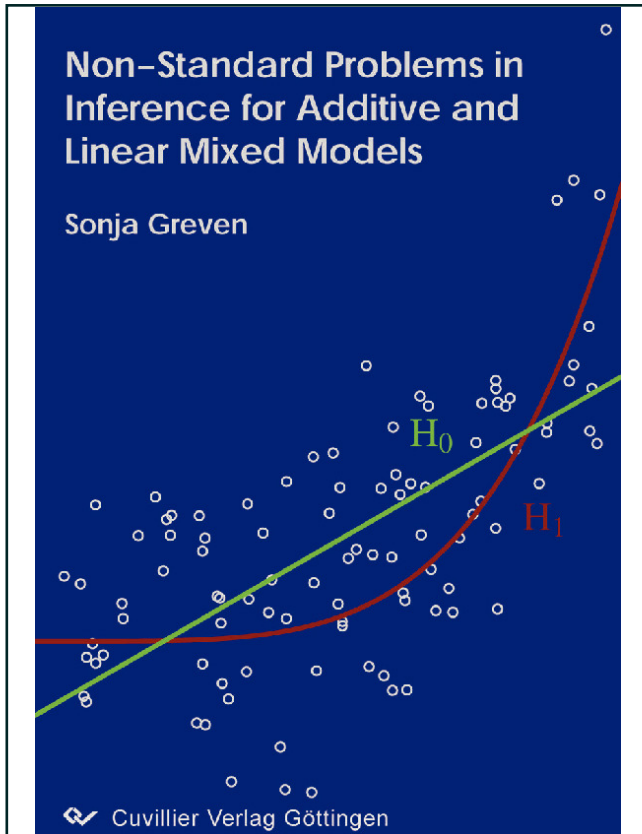




Sonja Greven (Autor)

## Non-Standard Problems in Inference for Additive and Linear Mixed Models



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