Preface

The topic of *routing through a network* has attracted considerable attention from both the combinatorial optimization community and practitioners due to a variety of applications in logistics and traffic control. In combinatorial optimization, such routing questions are typically modeled as network flow problems. Obviously, these models cannot reflect all practical requirements. In this work, we consider two aspects that cannot be dealt with at all, or at least not efficiently with standard techniques.

Efficient models define network flows as properties associated with nodes and edges. In doing so, we have no control over the number of paths in a solution, although this is a key value for the solution's applicability. In the first part of this thesis, we consider so–called *k–splittable flows*. These flows are required to be splittable into no more than a given number k of paths. In correspondence with the classical maximum s,t–flow problem, we investigate the problem of finding a maximum k–splittable s,t–flow in a network with edge capacities. We study the complexity of this question for given integer values of k and for k that is a function on graph parameters. We describe solution methods for polynomially solvable variants and prove approximation results for NP–hard problem configurations. For the class of graphs with bounded treewidth, a new approach enables us to solve problem variants that are hard on general graphs.

In the second part, we add a time dimension to classical static flows. Flow can then be seen as moving over time through a network, which is essential for practical purposes. We study such *flows over time* on grid graphs that typically arise in storage facilities. We investigate the problem of sending a given amount of flow to targets within a minimum possible time. For this so–called *quickest flow problem*, we prove complexity results and show polynomial algorithms or approximations. An entire landscape of problem variants is encompassed: We consider single and multicommodity environments. Path flows can wait on edges or have to be sent without disruption. A temporarily closure of edges can be allowed. Transit times along edges are considered to be constant or edge–specific.

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