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## Application of finite elements for computational aeroelasticity

The cover features the TU Braunschweig logo and the CAMPUS FORSCHUNGSFLUGHAFEN logo at the top. The central image shows a 3D finite element model of an aircraft wing in a flow field. Below the image is a mathematical equation:  $\hat{U}_k = \hat{U}_k^* \rightarrow l_k = \int_0^T \lambda^T (\hat{U}_k - \hat{U}_k^*) d\Gamma dt \rightarrow \int_0^T \lambda \lambda^T (\hat{U}_k - \hat{U}_k^*) d\Gamma = 0$ . The title 'Application of finite elements for computational aeroelasticity' is centered below the image. The author's name, 'Ralf Unger', and affiliation, 'TU Braunschweig, Institut für Flugzeugbau und Leichtbau', are listed. At the bottom, it says 'CFF - Forschungsbericht 2011-08' and includes the Cuvillier Verlag logo.

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