

# Contents

Zusammenfassung . . . . .	i
Abstract . . . . .	ii
Preface . . . . .	iii
Acknowledgments . . . . .	v
<b>1 Planning in Public Transit</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Classification of Planning Steps . . . . .	3
1.3 Basic Models . . . . .	3
1.3.1 Flow Based Models . . . . .	4
1.3.2 Path Based Models . . . . .	6
1.4 Network Design . . . . .	8
1.4.1 Description . . . . .	8
1.4.2 Models . . . . .	8
1.4.3 Applications . . . . .	9
1.5 Line Planning . . . . .	10
1.5.1 Description . . . . .	10
1.5.2 Models . . . . .	10
1.5.3 Approaches . . . . .	12
1.5.4 Integration . . . . .	13
1.6 Planning of Bus Stops . . . . .	13

---

1.7	Timetabling . . . . .	14
1.7.1	Description . . . . .	14
1.7.2	Models . . . . .	15
1.7.3	Algorithms . . . . .	16
1.7.4	Integration . . . . .	17
1.8	Planning of Public Tenderings . . . . .	17
1.9	Vehicle Scheduling . . . . .	18
1.9.1	Description . . . . .	18
1.9.2	Graph Theoretic Model . . . . .	19
1.9.3	Integer Programming Model . . . . .	21
1.9.4	Algorithms . . . . .	21
1.9.5	Algorithm VS-OPT . . . . .	22
1.9.6	Integration . . . . .	23
1.10	Duty scheduling . . . . .	24
1.10.1	Description . . . . .	24
1.10.2	Graph Theoretic Model . . . . .	25
1.10.3	Integer Programming Model . . . . .	27
1.10.4	Algorithms . . . . .	28
1.10.5	Integration . . . . .	28
1.11	Rostering . . . . .	28
1.11.1	Description . . . . .	29
1.11.2	Model . . . . .	30
1.11.3	Algorithms . . . . .	30
1.11.4	Integration . . . . .	31
1.12	Conclusion . . . . .	31

<b>2</b>	<b>Integration of Vehicle and Duty Scheduling</b>	<b>33</b>
2.1	Motivation . . . . .	33
2.1.1	Regional Public Transit . . . . .	34
2.1.2	Vehicle and Duty Schedule Efficiency . . . . .	35
2.1.3	Vehicle and Duty Costs . . . . .	36
2.2	Approaches to Integrated Scheduling . . . . .	37
2.2.1	Duty Scheduling with Vehicle Scheduling Constraints . . . . .	38
2.2.2	The Combined Approach . . . . .	39
2.2.3	Full Integration of Vehicle and Duty Scheduling . . . . .	42
2.3	Literature . . . . .	43
2.3.1	Ball, Bodin and Dial . . . . .	44
2.3.2	Vehicle Scheduling Centered Approaches . . . . .	44
2.3.3	Duty Scheduling Centered Approaches . . . . .	45
2.3.4	Fully Integrated Vehicle and Duty Scheduling . . . . .	47
2.4	IS-OPT . . . . .	50
2.4.1	Outline of our ISP-Algorithm . . . . .	50
2.4.2	Contributions . . . . .	52
<b>3</b>	<b>Basic Methodology</b>	<b>53</b>
3.1	Column Generation . . . . .	53
3.2	Lagrangian Relaxation . . . . .	55
3.2.1	Lagrangian Relaxation in General . . . . .	55
3.2.2	Linear Programming Duality . . . . .	56
3.2.3	Quadratic Programming Duality . . . . .	57
3.3	Lagrangian Relaxation for Column Generation . . . . .	58
3.3.1	Problem Class . . . . .	59
3.3.2	Restricted Problem . . . . .	59
3.3.3	Pricing Problem . . . . .	60
3.3.4	Lagrangian relaxation . . . . .	60
3.3.5	Reduced Cost Shifting . . . . .	62

---

<b>4</b>	<b>Proximal Bundle Method</b>	<b>65</b>
4.1	Description . . . . .	66
4.1.1	Idea and Properties . . . . .	66
4.1.2	Subgradients, Linearizations, and Cutting Plane Models	68
4.1.3	Quadratic Subproblem . . . . .	68
4.1.4	Algorithm . . . . .	72
4.1.5	Weight updating . . . . .	73
4.1.6	Notes On The Convergence . . . . .	73
4.2	Comparison with other Subgradient Methods . . . . .	74
4.3	Modifications and Extensions . . . . .	75
4.3.1	Separable Functions . . . . .	75
4.3.2	Primal Approximation of Linear Programs . . . . .	76
4.3.3	Handling of Bounded Functions . . . . .	78
4.4	Active Set Method . . . . .	80
4.4.1	Description . . . . .	81
4.4.2	Exact Active Sets . . . . .	83
4.5	Applications . . . . .	84
4.5.1	Approximating the Duty Scheduling Problem . . . . .	85
4.5.2	Approximating the Problem ISP . . . . .	87
4.6	Inexact Bundle Method . . . . .	88
4.6.1	Literature . . . . .	89
4.6.2	Vehicle Scheduling Component Function . . . . .	90
4.6.3	Duty Scheduling Component Function . . . . .	91
4.6.4	Combined Functions . . . . .	98
4.7	Computational Results . . . . .	100
4.7.1	Testbed . . . . .	100
4.7.2	Results . . . . .	101

---

<b>5</b>	<b>The Generation of Duties</b>	<b>107</b>
5.1	Motivation and Notation . . . . .	107
5.1.1	Master Problem . . . . .	108
5.1.2	Size of the Master Problem . . . . .	108
5.1.3	The Pricing Problem . . . . .	110
5.2	Description of Duties . . . . .	110
5.2.1	Duty Elements . . . . .	111
5.2.2	Duty types . . . . .	112
5.2.3	Resources . . . . .	114
5.2.4	Break rules . . . . .	115
5.2.5	Cost of a Duty . . . . .	115
5.3	Models for the Pricing Problem . . . . .	116
5.3.1	Pricing Networks . . . . .	117
5.3.2	Timelines . . . . .	118
5.3.3	IP Model . . . . .	121
5.3.4	Cost and Reduced Cost of Pairings and Links . . . . .	123
5.4	Literature . . . . .	124
5.5	Algorithm . . . . .	126
5.5.1	The Resource Constrained Shortest Path Problem . . . . .	126
5.5.2	Lagrangian Relaxation of all Resource Constraints . . . . .	127
5.5.3	Depth-First-Search . . . . .	130
5.6	Labeling . . . . .	131
5.6.1	Graph Construction . . . . .	132
5.6.2	Node Dominance . . . . .	134
5.6.3	Resource Scaling and Rounding . . . . .	136
5.6.4	Cost scaling . . . . .	139
5.7	Computational Results . . . . .	139

---

5.7.1	Testbed . . . . .	140
5.7.2	RCSP . . . . .	141
5.7.3	Lagrangian Relaxation . . . . .	143
5.7.4	Results of the Enumeration Algorithm . . . . .	146
5.8	Lower Bounds for the Duty Scheduling Problem . . . . .	147
5.8.1	RCSP-lower-bound . . . . .	147
5.8.2	LP lower bounds . . . . .	148
5.9	Conclusion . . . . .	149
<b>6</b>	<b>Rapid Branching</b>	<b>151</b>
6.1	Overview . . . . .	152
6.2	Branch-and-Bound . . . . .	152
6.3	Perturbation Branching Rule . . . . .	155
6.3.1	Motivation . . . . .	155
6.3.2	Determining the Main Branch . . . . .	155
6.3.3	Calculation of the Perturbed Objective Function . . . . .	157
6.3.4	Calculation of the Other Branches . . . . .	158
6.3.5	Comparison with Other Branching Rules . . . . .	159
6.4	Node Selection . . . . .	160
6.5	Lower Bounding . . . . .	161
6.6	Upper Bounding . . . . .	161
6.7	MIP-heuristics in the Literature . . . . .	162
6.7.1	Simplex Based Heuristics . . . . .	162
6.7.2	OCTANE . . . . .	163
6.7.3	Set Covering and Set Partitioning Heuristics . . . . .	163
6.7.4	Branch-and-Bound Based Heuristics . . . . .	163
6.7.5	Rounding Heuristics . . . . .	165
6.8	Computational Results . . . . .	166

---

6.8.1	Testbed . . . . .	166
6.8.2	Observations . . . . .	168
6.8.3	Conclusion . . . . .	169
<b>7</b>	<b>Computational Results</b>	<b>171</b>
7.1	Running Time . . . . .	171
7.2	Computation Times per Phase . . . . .	173
7.3	Algorithms . . . . .	174
7.4	RVB Instances . . . . .	174
7.5	RKH Instances . . . . .	176
7.6	Subcontractor Planning for an Regional Carrier . . . . .	178
7.7	ECOPT Instances . . . . .	180
7.8	Conclusion . . . . .	182
	<b>Glossary</b>	<b>185</b>
	<b>Symbols</b>	<b>187</b>
	<b>Bibliography</b>	<b>189</b>