1

Fundamentals of Optical Parametric Oscillators

"Reality is frequently inaccurate."

Douglas Adams, "The Restaurant at the End of the Universe"

An Optical Parametric Oscillator (OPO) transfers energy of an intense electromagnetic wave called the pump wave at frequency ν_p to two other waves of different frequency. These are called the signal (frequency ν_s) and idler wave (frequency ν_i). This effect opens up new spectral regimes for coherent radiation where it is difficult to provide suitable lasers. In the scope of this work, optical parametric oscillators have been built for spectroscopic applications. A simple theoretical treatment necessary to understand the effects will be given in this chapter*.

The interaction of waves is a consequence of the nonlinear response of the polarization to the electric field, which will be introduced in the first section. The following sections describe the propagation of electromagnetic waves in a nonlinear medium. The effect of parametric amplification is presented quantitatively with the help of the so-called coupled wave equations. A section about optical parametric oscillation is followed by considerations about matching the phase velocities of the relevant waves, which is important to achieve efficient frequency conversion. The derivation of the cou-

*Parts of this chapter have been published in:

- A continuous-wave optical parametric oscillator around 5 μm wavelength for high-resolution spectroscopy. J. Krieg, A. Klemann, I. Gottbehüt, S. Thorwirth, T. F. Giesen, S. Schlemmer, *Review of Scientific Instruments* **82** 063195 (2011) pled wave equations mainly follows Ref. [66], and solutions to those can be found with different ansatzes in Refs. [66, 100]. When absorption can be neglected, a comprehensive theoretical introduction to OPOs can also be found in Ref. [16].

1.1 The Nonlinear Optical Susceptibility

The superposition principle of electromagnetic waves is an implication of the linearity of Maxwell's equations. As a result, waves of different frequency interfere and do not interact with each other. If the waves travel in a dielectric medium, the polarization of the medium has to be taken into account. Usually, the relation between the polarization $\mathbf{P}$ and the electric field $\mathbf{E}$ is also linear:

$$P_i = \varepsilon_0 \sum_j \chi_{ij} E_j$$

with the electric susceptibility χ, which is a tensor of rank 2. In this context, Maxwell's equations are still linear and electric fields just superimpose and energy transfer between different waves is impossible.

The linear relation for the polarization does not hold in the case of strong electric fields, as for example in a focused laser beam. Then the polarization of the medium deviates from the linear expression above and has to be expanded in terms of the electric field:

$$P_i = \varepsilon_0 \left(\sum_j \chi^{(1)}_{ij} E_j + \sum_j \sum_k \chi^{(2)}_{ijk} E_j E_k + \mathcal{O}(\mathbf{E}^3) \right) , \tag{1.1}$$

where $\chi^{(2)}$ is a tensor of rank 3 and is referred to as the second-order susceptibility. The quadratic terms in the above expression including $\chi^{(2)}$ explain all nonlinear optical effects relevant for this work. Because of the nonlinearity of the polarization, waves of different frequency can now interact with each other.

$\chi^{(2)}$ as a tensor of rank 3 should have 27 elements, but symmetry considerations (see, e.g. Ref. [66, 104]) lead to a reduction of the elements. Often the tensor $d_{ijk} = \frac{1}{2}\chi_{ijk}$ is given instead of χ_{ijk} and can be written in a reduced notation, where the last two indices (j, k) form a new index l. d_{il} is in this notation a 3×6 matrix with the translation

l:	1	2	3	4	5	6
jk:	11	22	33	23, 32	31, 13	12, 21

The quantity of independent elements can be further reduced in a given material depending on its symmetry. For example, the crystal used in the scope of this work is Lithium-Niobate ($LiNbO_3$), and its nonlinear susceptibility in reduced notation is

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & -4.6 & -2.6 \\ -2.6 & 2.6 & 0 & -4.6 & 0 & 0 \\ -4.6 & -4.6 & -25.0 & 0 & 0 & 0 \end{pmatrix} \cdot 10^{-12} \, \frac{\mathrm{m}}{\mathrm{V}} \tag{1.2}$$

and has only three independent elements, of which d_{33} is the largest.

1.2 Electromagnetic Waves in Optically Nonlinear Media

In a nonlinear dielectric, where there are no free charges ($\rho = 0$), no electric current ($\mathbf{j} = 0$) and no magnetization ($\mathbf{M} = 0$), Maxwell's equations in matter take the form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{D} = 0 \tag{1.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial \mathbf{D}}{\partial t} \qquad \nabla \cdot \mathbf{B} = 0. \tag{1.4}$$

Here, $\mathbf{B}$ is the magnetic induction, and $\mathbf{D}$ is the electric displacement which can be related to $\mathbf{E}$:

$$\mathbf{D} = \varepsilon\varepsilon_0 \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}.$$

When we apply the operator ($\nabla\times$) to the first of Maxwell's equations and substitute the third equation in there, we obtain

$$\begin{aligned} \nabla \times \nabla \times \mathbf{E} &= -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \\ &= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) \\ &= -\mu_0 \frac{\partial^2}{\partial t^2}\mathbf{D} \\ &= -\varepsilon_0\mu_0 \frac{\partial^2}{\partial t^2}\mathbf{E} - \mu_0 \frac{\partial^2}{\partial t^2}\mathbf{P}. \end{aligned}$$

The vector calculus identity $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ can be simplified by arguing that $\nabla(\nabla \cdot \mathbf{E})$ vanishes in the case of plane waves (Ref. [66]), which is why we find

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}.$$

The linear term of the polarization can be split off using $\mathbf{P} = \mathbf{P}^{(1)} + \mathbf{P}^{(\mathrm{NL})} = \varepsilon_0(n^2-1)\mathbf{E} + \mathbf{P}^{(\mathrm{NL})}$, where $\mathbf{P}^{(\mathrm{NL})}$ is the nonlinear part of the polarization. It results in the wave equation in a nonlinear dielectric:

$$\left(\nabla^2 - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}^{(\mathrm{NL})}.$$

This equation forms the basis to describe nonlinear optical effects. Wave equations for different frequency components couple through the nonlinear polarization, which can be seen in the next section. The electric field can be written as a superposition of plane waves with a frequency ω_l and a wave-vector $\mathbf{k}$.

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{2}\sum_l \left[\boldsymbol{\mathcal{E}}_l(\mathbf{r})e^{-i(\omega_l t - \mathbf{k_l r})} + c.c.\right]. \tag{1.5}$$

$c.c.$ means the complex conjugate. Analogously, the polarization is expressed using the same frequency components:

$$\mathbf{P}^{(\mathrm{NL})} = \frac{1}{2}\sum_l \left[\boldsymbol{\mathcal{P}}(\omega_l,\mathbf{r})e^{-i\omega_l t} + c.c.\right]. \tag{1.6}$$

In this context, the wave equation takes the form

$$\left(\nabla^2 + \frac{n_l^2\omega_l^2}{c^2}\right)\boldsymbol{\mathcal{E}}_l(\mathbf{r})e^{i\mathbf{k_l r}} = -\frac{\omega_l^2}{\varepsilon_0 c^2}\boldsymbol{\mathcal{P}}(\omega_l,\mathbf{r}). \tag{1.7}$$

1.2.1 Coupled Wave Equations

We assume plane waves traveling in forward z-direction and the electric field oscillating in x-direction, which gives $\mathbf{k_l} = k_l\hat{\mathbf{e}}_z$ and $\boldsymbol{\mathcal{E}}_l(\mathbf{r}) = \mathcal{E}_l(z)\hat{\mathbf{e}}_x$:

$$\nabla^2\boldsymbol{\mathcal{E}}_l(\mathbf{r})e^{i\mathbf{k_l r}} = \hat{\mathbf{e}}_x\frac{\partial^2}{\partial z^2}\left[\mathcal{E}_l(z)e^{ik_l z}\right] \tag{1.8}$$

$$= \hat{\mathbf{e}}_x e^{ik_l z}\left[\underbrace{\frac{\partial^2}{\partial z^2}\mathcal{E}_l(z)}_{\approx 0} + 2ik_l\frac{\partial}{\partial z}\mathcal{E}_l(z) - k_l^2\mathcal{E}_l(z)\right]. \tag{1.9}$$

In the last equation, the term $\frac{\partial^2}{\partial z^2}\mathcal{E}_l(z)$ is neglected, because the amplitude is supposed to vary slowly with distance z. With $k_l^2 = n_l^2\omega_l^2/c^2$, the x-component of equation (1.7) reduces to

$$\frac{\partial}{\partial z}\mathcal{E}_l(z) = i\frac{\omega_l}{2\varepsilon_0 n_l c}\mathcal{P}_x(\omega_l,z)e^{-ik_l z}. \tag{1.10}$$

This equation has to be solved for every frequency component ω_l of the electric field relevant in the given nonlinear process.

1.3 Optical Parametric Amplification

The effect of optical parametric amplification (OPA) describes the enhancement of an optical wave at frequency ν_s (”signal”) at the cost of the attenuation of an intense wave called ”pump” at frequency $\nu_p > \nu_s$. For reasons of energy conservation, a third wave is generated at frequency ν_i, so that the following condition is fulfilled:

$$\nu_p = \nu_s + \nu_i. \tag{1.11}$$

In this process, the electric field can be expressed using three frequency components. Assuming plane waves which are all linearly polarized in x-direction and traveling in $+z$-direction ($\mathbf{k_l} = k_l \hat{\mathbf{e}}_z$), we have:

$$\mathbf{E} = \frac{\hat{\mathbf{e}}_x}{2}\left[\mathcal{E}_p e^{-i(\omega_p t - k_p z)} + \mathcal{E}_s e^{-i(\omega_s t - k_s z)} + \mathcal{E}_i e^{-i(\omega_i t - k_i z)} + c.c.\right]. \tag{1.12}$$

The three waves do not necessarily have to have the same polarization axis to couple via the nonlinear polarization, because the tensor $\chi^{(2)}$ is in general not diagonal. However, the assumption of one single polarization direction for all involved waves is more illustrative and also holds for the parametric process used in this work.

In the following sections, the coupled wave equations (1.10) are derived for this explicit case and solved to get a quantitative understanding.

1.3.1 Coupled Wave Equations for Parametric Processes

The x-component of the nonlinear polarization $P_x^{(\mathrm{NL})} = 2\varepsilon_0 d E_x^2$ contains many linear combinations of the frequencies $\omega_p, \omega_s, \omega_i$. Due to interference, only waves which have a fixed phase relationship to their driving polarization build up to a macroscopic scale. This issue will be addressed in section 1.5. Now it is assumed that only the following polarization components are relevant as in a parametric process:

$$\mathcal{P}_x(\omega_p = \omega_s + \omega_i, z) = 2\varepsilon_0 d\, \mathcal{E}_s \mathcal{E}_i \quad e^{(k_i + k_s)z} \tag{1.13}$$
$$\mathcal{P}_x(\omega_s = \omega_p - \omega_i, z) = 2\varepsilon_0 d\, \mathcal{E}_p \mathcal{E}_i^* \ \ e^{(k_p - k_i)z} \tag{1.14}$$
$$\mathcal{P}_x(\omega_i = \omega_p - \omega_s, z) = 2\varepsilon_0 d\, \mathcal{E}_p \mathcal{E}_s^* \, e^{(k_p - k_s)z}. \tag{1.15}$$

By substituting these polarizations into equation (1.10), one obtains the coupled wave equations for three-wave-mixing:

$$\frac{\partial}{\partial z}\mathcal{E}_p(z) = i\gamma_p\,\mathcal{E}_s(z)\mathcal{E}_i(z)\,e^{-i\Delta kz} - \frac{\alpha_p}{2}\mathcal{E}_p(z) \tag{1.16}$$

$$\frac{\partial}{\partial z}\mathcal{E}_s(z) = i\gamma_s\,\mathcal{E}_p(z)\mathcal{E}_i^*(z)\,e^{+i\Delta kz} - \frac{\alpha_s}{2}\mathcal{E}_s(z) \tag{1.17}$$

$$\frac{\partial}{\partial z}\mathcal{E}_i(z) = i\gamma_i\,\mathcal{E}_p(z)\mathcal{E}_s^*(z)\,e^{+i\Delta kz} - \frac{\alpha_i}{2}\mathcal{E}_i(z). \tag{1.18}$$

Here, γ_l ($l = p$,s,i) is the nonlinear coupling factor $\gamma_l = (2\pi\nu_l d)/(n_l c)$ and $\Delta k = k_p - k_s - k_i$ is called the phase mismatch of the process. Absorption of the waves in the crystal was added using a term $-\frac{\alpha_l}{2}\mathcal{E}_l$ in the coupled wave equations, because in this work the absorption cannot be neglected. α_l is then the absorption coefficient from the Lambert-Beer law for the intensity of a wave $I_l(z) = I_l(0)e^{-\alpha_l z}$, where I_l can be expressed as $I_l = c\varepsilon_0 n_l|\mathcal{E}_l|^2/2$. The intensity of the plane wave is connected to its power P by the beam area A: $I_l = P_l/A$.

By comparing $\frac{\partial}{\partial z}|\mathcal{E}_l|^2 = \mathcal{E}_l^*\frac{\partial}{\partial z}\mathcal{E}_l + \mathcal{E}_l\frac{\partial}{\partial z}\mathcal{E}_l^*$ of every of the above equation, one obtains the Manley-Rowe relations including correction terms for the absorption in the nonlinear medium:

$$-\frac{1}{\nu_p}\left(\frac{\partial}{\partial z} - \alpha_p\right) I_p = \frac{1}{\nu_s}\left(\frac{\partial}{\partial z} - \alpha_s\right) I_s = \frac{1}{\nu_i}\left(\frac{\partial}{\partial z} - \alpha_i\right) I_i. \tag{1.19}$$

1.3.2 Solution of the Coupled Wave Equations for a Parametric Process

To solve the coupled wave equations including absorption, we assume that the incident pump wave has a constant amplitude over the length of the nonlinear medium: $\mathcal{E}_p(z) = \mathcal{E}_p(0) = \mathcal{E}_p$. This approximation holds when the pump wave is much stronger than the other waves and the absorption of the pump in the crystal can also be neglected. Because of $\frac{\partial}{\partial z}\mathcal{E}_p(z) = 0$, the problem reduces to two coupled equations:

$$\frac{\partial}{\partial z}\mathcal{E}_s(z) = i\gamma_s\,\mathcal{E}_p\mathcal{E}_i^*(z)\,e^{+i\Delta kz} - \frac{\alpha_s}{2}\mathcal{E}_s(z) \tag{1.20}$$

$$\frac{\partial}{\partial z}\mathcal{E}_i(z) = i\gamma_i\,\mathcal{E}_p\mathcal{E}_s^*(z)\,e^{+i\Delta kz} - \frac{\alpha_i}{2}\mathcal{E}_i(z). \tag{1.21}$$

With the ansatz $\mathcal{E}_l(z) = \tilde{\mathcal{E}}_l e^{(\Gamma + i\frac{\Delta k}{2})z}$, where $\tilde{\mathcal{E}}_l$ are constant amplitudes independent of z, we find a characteristic equation for Γ:

$$\left[\left(\Gamma + i\frac{\Delta k}{2} + \frac{\alpha_s}{2}\right)\left(\Gamma - i\frac{\Delta k}{2} + \frac{\alpha_i}{2}\right) - \gamma_s\gamma_i|\mathcal{E}_p|^2\right]\tilde{\mathcal{E}}_s = 0. \tag{1.22}$$

For the non-trivial case $\tilde{\mathcal{E}}_s \neq 0$, the term in the square brackets vanishes, which results in a quadratic equation for Γ with two independent results $\Gamma_\pm$:

$$\Gamma_\pm = \underbrace{-\frac{1}{4}(\alpha_i + \alpha_s)}_{=\alpha} \pm \underbrace{\sqrt{\gamma_s\gamma_i|\mathcal{E}_p|^2 + \delta^2}}_{=g}, \tag{1.23}$$

where the parameter $\delta = (\alpha_i - \alpha_s - 2i\Delta k)/4$ has been introduced. Now the coupled wave equations take the shape

$$\mathcal{E}_s(z) = \left(\tilde{\mathcal{E}}_s^+ e^{+gz} + \tilde{\mathcal{E}}_s^- e^{-gz}\right) e^{-\alpha z} e^{i\frac{\Delta k}{2}z} \tag{1.24}$$

$$\mathcal{E}_i(z) = \left(\tilde{\mathcal{E}}_i^+ e^{+gz} + \tilde{\mathcal{E}}_i^- e^{-gz}\right) e^{-\alpha z} e^{i\frac{\Delta k}{2}z}, \tag{1.25}$$

with the abbreviations $\alpha = (\alpha_s + \alpha_i)/4$ as a mean value for signal and idler absorption coefficients, and $g^2 = \gamma_s\gamma_i|\mathcal{E}_p|^2 + \delta^2 = g_\star^2 + \delta^2$ as a driving term including pump intensity, nonlinear coupling factors, and also absorption coefficients and phase mismatch. For an OPO being pumped at 1064 nm wavelength, g is dominated by the absorption parameter δ for long idler wavelengths $\lambda_i > 4.5$ μm, as an example may illustrate: Using a 1 W pump laser focused down to a beam radius of 100 μm producing perfectly phase matched 5 μm wavelength of idler radiation, $\gamma_s\gamma_i|\mathcal{E}_p|^2 \approx 40$ m^{-1} and $\delta^2 \approx 625$ m^{-1}.

The amplitude coefficients $\tilde{\mathcal{E}}_{s,i}^\pm$ have to be determined by boundary conditions. Hence, we set the amplitudes of signal and idler at the front facet of the nonlinear medium $\mathcal{E}_i(0) = \mathcal{E}_{i0}$ and $\mathcal{E}_s(0) = \mathcal{E}_{s0}$, respectively. At position $z = 0$, equations (1.24) and (1.25) give the relations $\tilde{\mathcal{E}}_s^- = \mathcal{E}_{s0} - \tilde{\mathcal{E}}_s^+$ and $\tilde{\mathcal{E}}_i^- = \mathcal{E}_{i0} - \tilde{\mathcal{E}}_i^+$. By further comparison of $\frac{\partial}{\partial z}\mathcal{E}_s(z)\big|_{z=0}$ and $\frac{\partial}{\partial z}\mathcal{E}_i(z)\big|_{z=0}$ with the right sides of equations (1.20) and (1.21) at position $z = 0$, one finally obtains for the amplitude coefficients:

$$\tilde{\mathcal{E}}_s^\pm = \pm\frac{1}{2g}\left[i\gamma_s\mathcal{E}_p\mathcal{E}_{i0}^* - \left(\Gamma_\mp + i\frac{\Delta k}{2} + \frac{\alpha_s}{2}\right)\mathcal{E}_{s0}\right] \tag{1.26}$$

$$\tilde{\mathcal{E}}_i^\pm = \pm\frac{1}{2g}\left[i\gamma_i\mathcal{E}_p\mathcal{E}_{s0}^* - \left(\Gamma_\mp + i\frac{\Delta k}{2} + \frac{\alpha_i}{2}\right)\mathcal{E}_{i0}\right]. \tag{1.27}$$

The solutions can now be expressed as follows:

$$\mathcal{E}_s(z) = \left[\mathcal{E}_{s0}\cosh(gz) + \frac{1}{g}\left(i\gamma_s\mathcal{E}_p\mathcal{E}_{i0}^* + \delta\mathcal{E}_{s0}\right)\sinh(gz)\right] e^{-\alpha z} e^{i\frac{\Delta k}{2}z} \tag{1.28}$$

$$\mathcal{E}_i(z) = \left[\mathcal{E}_{i0}\cosh(gz) + \frac{1}{g}\left(i\gamma_i\mathcal{E}_p\mathcal{E}_{s0}^* + \delta\mathcal{E}_{i0}\right)\sinh(gz)\right] e^{-\alpha z} e^{i\frac{\Delta k}{2}z}. \tag{1.29}$$

In the parametric process of a singly resonant OPO, there is no idler wave incident at the front facet of the nonlinear crystal, which gives $\mathcal{E}_{i0} = \mathcal{E}_{i0}^* = 0$, while the signal wave amplitude is non-zero because of the constant feedback in the optical cavity: $\mathcal{E}_{s0}, \mathcal{E}_{s0}^* \neq 0$. These two conditions lead to:

$$\mathcal{E}_s(z) = \mathcal{E}_{s0}\left[\cosh(gz) + \frac{\delta}{g}\sinh(gz)\right] e^{-\alpha z} e^{i\frac{\Delta k}{2}z} \tag{1.30}$$

$$\mathcal{E}_i(z) = \mathcal{E}_{s0}^*\left[i\frac{\gamma_i \mathcal{E}_p}{g}\sinh(gz)\right] e^{-\alpha z} e^{i\frac{\Delta k}{2}z}. \tag{1.31}$$

Note that these relations have been derived using the approximation of a constant pump field over the entire nonlinear crystal, thus they are only valid in the low conversion efficiency regime. They can be used to derive the threshold condition (see section 1.4), because the signal field present in the cavity close to threshold is small.

Neglecting absorption by setting $\alpha_i = \alpha_s = 0$, we find for the intensity $I_l(z)$ of the waves:

$$I_s(z) = I_{s0}\left[1 + (g_\star z)^2 \frac{\sinh^2(gz)}{(gz)^2}\right] \tag{1.32}$$

$$I_i(z) = I_{s0}\frac{\nu_i}{\nu_s}(g_\star z)^2 \frac{\sinh^2(gz)}{(gz)^2}. \tag{1.33}$$

And for perfect phase matching $\Delta k = 0$, we find

$$I_s(z) = I_{s0}\left(\cosh(gz) + \frac{\delta}{g}\sinh(gz)\right)^2 e^{-2\alpha z} \approx I_{s0}e^{2(g-\alpha)z} \tag{1.34}$$

$$I_i(z) = I_{s0}\frac{\nu_i}{\nu_s}(g_\star z)^2 \frac{\sinh^2(gz)}{(gz)^2} e^{-2\alpha z}. \tag{1.35}$$

The approximation for the signal intensity is valid for strong idler absorption $\alpha^2 \gg \gamma_s \gamma_i |\mathcal{E}_p|^2$.

1.3.3 Parametric Gain

The gain G_s the signal wave experiences upon a single pass through the nonlinear medium of length L can be expressed as follows:

$$G_s = \frac{I_s(L) - I_s(0)}{I_s(0)} = \frac{I_s(L)}{I_s(0)} - 1 = \frac{|\mathcal{E}_s(L)|^2}{|\mathcal{E}_s(0)|^2} - 1. \tag{1.36}$$

Substituting the solutions found above results in the relation

$$G_s = \left(\cosh(gL) + \frac{\delta}{g}\sinh(gL)\right)^2 e^{-2\alpha L} - 1. \tag{1.37}$$

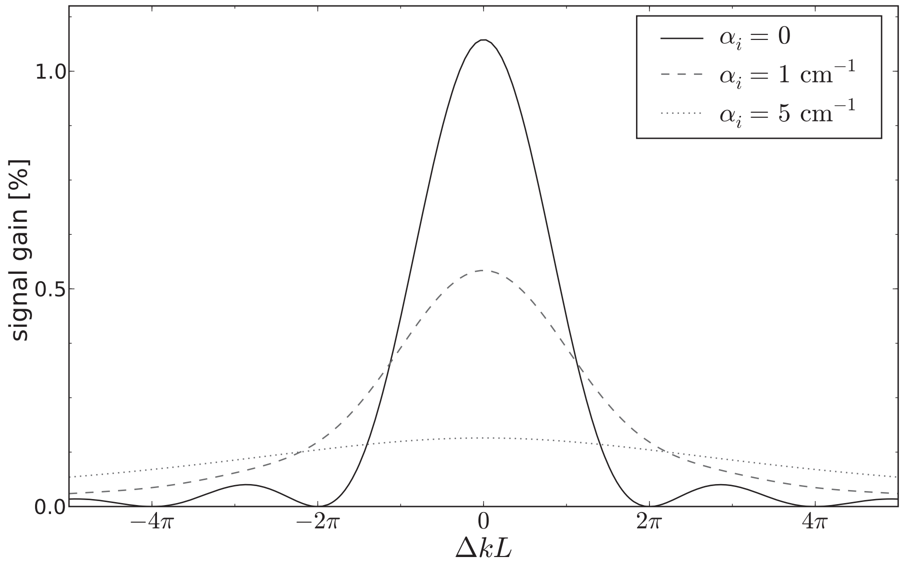

Figure 1.1: Signal gain profiles for different absorption coefficients of the idler wave. With increasing absorption, the maximum decreases and the profile gets broader. Side-lobes vanish. The pump laser has a wavelength of $\lambda_p = 1064$ nm, a power of 1 W and a beam radius of 100 μm. Idler wavelength is $\lambda_i = 5000$ nm, the nonlinear crystal has a length of 5 cm and an effective nonlinear coefficient $d = 17$ pm/V. Myers *et al.* reported an idler absorption coefficient of $\alpha_i \approx 1$ cm^{-1} for this idler frequency [74], corresponding to the dashed red line.

Figure 1.1 shows the gain curve $G_s(\Delta kL)$ calculated for different idler absorption coefficients. Note that the gain in the signal wave gets smaller, even though absorption is only present in the idler wave. Also the gain curve gets broader when the idler absorption increases.

When we neglect absorption ($\alpha_i = \alpha_s = 0$) and assume perfect phase matching ($\Delta k = 0$), the parameter δ equals zero and the signal gain reduces to

$$G_s = \sinh^2(g_\star L) \approx g_\star^2 L^2 + \mathcal{O}(g_\star^4 L^4) \tag{1.38}$$

with the abbreviation $g_\star^2 = g^2|_{(\alpha=\Delta k=0)} = (2\omega_s\omega_i d^2 I_p)/(\varepsilon_0 n_p n_s n_i c^3)$.

For perfect phase matching ($\Delta k = 0$) but strong idler absorption ($\alpha^2 \gg \gamma_s\gamma_i|\mathcal{E}_p|^2$), the gain can be approximated as

$$G_s \approx e^{2(g-\alpha)L} - 1 \tag{1.39}$$

$$\approx 2(g-\alpha)L. \tag{1.40}$$

In this case, the gain increases only linear with length instead of quadratically. Figure 1.2 illustrates this behavior.

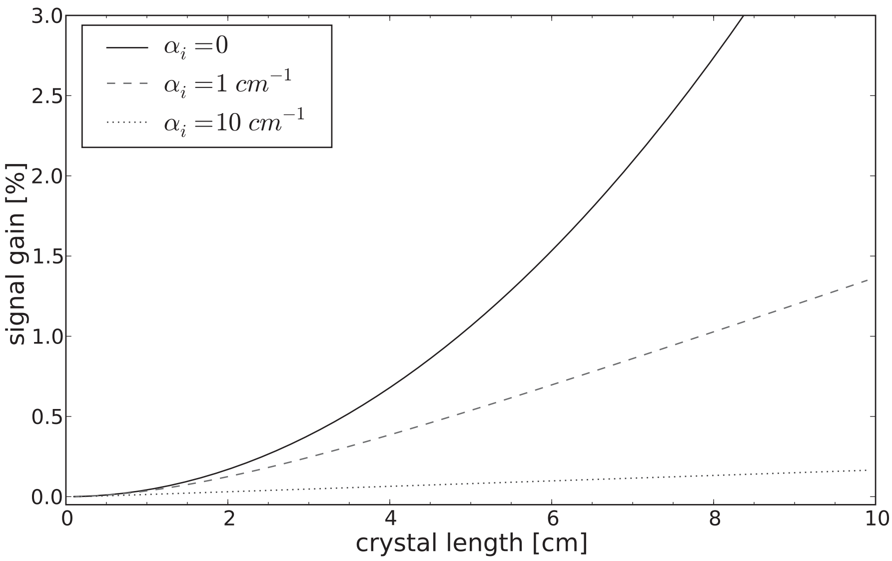

Figure 1.2: Signal gain dependency on crystal length for perfect phase matching and zero absorption of signal radiation for different idler absorptions. In the limit of zero absorption, gain grows quadratically with length. With increasing idler absorption, gain grows only linearly. The pump laser has a wavelength of $\lambda_p = 1064$ nm, a power of 1 W and a beam radius of 100 μm. Idler wavelength is $\lambda_i = 5000$ nm, the nonlinear crystal has an effective nonlinear coefficient $d = 17$ pm/V.

1.4 Optical Parametric Oscillation

An optical parametric oscillator (OPO) bases on the optical parametric amplification (OPA) as described above. Thus in an OPO, energy is converted from an intense field of radiation at frequency ν_p — called the pump wave — to two other waves of different frequency, which are called signal wave (frequency ν_s) and idler wave (frequency ν_i). The designation is made in a way so that $\nu_p > \nu_s > \nu_i$. In this process, the energy is conserved, which results in the frequency relation (1.11).

If the signal wave is amplified as shown in Sec. 1.3.3, it can also start to oscillate when the amplified signal is fed back to the front side of the nonlinear medium. Since the conversion efficiency depends on the intensity of all three waves in the parametric process, a feedback (and oscillation) can enhance the process by orders of magnitude. The following sections give an insight to effects in OPOs.

1.4.1 Threshold

With increasing pump power, the signal gain grows approximately linear according to equation (1.37). The feedback in the signal wave incorporates