Nach der klassischen Logik kann dann das Atom entweder in der linken oder in der rechten Hälfte des Kastens sein. Es gibt keine dritte Möglichkeit; ' tertium non datur'.

Werner Heisenberg [Heisenberg, 1990]

1 Introduction

1.1 A brief history of Quantum Computing

Werner Heisenberg early on noted the dilemma of quantum mechanics illustrated in the quotation above[1]. The ≪tertium non datur≫ of classical logic was apparently in stark contradiction to the superposition principle of quantum mechanics. His former student Carl-Friedrich von Weizsäcker postulated - in the sense of Bohr-complementarity - that classical logic was the classical limit and the a-priori preliminary of some yet-to-be-definded quantum logic [von Weizsäcker, 1990]. In an article ≪Komplementarität und Logik ≫- dedicated to Niels Bohr on the occasion of Bohrs 70th birthday in 1955 - von Weizsäcker breaks with two-valued logic, and he anticipates the idea of a qubit-based quantum logic by applying the Kopenhagen interpretation of quantum mechanics not to measurements, but to logical statements:

> Für jede einfache *Aussage* bezüglich einer Alternative [...] wird eine komplexe Zahl als ein Maß für ihren 'Wahrheitswert' eingeführt. [...] Das Absolut-Quadrat der komplexen Zahl gibt die Wahrscheinlichkeit dafür an, daß die *Aussage* richtig ist [2].

[1] "Following classical logic the atom can then either be in the left or the right half of the box. There is no third possibility; 'tertium non datur'."

[2] "For every simple *statement* with regard to an alternative a complex number is introduced as a measure

The «simple statement with regard to an alternative» means a two-valued alternative 'true - false' as known from classical information theory, albeit now with the expanded possibilities stemming from the super-position principle. von Weizsäcker tried to formalize this idea further by breaking down the whole quantum mechanical theory into these elementary decisions - or *Ur* as he called it - represented by a 2-dimensional Hilbert-space. In that sense he also anticipated the universality claim of the quantum computer - the development of which will be described in the following.

Interestingly enough todays quantum information theory was arrived at a good 25 years later via quite a different reasoning: the question of simulating physics. The philosophically motivated ansatz of von Weizsäcker seems to have been too 'heavy-weighted' to gain acknowledgment and popularity as fast as the pragmatic approach of how to efficiently compute physical phenomena. The starting point is usually attributed to a keynote given by *Feynman* at a conference 'Simulating physics with computers' in 1982 where he addresses the incomputability of certain problems, specifically the impossibility to efficiently simulate entanglement of a two-partite system by classical numerical computation. But he proposes an alternative approach [Feynman, 1982]:

> Now it turns out, as far as I can tell, that you can simulate this [quantum effect] with a quantum system, with quantum computer elements. [...] It has been found that there is a kind of (classical) universal computer that can do anything, and it doesn't make much difference specifically how it's designed. The same way we should try to find out what kinds of quantum mechanical systems are mutually intersimulatable, and try to find a specific class, or a character of that class which will simulate everything. [...] I believe it's rather simple to answer that question and to find the class, but I just haven't done it.

David Deutsch took to the call. His paper "Quantum theory, the Church-Turing principle and the universal quantum computer" [Deutsch, 1985] can be considered the starting point of a formalized quantum information theory. He shows how 'every finitely realizable physical system' - including quantum system - can be efficiently simulated by a finite-element *quantum computer*, the quantum mechanical analog of a (classical) universal *Turing*-machine, and delivers a proof of the quantum mechanical Church-Turing hypothesis. Just as the Turing-machine calculates any computable function by a series of operations on one or two bits - and hence the name universal - Deutsch constructs a universal quantum computer that simulates any unitary evolution of a finite quantum system by a series

for its 'truth value'. The absolute square of the complex number states the probability for the truthfulness of the *statement*."

of operations on one or two quantum bits, or *qubits*. Like its classical counter-part the qubit comprises two distinguishable states, $|0\rangle$ and $|1\rangle$. In contrast to the classical bit though, the general state of the qubit is a super-position of its basis states (also called the computational basis):

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{1.1}$$

with $|\alpha|^2 + |\beta|^2 = 1$. Eq. 1.1 also nicely illustrates the coup against the "tertium non datur".

So now the 'class which will simulate everything' has been found, and quantum mechanical phenomena including entanglement can be efficiently calculated. This in itself is an exciting field of research for basal quantum mechanical questions, but David Deutsch went one step further and showed a surprising property of the new device: the quantum computer can efficiently calculate certain classical functions that are *not* efficiently computable on a classical Turing machine. By exploiting the super-position capability of the qubit the quantum computer can calculate a function of all possible input states in parallel. Cunning unitary evolutions applied to the whole super-position state then use this massive 'quantum parallelism' to obtain an exponential speed-up for the calculation of certain problems. This quantum parallelism will be briefly sketched in the following.

Let an input register with N qubits be described by the basis states

$$|n\rangle = |b_1\rangle \otimes |b_2\rangle \otimes \cdots \otimes |b_N\rangle = |b_1 b_2 \dots b_N\rangle, \tag{1.2}$$

where $b_k \in \{0, 1\}, k = 1 \dots N$ and thus $n = 0 \dots 2^N - 1$. A classical register would hold one of the values n, whereas the quantum register can hold all values n as a super-position (ignoring normalization factors):

$$|\Psi\rangle = \sum_{n=0}^{2^N-1} |n\rangle. \tag{1.3}$$

Consequently, if we were to calculate the function $f(|n\rangle)$ of an input state - represented by a unitary transformation U_f - the linearity of quantum mechanics ensures that this can be done in parallel by applying U_f to the super-position $|\Psi\rangle$:

$$U_\mathrm{f}(|\Psi\rangle) = U_\mathrm{f}\left(\sum_{n=0}^{2^N-1} |n\rangle\right) = \sum_{n=0}^{2^N-1} U_\mathrm{f}(|n\rangle). \tag{1.4}$$

In one time step the quantum computer has evaluated all possible results of f. Of course, the result is in form of a super-position, which is not a usable (classical) result. If we were

to measure $U_{\mathrm{f}}(|\Psi\rangle)$ the wave-function would collapse, not only giving only one solution $U_{\mathrm{f}}(|n\rangle)$, but this value would furthermore be a random choice between the N different input states $|n\rangle$. The real ingenuity is the realization that this massive parallelism can be accessed by subsequently performing interference experiments on $U_{\mathrm{f}}(|\Psi\rangle)$ that enhance the measurement probability of the desired result state. To accurately describe this process for a real quantum algorithm is beyond the scope of this introduction.

The first true quantum algorithm that exhibits a computational speed-up is the *Deutsch-Jozsa* algorithm that determines if a given function f is even or odd [Deutsch & Jozsa, 1992]. As it operates on just two qubits it is also is the first algorithm that was experimentally implemented [Jones & Mosca, 1998], and it still is used as a demonstration for new experimental systems [Gulde et al., 2003]. Then a discovery by *Peter Shor* incited an interest that went far beyond that of quantum physicists: a quantum algorithm for prime factorization of large numbers with an exponential speed-up over its classical pendant [Shor, 1994]. The large interest stems from the fact that current classical encryption schemes - like the wide-spread RSA [Rivest et al., 1978] - rely on the 'hardness' of the problem of prime factorization. Shor's algorithm thus jeopardizes (at least theoretically) a large part of todays encryption industry. The basis for Shor's algorithm is the quantum Fourier transformation (QFT) that operates exponentially faster then the fastest classical Fourier transform (FFT): $\mathcal{O}(n^2)$ to $\mathcal{O}(n \cdot 2^n)$ [Chuang, 2003].

Quantum computing is now just one field of many different and interesting applications of quantum information theory. Quantum cryptography, quantum teleportation and dense coding are just a sample. Many different experimental implementations have been proposed for a quantum computer. Liquid-state nuclear magnetic resonance (NMR) [Jones & Mosca, 1998], ion traps or super-conducting flux qubits (Josephson junctions) [Devoret & Martinis, 2003] are but an excerpt of the proposal list currently being tested for the 'transistor of the future'. This thesis is an experimental work that researches a new approach to quantum computing with trapped ions. In the following, 'traditional' ion trap quantum computing is presented first in Sec. 1.2. In Sec. 1.3 the new proposal for ion trap quantum computing with a magnetic gradient field will be introduced.

1.2 Ion trap quantum computing

The first proposal to use a linear ion trap as 'a realistic physical system to implement a quantum computer' appeared in 1995 by *Ignacio Cirac* and *Peter Zoller* [Cirac & Zoller, 1995]. The authors suggest to compose a qubit of two internal energy levels of an ion. The com-

munication between the ions is mediated by the Coulomb repulsion, which together with the confining trap potential forms a set of common (harmonic) oscillator modes of the ion string. By using one particular oscillation mode as a 'bus-qubit' conditional quantum dynamics between two ions is possible. For this purpose the state of one qubit is copied into the bus-qubit by coupling the internal dynamics of the ion with the external dynamics of the oscillation mode. In the article a quantum controlled-not operation (the 'c-not' gate) is explicitly constructed from a series of laser pulses acting on the ions. It had been shown before that 'there exists a 2-bit quantum gate that is sufficient to build any quantum logic network' [Sleator & Weinfurter, 1995]: the aforementioned c-not gate. This is not an exclusive universality. The c-not is just one possible realization of an universal conditional 2-qubit gate. It seems worth to stress the universality of that claim [Deutsch, 1985]: the c-not gate together with a set of one-qubit operations allows to simulate *every* finite physical system! The same year the c-not gate was experimentally implemented between an ion and an oscillation mode by the group of Dave Wineland [Monroe et al., 1995], and later the full Cirac-Zoller c-not gate between two ions was experimentally realized by the group of Rainer Blatt [Schmidt-Kaler et al., 2003b].

Since then the last decade has seen considerable progress of quantum information science in linear ion traps. Enhanced rotation angle estimation [Meyer et al., 2001] and robust geometric phase gates [Leibfried et al., 2003], Bell-inequality measurements [Rowe et al., 2001], self-learning estimation of quantum states [Hannemann et al., 2002], the realization of quantum error correction [Chiaverini et al., 2004] and the implementation of the full Deutsch-Josza-algorithm [Gulde et al., 2003] show the potential of quantum information science and quantum computing with only a single or two ions. As of late the spectacular quantum teleportation of atomic qubits [Riebe et al., 2004, Barrett et al., 2004] or the demonstration of the first Qubyte with eight ions [Haffner et al., 2005] have pushed the limits. On the theoretical side, an article by Soerensen and Molmer [Sorensen & Molmer, 2000] proposed a similar c-not gate relying on two-photon transitions that is distinctly less stringent in its demand on the state of the bus-qubit, i.e. the Sorensen-Molmer gate does not require ground-state cooling of the coupling oscillator mode. Cirac and Zoller eliminated the direct dependence of the oscillation mode with a proposal involving short laser pulses, allowing arbitrarily fast gate times [Garca-Ripoll et al., 2003]. The new Cirac-Zoller gate has until now not been experimentally realized.

Notwithstanding these successes several problems remain that prohibit scaling to a larger number of qubits. The coherence time of optical qubits (i.e. qubits where the energy difference between the two states is on the order of an optical transition) is limited by the life-time of the upper level. For $^{40}Ca^{+}$ used in Innsbruck [Schmidt-Kaler et al., 2003a] and

Oxford [Lucas et al., 2003] the $D_{5/2}$ state has a life-time of approximately 1 s, ultimately limiting the length of a quantum algorithm. The demands on the laser stability both in frequency and amplitude control require state-of-the-art equipment. In comparison to coherent manipulation of qubits in the radio-frequency domain this is a substantial experimental disadvantage. The addressing of the ions is the direct and exclusive illumination of only one ion by a tightly focused laser beam. For larger ion strings with a small inter-ion distance this addressing becomes increasingly difficult. A rest illumination of neighboring ions is unavoidable, inducing unwanted residual transitions.

To counter theses problems a different trap design has been envisaged. The new type of trap is micro-fabricated by standard lithographic techniques, and comprises many different trapping zones [Kielpinski et al., 2002]. Only two ions at a time interact in special interaction zones. This avoids the addressing problems arising for longer ion strings. After interaction, the ions are shuttled to cooling zones, where they are sympathetically cooled. The short maximal coherence time for optical transitions is avoided using hyper-fine states instead. This reduces the coherence requirement of the driving laser from the optical to the micro-wave regime, implemented in form of a Raman-laser setup.

A large effort in the community is underway to realize this new type of trap. In the meantime, we propose a modification of the existing form of the linear trap QC that simplifies both the addressing and the coherent manipulation of the qubits even more. As such, we hope to fill the gap between the few-qubit QC already demonstrated and the large-scale (hundreds of qubits) architecture described above. Furthermore, no fundamental reason prohibits the combination of the micro-array trap design with the gradient scheme presented in this work. Therefore, the gradient scheme may offer advantages for all ion trap quantum computers.

1.3 Magnetic field gradient proposal

The scheme to use ion traps with a static, magnetic gradient field for quantum computation [Mintert & Wunderlich, 2001, Wunderlich, 2001, Wunderlich et al., 2005] aims at both increasing the scalability of the system and at opening up new possibilities for it. The scalability is mainly increased by simplifying the experimental complexity. A solution for the problems inherent to the addressing and the coherent manipulation discussed above is presented. All methods requiring so far optical radiation - e.g. side-band cooling, c-not gate - will become feasible with MW- or RF-radiation alone. In the end the goal is to combine the present advantages of the ion trap with those of a NMR quantum computer.

The first part of the gradient scheme is the addressing of the ions. In the presence of a magnetic gradient field the Zeeman-splitting of the energy levels now depends on the position of the ions. If we assume for simplicity a magnetic field that is linearly increasing in the z-direction (parallel to the trap axis) the Larmor-frequency of the Zeeman-splitting for a (linear) string of ions increases linearly as well. Using rf-radiation the addressing of the qubits then is straight-forward by changing the rf-frequency to the resonance frequency of the desired ion. But a larger obstacle remains: to be able to use rf-radiation (or micro-wave radiation, for that matter) for the addressing in the first place we need to be able to perform conditional quantum logic gates with rf-radiation as well. This forms the second part of the gradient scheme.

The ultimate reason why so far coherent manipulation of qubits in ion traps is performed with laser sources is the necessity to couple the internal dynamics of the ion (driven by the laser source) with the external dynamics, the oscillatory movement of the ion in the trap potential. The Circac-Zoller-proposal uses the common oscillation mode of the ions as a bus-mode that mediates the communication between the ions. The measure for the strength of the coupling between the internal and the external dynamics is given by the so-called Lamb-Dicke-parameter (LDP):

$$\eta = k \cdot \Delta X = \frac{\hbar k}{2\Delta P}. \tag{1.5}$$

k is the wave-vector of the exciting electro-magnetic radiation and $\Delta X = \sqrt{\frac{\hbar}{2m\omega}}$ and $\Delta P = \sqrt{\frac{1}{2}m\hbar\omega}$ are the spread of the position and momentum operator, respectively, of the ground-state wave-function of the quantum harmonic oscillator. In other words, the LDP is measure of how much the amplitude of the electro-magnetic wave varies over the spatial extent of the ion, or how much momentum the wave carries with respect to the momentum of the ion. Inserting the numbers for a typical secular trap frequency of $\omega = 2\pi \cdot 1\,\mathrm{MHz}$ it becomes clear that only for optical radiation does η take on a appreciable value $> 10^{-3}$, and a relevant coupling between the internal and external dynamics of the ion arises. Following this logic, micro-wave or even radio-frequency radiation never was a choice for ion trap quantum computing. This explains why even for the hyper-fine qubit with a transition frequency in the micro-wave regime a laser source is required (in form of a Raman-laser setup), instead of using the experimentally much less demanding direct irradiation with micro-waves.

Central to the second step of the gradient scheme is the emergence of a new, effective Lamb-Dicke-parameter η_{eff} due to the interaction of the ions with the magnetic gradient field. Quite generally, for a conservative potential ϕ a force stems from the gradient of

that potential, i.e. $F = -\nabla\phi$. The magnetic dipole moment $\vec{\mu} = g_j m_j \vec{\mu_B}$ of the ion in an external magnetic field $\vec{B}$ has the potential energy $\phi = -\vec{\mu} \cdot \vec{B}$. For the case of our magnetic gradient field this implies a state-dependent force:

$$F = \mu \cdot \nabla B. \tag{1.6}$$

Graphically speaking, the ion is displaced to a new equilibrium position by this force, with the sign of the displacement - with respect to the original equilibrium position - depending on the internal state m_j of the ion. Accordingly, changing the internal state of the ion (e.g. by a π-pulse using rf-radiation) pushes it to the other equilibrium position. The Coulomb repulsion between the ions now lets the other ions of the whole string 'feel' this displacement. In effect, there is a chance that the state of the oscillation mode of the ion string is excited. But this is just the same as saying that the effective LDP η_{eff} has an appreciable value. This admittedly rather illustrative description of η_{eff} will be motivated (and calculated) more thoroughly in Sec. 2.4.2.

The implications of the new, effective LDP are important for ion trap quantum computing. Using η_{eff}, side-band cooling with micro-waves becomes feasible as side-bands in the rf- or mw-spectrum emerge. But the coupling between internal and external dynamics further allows for coherent interaction on side-band transitions, basic ingredient of the Cirac-Zoller c-not gate. In conclusion, η_{eff} eliminates the need for extremely stable laser-sources, as the coherent manipulation of the qubit states can now be performed with mw- or rf-radiation directly.

The theory pertaining to this concept has been developed in detail in the references given above. This thesis is the first experimental step to implement the gradient scheme. During the course of this work a new linear trap has been put in operation with alternative radio-frequency trap drives, photo-ionization of ytterbium for ion trap purposes has been implemented for the first time, and a magnetic gradient field has been setup with permanent magnets. The method of rf-optical double-resonance spectroscopy has been adapted for ion trap purposes to have an experimental probe that is magnetically sensitive. The measurements characterizing the magnetic field gradient by rf-spectroscopy form the central aspect of this work.

The long-term goal is to perform measurements with the ^{171}Yb isotope of ytterbium. It has a nuclear spin of $I = 1/2$ with a convenient hyper-fine structure for quantum information processing. Nonetheless, it is experimentally more challenging than its 'sibling', the ^{172}Yb isotope, as the fluorescence rate and hence the cooling rates are lower [Balzer, 2003], making it more difficult to cool and localize. Therefore, the Zeeman sub-levels of the ^{172}Yb isotope are employed instead, in a first step to demonstrate the addressing ability of the

gradient scheme. The rf-optical double resonance spectroscopy is an adequate tool to probe this magnetic four-level sub-structure, both incoherently and coherently.

After this introduction Chapter 2 presents the theory relevant for this work. It is grouped in 'interactions': the interaction of the ions with the trapping potential (2.1), with laser light (2.2), with the exciting rf-radiation used for the double-resonance spectroscopy (2.3), and finally with the magnetic gradient field (2.4). Next, following the theory, Chapter 3 describes the experimental issues pertaining to trapping single ytterbium ions. Its sections are formed by functionality: the linear trap itself (3.1), lasers and optics (3.2), the detection systems (3.3), magnetic fields (3.4) and experimental control (3.5). The experimental results are split into two chapters. First, the photo-ionization experiments are discussed in Chapter 4. Using an oven with natural abundance isotope distribution the isotope shift of the atomic $^1S_0 \rightarrow {}^1P_1$ transition is determined from atomic excitation spectra in (4.2). Photo-ionization furthermore allows for the deterministic loading of a desired number of ions, shown in (4.3). Finally, Chapter 5 comprises the main experimental results of this work. First, the double-resonance spectroscopy itself is established (5.1). With it, the static magnetic fields creating a well-defined quantization axis for the following measurements are characterized (5.2). Qubits are manipulated by coherent interactions. That the double-resonance spectroscopy allows for these types of measurements is shown in (5.3), where coherent Rabi-oscillations are presented. And last but not least, the permanent magnets are put in place to create the magnetic gradient field (5.4). This section contains the actual measurements showing the individual addressing of single ytterbium ions in a linear ion crystal. The main body of this thesis closes with the conclusion and outlook in Chapter 6.