Contents

1 Introduction **1**

2 Preliminaries **5**
2.1 Basics . . . 5
2.2 Complexity . . . 8
2.2.1 A Basic Model . . . 8
2.2.2 Complexity Classes . . . 9
2.2.3 Reductions . . . 11
2.3 Constraints . . . 14
2.4 Closure Properties and Post's Lattice . . . 16
2.4.1 Functions and Clones . . . 16
2.4.2 Relations and Co-Clones . . . 16

3 The Complexity of Satisfiability Problems **23**
3.1 Introduction . . . 23
3.2 A Complete Classification . . . 24
3.2.1 Upper Bounds . . . 24
3.2.2 Lower Bounds . . . 28
3.3 Conclusion . . . 30

4 Quantified Constraints **31**
4.1 Introduction . . . 31
4.2 Prerequisites . . . 31
4.2.1 Quantified Formulae . . . 32
4.2.2 Counting . . . 33
4.3 Quantified Constraint Satisfaction Problems . . . 34
4.4 Complementive Reductions . . . 37
4.5 Counting Problems for Quantified Constraints . . . 40
4.6 Conclusion . . . 47

5 Graph Related Constraint Problems **49**
5.1 Introduction . . . 49
5.2 Prerequisites . . . 50
5.3 Results for Two Constraint Languages . . . 51
5.3.1 Equivalence . . . 51
5.3.2 Implication . . . 55

5.4 Isomorphic Implication 56
5.4.1 Upper Bounds 58
5.4.2 Lower Bounds 60
5.4.3 Connection to GI 68
5.5 Isomorphic Implication Revisited 69
5.6 Conclusion 81

Bibliography **87**