Contents

1 Introduction **1**

2 Static k-splittable flows **7**

2.1 Introduction 7

2.2 Complexity in case of a constant number of paths 15

2.3 Complexity if the number of paths depends on the graph . 22

2.3.1 Polynomially solvable cases 23

2.3.2 Proofs of NP–hardness 26

2.4 Solving the problem 31

2.4.1 The packing step in general graphs 31

2.4.2 The routing step in graphs of bounded treewidth . 42

2.5 Conclusion and open questions 49

3 Flows over time in grid graphs **51**

3.1 Introduction 51

3.1.1 Related work 52

3.1.2 Our model 55

3.1.3 Notation 59

3.1.4 Outline of the chapter 60

3.2 Quickest s,t–flows without time windows 61

3.2.1 Uniform transit times 61

3.2.2 Non–uniform transit times 68

3.3 Quickest s,t–flows with time windows 73
3.3.1 Uniform transit times 76
3.3.2 Non–uniform transit times 76
3.4 Conclusion and open questions 79

4 Multicommodity flows over time in grid graphs 81
4.1 Introduction 81
4.1.1 Time windows 81
4.1.2 Waiting 83
4.1.3 Integrality 85
4.1.4 Outline of the chapter 87
4.2 Uniform transit times 87
4.2.1 Proofs of NP–hardness 87
4.2.2 Approximation algorithms 99
4.3 Non–negative integral transit times 105
4.4 Conclusion and open questions 109

Bibliography 111

Symbol Index 117

Index 119