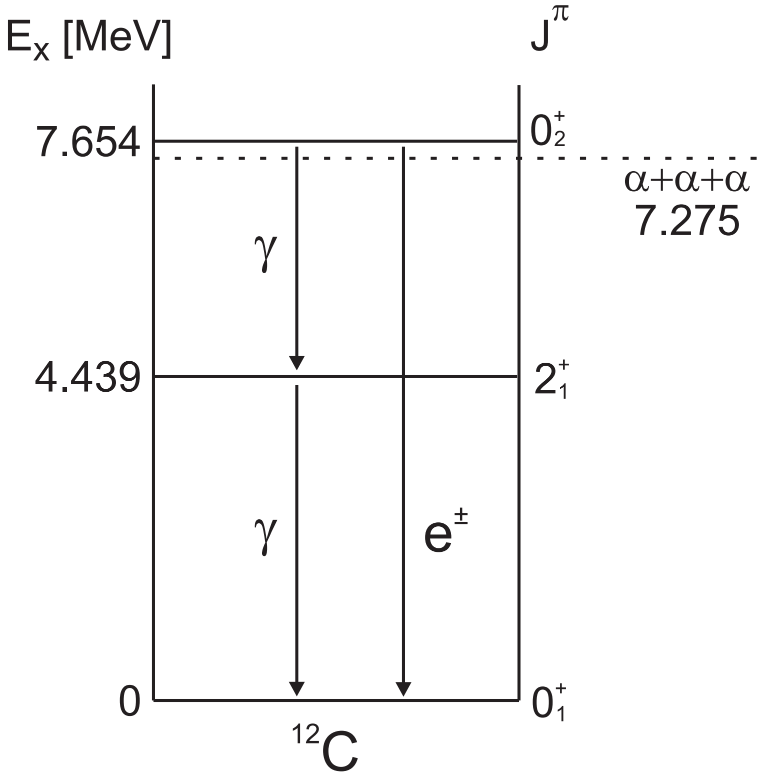

Fig. 1.1: Low-lying states in ^{12}C. Data are taken from Ref. [4]. After the triple-alpha fusion process the ground state is populated by decay of the Hoyle state ($E_x = 7.654$ MeV) through a gamma emission cascade (γ) or internal pair production ($e^{\pm}$).

with

$$Q_{3\alpha} = [M(^{12}\mathrm{C})\,c^2 + E_x] - 3\,M(^4\mathrm{He})\,c^2, \tag{1.4}$$

where Γ_{rad} is the radiative width of the Hoyle state, k the Boltzmann constant, T the temperature, c the velocity of light, $M(^{12}\mathrm{C})$ and $M(^4\mathrm{He})$ the atomic mass excesses of ^{12}C and ^{4}He, respectively, E_x the excitation energy of the Hoyle state, and $Q_{3\alpha}$ the energy released in the decay

$$^{12}\mathrm{C}^*(7.654\ \mathrm{MeV}) \rightarrow 3\,^4\mathrm{He}. \tag{1.5}$$

As seen from Eq. (1.3), for a precise determination of $r_{3\alpha}$ only Γ_{rad} and $Q_{3\alpha}$ need to be known. However, Γ_{rad} cannot be measured directly but depends on different experimental quantities discussed below. In Tab. 1.1 quantities which contribute to $r_{3\alpha}$ and their corresponding errors are summarized. The released

Tab. 1.1: Parameters contributing to the 3α reaction rate and their relative uncertainties.

Quantity	Value	Error (%)	Refs.
$Q_{3\alpha}$	379.38 ± 0.20 keV	1.2^a	[7–12]
Γ_{rad}/Γ	$(4.12 \pm 0.11)\times 10^{-4}$	2.7	[13–19]
Γ_π/Γ	$(6.74 \pm 0.62)\times 10^{-6}$	9.2	[20–22]
$\Gamma_\pi{}^b$	$(62.0 \pm 6.0)\times 10^{-6}$ eV	9.7	[24]
Γ_π	$(59.4 \pm 5.1)\times 10^{-6}$ eV	8.6	[25]
Γ_π	$(52.0 \pm 1.4)\times 10^{-6}$ eV	2.7	[26]

[a] Contribution to the 3α reaction rate at $T_9 = 0.2$ (in units of 10^9 K).
[b] For the extraction of the pair width a high-energy approximation [23] was used, whose application is questionable for the experimental energies. A generally valid expression is presented in Sec. 2.1.1. Its application would reduce the quoted value to $\Gamma_\pi = (58.9 \pm 5.6)\times 10^{-6}$ eV.

energy $Q_{3\alpha} = 379.38$ keV is known with an uncertainty ± 0.20 keV (see Ref. [12] and references therein). The uncertainty of the 3α process due to this factor is very small and corresponds to only $\pm 1.2\%$ at 2×10^8 K. This uncertainty decreases as $1/T$ with increasing temperature. Consequently, the main uncertainty in $r_{3\alpha}$ is due to the uncertainty in Γ_{rad} which is experimentally determined as a product of three independently measured quantities

$$\Gamma_{rad} = \Gamma_\gamma + \Gamma_\pi = \frac{\Gamma_\gamma + \Gamma_\pi}{\Gamma} \cdot \frac{\Gamma}{\Gamma_\pi} \cdot \Gamma_\pi, \tag{1.6}$$

where $\Gamma = \Gamma_\alpha + \Gamma_\gamma + \Gamma_\pi$ is the total decay width taking into account α, γ and $e^\pm$ decay of the Hoyle state. At present these three quantities are known with an accuracy (left to right) of $\pm 2.7\%$ [13–19], $\pm 9.2\%$ [20–22], and $\pm 6.4\%$ (from combination of values quoted in Refs. [24,25]), that leads to a total $r_{3\alpha}$ uncertainty of $\pm 11.6\%$. A new experiment for a precise measurement of the second quantity is discussed in Refs. [5, 27]. Additionally, a new result for the third quantity, *i.e.* the pair width Γ_π, with a quoted accuracy of $\pm 2.7\%$ is given by Crannell *et al.* [26]. It would reduce the total uncertainty of the triple-alpha process to $\pm 10.0\%$. Unfortunately, the new value is inconsistent within error bars with the earlier values of Γ_π [24, 25].

The present uncertainty of the triple-alpha reaction rate leads to a large uncertainty in the determination of the size of the pre-collapse iron core of the core-collapse supernova [5, 6]. For example, for a $25M_{sun}$ star the core mass has an uncertainty of about $0.2M_{sun}$. This uncertainty is important for the behavior of the supernova explosion, as it takes 3×10^{51} erg to dissociate $0.2M_{sun}$ into nucleons, which is similar to the energy released in the explosion. This example shows the importance of the precise determination of the triple-alpha reaction rate.

Thus, a main purpose of the present work is to resolve the existing inconsistency in the partial decay width Γ_π of the Hoyle state using the high-resolution electron scattering technique. Detailed formulae for the extraction of the pair width from such data are presented in Sec. 2.1.

On the other side the Hoyle state is playing a prominent role in nuclear structure as a prototype of α-cluster states in light nuclei. Unlike the ground state its description poses a continuing challenge to shell-model approaches. Even the most advanced no-core calculations using very large model spaces fail [28]. In fact, this state is not tangible in models using a harmonic oscillator basis. On the other hand, cluster models have been popular for describing the spectrum of ^{12}C [29–32]. Recently it has been pointed out that the Hoyle state can be viewed as a dilute gas of weakly interacting α particles resembling the properties of a Bose-Einstein condensate [33–38].

Thus, the second main goal of this work is the study of the structure of the both ground and the Hoyle state in ^{12}C by comparing high-precision electron scattering data with predictions of modern theoretical approaches, which are able to reproduce the main nuclear properties and thereby provide an opportunity to understand the intrinsic structure of the studied states. In the present work the calculations were performed using fermionic molecular dynamics, α cluster and Bose-Einstein condensate models. A detailed description of the models is given in Sec. 2.2.

Additionally, Part I of the thesis deals with another astrophysical topic, namely an investigation of the nucleosynthesis of fluorine. Its nucleosynthesis has been a subject of continuing discussions. This element, with one stable isotope ^{19}F, is located in the middle of very complex reaction networks and thus sensitive to various competing processes of production and destruction. Its solar abundance is a very small fraction of neighboring elements such as carbon and oxygen. As this element is near the heavy-end of CNO cycle, it can play an important intermediary role in the cold and hot CNO cycles and also in the breakout from the CNO cycle via the ^{19}F(p,γ) reaction [39]. The suggestions for the nucleosynthesis of this element vary from neutrino processes in explosive supernovae [40, 41] to helium burning in massive stars and thermal pulses [42]. The discovery of Jorissen *et al.* [42], that fluorine occurs in red giant stars with abundances of up to 30 times the solar value, has spurred a lot of activity on this topic. More recently, varying amounts of fluorine were found [43] in the Large Magellanic Cloud and ω Centauri, indicating that the current-day theoretical description of the fluorine abundance is far from being adequate. However, the clear trend between the oxygen and fluorine abundances seems to suggest pre-explosive nucleosynthesis involving a series of thermonuclear reactions in massive stars [43]. This, in turn, led to many model calculations re-evaluating the nuclear reaction networks but the issue of sources and abundances of ^{19}F is still unresolved [41, 44, 45].

Nuclear reactions which are relevant for the production of ^{19}F and its subsequent decay are shown in Fig. 1.2. Some earlier models [46], but also modern network calculations [47] include the population of ^{19}F via beta decay of ^{19}O as a possible channel during the *r*-process nucleosynthesis. This process is not considered to be very important on the ground that the ^{18}O(n,γ)^{19}O cross section is very small, *e.g.* $\langle\sigma\rangle \sim 8\ \mu$b for 25 keV neutrons [48]. However, an contribution of this channel can increase significantly at higher temperatures for a few reasons. Firstly, in an *r*-process scenario, the ^{16}C(α, n)^{19}O reaction might contribute as a source for ^{19}O, since ^{16}C is long lived with a half-life of 0.75 seconds and the reaction is exoergic with a Q value of 4.7 MeV. At an *r*-process site, nuclides with A = 19 and Z < 8, such as ^{19}N may be produced following neutron-capture reactions. Secondly, due to resonant contributions the neutron capture cross section increases by a factor 12 for neutrons at energies of about 400 keV [48]. Thirdly, as will be shown in this work the weak decay of ^{19}O may be enhanced by a factor of three at higher temperatures due to contributions from beta decay of the thermally populated first excited level at $E_x = 96$ keV in ^{19}O.

Part I of this thesis is structured as follows. Section 2 deals with the basics of the electron scattering formalism and gives a short introduction to the theoretical ap-

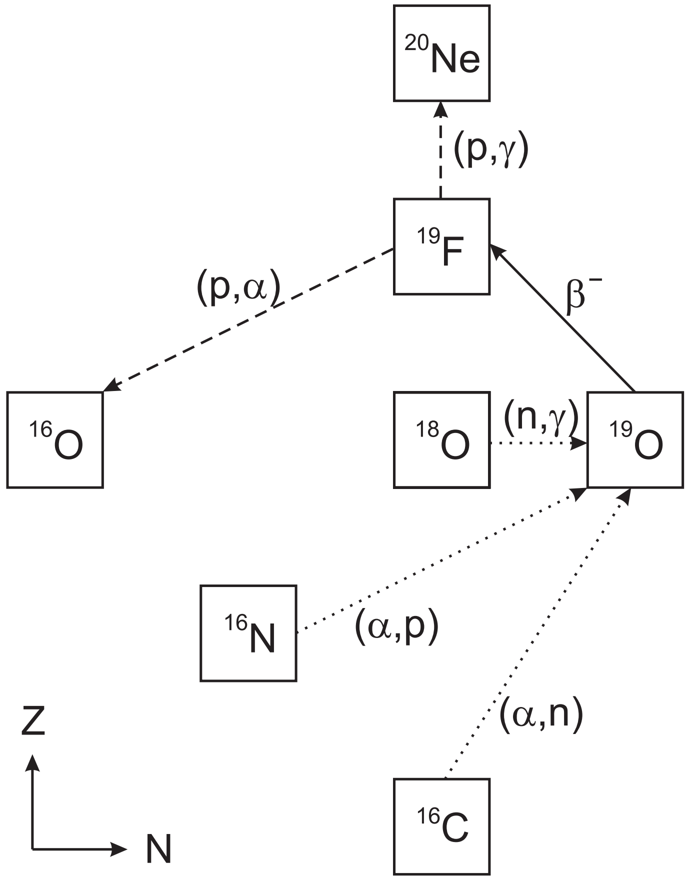

Fig. 1.2: Possible mechanisms relevant for production of ^{19}F via ^{19}O(β^-) reaction and breakout from the CNO cycle. The leakage from the CNO cycle depends on the abundance of ^{19}F, reaction rates of ^{19}F(p,γ)^{20}Ne, and the competing back-processing reaction ^{19}F(p,α)^{16}O.

proaches used. The high-resolution electron scattering facility at the S-DALINAC and details of the experiment are described in Sec. 3. Section 4 deals with the analysis of the measured spectra and the extraction of physical quantities. Results and their comparison to the model predictions are presented in Sec. 5. The role of β decay from the first excited state in ^{19}O for the abundance of ^{19}F is presented in Sec. 6. Finally, a short summary and outlook are given in Sec. 7. Parts of the results of this thesis are published in Refs. [49] and [50].