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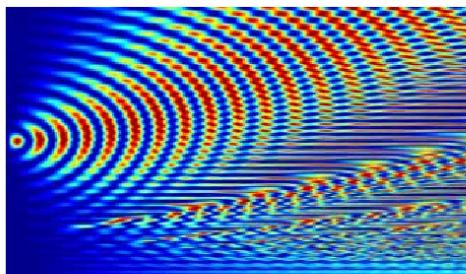
# Quantum Optimal Control Theory: Filter Techniques, Time-Dependent Targets, and Time-Dependent Density-Functional Theory

Jan Werschnik

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Filter Techniques, Time-Dependent Targets,  
and Time-Dependent Density-Functional  
Theory

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