Contents

Introduction **1**
Outline of the Thesis 3

I Packet Routing **9**

1 Trees and Direct Schedules **11**
1.1 Introduction 11
1.1.1 Problem Definition 12
1.1.2 Related Work 12
1.1.3 Outline of the Chapter 14
1.2 Schedules for Undirected Trees 15
1.3 Schedules for Directed Trees 19
1.3.1 Path Coloring 19
1.3.2 Time-Dependent Edge Coloring 21
1.3.3 Routing Schedule 21
1.4 Direct Schedules 22
1.5 Conclusion 24

2 Schedules for General Graphs **27**
2.1 Introduction 27
2.1.1 The Model 28
2.1.2 Outline of the Chapter 29
2.2 Tight Bound for Small Dilation 30
2.3 High Level Ideas for General Bounds 34
2.4 Technical Analysis 37
2.4.1 Framework 43
2.4.2 High Values for τ 49
2.4.3 Unit Transit Times and Unit Bandwidths 50
2.4.4 Algorithmic Bounds 50
2.5 Conclusion 51

3 Complexity of Packet Routing **53**
3.1 Introduction . . . 53
3.1.1 Outline of the Chapter . . . 54
3.2 General Graphs . . . 54
3.3 Trees . . . 58
3.4 Absolute Approximation . . . 63
3.5 Conclusion . . . 67

4 Periodic Packet Routing **69**
4.1 Introduction . . . 69
4.1.1 Definitions . . . 70
4.1.2 Related Work . . . 72
4.1.3 Outline of the Chapter . . . 73
4.1.4 Comparison of Template- and Priority-Schedules . . . 74
4.2 Necessary Bound on Congestion . . . 75
4.3 Template Schedules . . . 76
4.3.1 Directed Trees . . . 77
4.3.2 Bidirected Trees . . . 82
4.3.3 Undirected Trees . . . 92
4.4 Global- and Edge-priority Schedules . . . 96
4.4.1 Lower Bounds . . . 100
4.4.2 Strict Periodic Setting . . . 105
4.5 Imitation Theorems . . . 108
4.5.1 Template Schedules vs. Global-Priority Schedules . . . 108
4.5.2 Template Schedules vs. Edge-Priority Schedules . . . 110
4.6 Conclusion . . . 113

II Scheduling **115**

5 Increasing Speed Scheduling **117**
5.1 Introduction . . . 117
5.1.1 Definitions . . . 118
5.1.2 Related Work . . . 119
5.1.3 Outline of the Chapter . . . 120
5.2 From Flows to Scheduling . . . 121
5.3 Polynomial Time Approximation Scheme . . . 123
5.4 Tractable Cases of ISS . . . 130
5.5 A Tight Analysis of Smith's Rule . . . 134
5.6 Blind Algorithms . . . 143
5.7 Online Algorithms . . . 144
5.7.1 Lower Bound for Online Algorithms . . . 147
5.7.2 Unit Weight Case . . . 148
5.8 Conclusion . . . 148

6 Periodic Maintenance Problem 151
6.1 Introduction . . . 151
6.1.1 Problem Definition . . . 153
6.1.2 Related Work . . . 154
6.1.3 Outline of the Chapter . . . 154
6.2 General Periodic Maintenance Problem . . . 155
6.2.1 First-Fit Algorithm . . . 156
6.2.2 Enumeration and First-Fit . . . 156
6.2.3 Complexity . . . 160
6.3 Harmonic Periodic Maintenance Problem . . . 161
6.3.1 Bin-Trees . . . 162
6.3.2 First-Fit Algorithm . . . 163
6.3.3 Complexity . . . 167
6.3.4 APTAS for Constant Number of Periods . . . 168
6.4 Conclusion . . . 169

7 Scheduling on Unrelated Machines 171
7.1 Introduction . . . 171
7.1.1 The Minimum Makespan Problem . . . 172
7.1.2 The MaxMin-Allocation Problem . . . 174
7.1.3 Outline of the Chapter . . . 175
7.2 LP-Based Approaches . . . 176
7.3 Integrality Gap of the Configuration-LP . . . 181
7.3.1 Integrality Gap of the Configuration-LP . . . 181
7.3.2 Integrality Gap for Unrelated Graph Balancing . . . 183
7.4 Cases with Better Approximation Factors . . . 186
7.4.1 Bounded GCD of Processing Times . . . 186
7.4.2 Bounded Range of Processing Times . . . 188
7.4.3 Big Machines/Small Machines . . . 189
7.5 MaxMin-Allocation Problem . . . 190
7.5.1 2-Approximation for MaxMin-Balancing . . . 191
7.5.2 Half-Integral Solutions . . . 193
7.5.3 Tractable Cases . . . 195
7.6 Conclusion . . . 196

Bibliography 198