Chapter 1

Introduction

Although Polarization Mode Dispersion (PMD) in optical fibers has been investigated since the late 70s, it attracted serious attention since approximately 1997 due to the commercial advent of 10 Gbit/s optical transmission systems. After tackling fiber impairments in optical WDM systems such as chromatic dispersion and non-linear effects, the remaining impairments caused by PMD still pose an ultimate limit for optical high-speed transmission.

The main reason for the difficulties in compensating PMD-induced distortions probably is that PMD changes statistically with time at different time scales. Therefore, unlike chromatic dispersion, PMD has to be compensated dynamically, e.g. by using an active control circuitry. Hence, the main goal of compensating PMD is not only to find the optimal compensator settings but also to keep track of this optimum according to the temporal fluctuations of the transmission link. This gives rise to numerous questions regarding response time, reliability, complexity and cost of such a solution.

Key parameter of a PMD-disturbed fiber link is the so-called *differential group delay* (DGD). The DGD defines the delay difference between two orthogonal polarization states

Figure 1.1: The effect of PMD: A propagating signal splits up into two components traveling at different group velocities. At the fiber output they are separated by the *differential group delay* (DGD). The PMD compensator delays the faster component by the appropriate amount to recover the original signal.

of a birefringent fiber. If a fiber's DGD stays significantly below the duration of a single bit, the PMD-induced impairments can be neglected. However, upcoming 40 Gbit/s systems and particularly OTDM systems with bit rates of 160 Gbit/s (here the bit duration is less than 6.25 ps) will be severely disturbed by the DGD levels of common deployed fibers. At these bit rates, it will hence be unavoidable to actively counteract PMD-induced impairments since the common approach, namely selecting a low-PMD fiber, or inserting a regenerator will not always be an option.

PMD is a consequence of fiber birefringence mainly caused by mechanical stress and elliptical core profile of the waveguide. Due to the cabling and deploying process the orientation of the waveguide as well as the mechanical stress varies randomly along the fiber link. In order to describe this stochastic behavior the so-called *waveplate model* is commonly used to investigate the properties of real fiber links. The main approach of the waveplate model is to divide the fiber into several randomly concatenated linearly birefringent elements (i.e. waveplates).

Considering a single waveplate, a polarized signal propagating through such a (linearly birefringent) element is split into its two components coinciding with the element's birefringent axes. These two components are traveling at different propagating speeds. Thus, a single waveplate can cause two-path propagation and therefore may induce inter-symbol interference if the time delay between the two signal components (i.e. the DGD) is large enough. In the waveplate model, these two components pass the second waveplate, which has a different orientation compared to the first. Thus, the two components decompose into four signals. These four signals, again, are divided into eight signals when passing the third waveplate and so forth. Having this model in mind, PMD can be interpreted as a problem of multipath propagation. As explained in the next chapters, the complex model of concatenated waveplates can be simplified by modeling a fiber as a (single) birefringent element whose DGD and birefringent axes (the so-called *Principal States of Polarization*, PSPs) are wavelength dependent. Hence, compensating PMD means to compensate the DGD of the system at each signal wavelength. This thesis presents different approaches for optically compensating PMD and discusses the prospects of the proposed solutions.

Structure of this Work

The main objective of this work was to investigate different approaches of optical PMD compensation in theory and in practice. The different PMD compensators described in this thesis all consist of the following building blocks (see Fig. 1.2):

- Polarization Controller (PC)
- PMD Equalizer
- Error Signal Detector

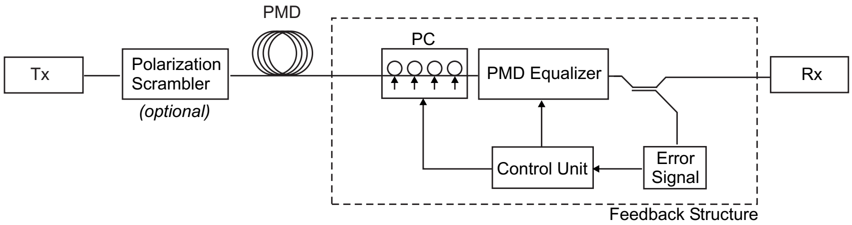

Figure 1.2: Commonly used setup for compensating PMD.

- Control Unit

The basic concept of compensating the fiber's PMD is to add a controlled amount of birefringence produced by a *PMD equalizer*. Ideally, this equalizer is tuned to exhibit the same amount of DGD as the transmission link, and the polarization controller is adjusted in a way that the faster signal component is delayed with respect to the slower signal component.

This work is divided into the following main topics and addresses all of the abovementioned building blocks:

- Chapters 2 and 3 explain basics about PMD in optical communication systems. The central terms first-order/second-order PMD as well as the Principal States model are introduced and the PMD tolerance of different optical communication systems is investigated.
- Chapters 4 and 5 focus on technologies for measuring polarization and PMD in optical fibers. Experimental work is presented applying polarimetric and interferometric approaches to measure PMD with high accuracy.
- Chapter 6 focuses on the compensation of PMD. This includes implementation of different error detectors based on polarimetric detection as well as different implementations of PMD equalizers. All presented components are demonstrated in compensation experiments conducted in collaboration with the Fraunhofer Institut für Nachrichtentechnik, Heinrich-Hertz-Institut.

Chapter 2

Representation of Polarized Optical Signals

2.1 Jones Representation

The *Jones* and *Stokes* representations are two established ways for describing polarization effects in optical systems. Particularly in single-mode fiber-optic systems, the Jones formalism can be seen as a vectorized form of the system theory known from electrical systems. The following sections introduce the basics of the Jones representation. The Stokes representation is explained in section 2.2.

2.1.1 Optical Signals in Jones Space

Physically an optical signal is described by the amplitude and direction of the electrical field vector at a certain position and time ($\vec{p}$ denotes a unity vector in three-dimensional space):

$$\vec{E}(t,x,y,z) = E(t,x,y,z) \cdot \vec{p}(t,x,y,z) \tag{2.1}$$

For plane optical waves propagating in z-direction, the z-component of the electrical field disappears and just the x- and y-components are remaining. The electrical field vector at a position z can then be written as follows:

$$\vec{E}(t) = E(t) \cdot \vec{p}(t) = E(t) \cdot \begin{bmatrix} p_x(t) \\ p_y(t) \\ 0 \end{bmatrix} = \begin{bmatrix} E_x(t) \\ E_y(t) \\ 0 \end{bmatrix} \tag{2.2}$$

Note that $E_x(t)$ and $E_y(t)$ are non-complex scalar electrical signals. The optical signal can also be represented in Fourier space:

$$\underline{\vec{E}}(\omega) = \begin{bmatrix} \underline{E}_x(\omega) \\ \underline{E}_y(\omega) \\ 0 \end{bmatrix} \tag{2.3}$$

This representation implies that the resulting electrical field can be formed by the superposition of an x-polarized component and an y-polarized component. The Jones formalism generalizes this implication and forms the electrical field out of two orthogonally polarized components each of which are described by its Fourier transform $\underline{J}_x(\omega)$ and $\underline{J}_y(\omega)$. The polarization of these components is not necessarily identical to the axes of the physical coordinate system and can be described by the two base polarization modes $\vec{e}_x(x, y)$ and $\vec{e}_y(x, y)$. $\underline{J}_x(\omega)$ and $\underline{J}_y(\omega)$ are the elements of the *Jones vector*. It is often useful to transform this formalism to the time domain:

$$\begin{aligned} \underline{\vec{J}}(\omega) &= \begin{bmatrix} \underline{J}_x(\omega) \\ \underline{J}_y(\omega) \end{bmatrix} & \bullet\!\!-\!\!\circ \quad \vec{J}(t) &= \begin{bmatrix} J_x(t) \\ J_y(t) \end{bmatrix} \\ \underline{\vec{E}}(\omega) &= \underline{J}_x(\omega) \cdot \vec{e}_x + \underline{J}_y(\omega) \cdot \vec{e}_y & \bullet\!\!-\!\!\circ \quad \vec{E}(t) &= J_x(t) \cdot \vec{e}_x + J_y(t) \cdot \vec{e}_y \end{aligned} \tag{2.4}$$

In this general definition, $\underline{\vec{J}}(\omega)$ describes an arbitrary optical signal. This can be any kind of modulated data signal but also a monochromatic CW signal. For the latter case the Jones representation is given by[1]:

$$\underline{\vec{J}}_{cw}(\omega) = \begin{bmatrix} \underline{J}_x^* \cdot \delta(\omega + \omega_0) + \underline{J}_x \cdot \delta(\omega - \omega_0) \\ \underline{J}_y^* \cdot \delta(\omega + \omega_0) + \underline{J}_y \cdot \delta(\omega - \omega_0) \end{bmatrix} \bullet\!\!-\!\!\circ \; \vec{J}_{cw}(t) = \begin{bmatrix} |\underline{J}_x| \cos(\omega_0 t + \arg \underline{J}_x) \\ |\underline{J}_y| \cos\left(\omega_0 t + \arg \underline{J}_y\right) \end{bmatrix} \tag{2.5}$$

Note that the spectral representation is the Fourier transform of a non-complex signal. Thus the negative spectral components are the conjugate of the positive spectral components. Since the negative spectral components do not contain additional information, it is often suitable just to consider the analytical signal which eliminates negative spectral components[2]:

$$\underline{\vec{J}}^{+}(\omega) = \frac{1}{2}\underline{\vec{J}}(\omega) - j\frac{1}{2}\mathcal{H}\left\{\underline{\vec{J}}(\omega)\right\} \tag{2.6}$$

For the monochromatic signal this equates to:

$$\underline{\vec{J}}_{cw}^{+}(\omega) = \begin{bmatrix} \underline{J}_x \cdot \delta(\omega - \omega_0) \\ \underline{J}_y \cdot \delta(\omega - \omega_0) \end{bmatrix} \bullet\!\!-\!\!\circ \; \underline{\vec{J}}_{cw}^{+}(t) = \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} \cdot e^{j\omega_0 t} \tag{2.7}$$

[1] $\delta(x)$ denotes the Dirac function

[2] $\mathcal{H}\{.\}$ denotes the Hilbert transform.

Obviously this signal is entirely defined by its frequency and the complex amplitudes $\underline{J}_x$ and $\underline{J}_y$. Therefore, Jones vectors describing monochromatic signals are often written as

$$\vec{\underline{J}}_{cw} = \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} \tag{2.8}$$

To treat this simplification in a more systematic way, one can adopt the *equivalent baseband representation* known from modulation theory. This representation moves a bandwidth-limited signal modulated onto a carrier with the frequency ω_0 into the baseband:

$$\vec{\underline{J}}^{+}_{BP}(\omega) = \vec{\underline{J}}^{+}(\omega + \omega_0) \;\bullet\!\!-\!\!\circ\; \vec{\underline{J}}^{+}_{BP}(t) = \vec{\underline{J}}^{+}(t) \cdot e^{-j\omega_0 t} \tag{2.9}$$

Using the aforementioned calculus, a polarized amplitude-modulated signal (modulating signal: $m(t)$), as it is often used in optical communication, can be represented as follows:

$$\begin{aligned}
\vec{J}(t) &= m(t) \cdot \begin{bmatrix} |\underline{J}_x| \cos\left(\omega_0 t + \arg \underline{J}_x\right) \\ \left|\underline{J}_y\right| \cos\left(\omega_0 t + \arg \underline{J}_y\right) \end{bmatrix} && (2.10)\\
\vec{\underline{J}}^{+}(t) &= m(t) \cdot \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} \cdot e^{j\omega_0 t} && (2.11)\\
\vec{\underline{J}}^{+}_{BP}(t) &= m(t) \cdot \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} && (2.12)\\
\circ\!\!-\!\!\bullet\; \vec{\underline{J}}^{+}_{BP}(\omega) &= m(\omega) \cdot \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} && (2.13)
\end{aligned}$$

For monochromatic signals $m(t)$ equals 1 and the equivalent baseband signal equates to:

$$\vec{\underline{J}}^{+}_{BP,cw}(t) = \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} \;\circ\!\!-\!\!\bullet\; \vec{\underline{J}}^{+}_{BP,cw}(\omega) = \begin{bmatrix} \underline{J}_x \\ \underline{J}_y \end{bmatrix} \cdot \delta(\omega) \tag{2.14}$$

An optical signal is often received using a photodiode. This receiver is known as envelope detector since its output is proportional to the squared envelope of the electrical field. This can be easily expressed by means of the analytical signal or equivalent baseband signal:

$$\begin{aligned}
I(t) &= \left|\underline{J}_x^{+}(t)\right|^2 + \left|\underline{J}_y^{+}(t)\right|^2 &&= \underline{J}_x^{+}(t)\underline{J}_x^{+*}(t) + \underline{J}_y^{+}(t)\underline{J}_y^{+}(t)^{*} \\
&= \left|\underline{J}^{+}_{BP,x}(t)\right|^2 + \left|\underline{J}^{+}_{BP,y}(t)\right|^2 &&= \underline{J}^{+}_{BP,x}(t)\underline{J}^{+}_{BP,x}{}^{*}(t) + \underline{J}^{+}_{BP,y}(t)\underline{J}^{+}_{BP,y}{}^{*}(t)
\end{aligned} \tag{2.15}$$

2.1.2 Optical Systems in Jones Space

In the Jones representation an optical system is given by its *Jones matrix*. The Jones matrix comprises four elements describing the coupling between the two base polarization modes:

$$\underline{\mathbf{J}}(\omega) = \begin{bmatrix} \underline{J}_{11}(\omega) & \underline{J}_{12}(\omega) \\ \underline{J}_{21}(\omega) & \underline{J}_{22}(\omega) \end{bmatrix} \tag{2.16}$$

The Jones matrix is the counterpart to the transfer function in electrical signal theory. Each of the elements are complex scalar transfer functions in the Fourier sense. An optical signal passing through an optical system can then be described by a simple matrix/vector multiplication:

$$\underline{\vec{J}}_{out}(\omega) = \underline{\mathbf{J}}(\omega) \cdot \underline{\vec{J}}_{in}(\omega) \tag{2.17}$$

The concatenation of two optical systems given by $\underline{\mathbf{J}}_1(\omega)$ and $\underline{\mathbf{J}}_2(\omega)$ can be described by a matrix multiplication:

$$\underline{\mathbf{J}}(\omega) = \underline{\mathbf{J}}_2(\omega) \cdot \underline{\mathbf{J}}_1(\omega) \tag{2.18}$$

Note that since the matrix multiplication is not commutative, reordering the partial systems changes the result.

The formalism also holds for analytical systems:

$$\underline{\mathbf{J}}^{+}(\omega) = \frac{1}{2}\underline{\mathbf{J}}(\omega) - j\frac{1}{2}\mathcal{H}\left\{\underline{\mathbf{J}}(\omega)\right\} \tag{2.19}$$

$$\underline{\vec{J}}^{+}_{out}(\omega) = \underline{\mathbf{J}}(\omega) \cdot \underline{\vec{J}}^{+}_{in}(\omega) \tag{2.20}$$

$$\underline{\vec{J}}^{+}_{out}(\omega) = \underline{\mathbf{J}}^{+}(\omega) \cdot \underline{\vec{J}}^{+}_{in}(\omega) \tag{2.21}$$

The Jones matrix has to be transformed to the equivalent baseband representation when applied to baseband Jones vectors:

$$\underline{\mathbf{J}}^{+}_{BP}(\omega) = \underline{\mathbf{J}}^{+}(\omega + \omega_0) \tag{2.22}$$

$$\underline{\vec{J}}^{+}_{BP,out}(\omega) = \underline{\mathbf{J}}^{+}_{BP}(\omega) \cdot \underline{\vec{J}}^{+}_{BP,in}(\omega) \tag{2.23}$$

In systems without polarization dependent loss (PDL) the Jones matrix can be split into a common phase term and a unitary matrix:

$$\underline{\mathbf{J}}(\omega) = e^{j\beta(\omega)L} \cdot \underline{\mathbf{J}}_U(\omega) \qquad \underline{\mathbf{J}}_U(\omega) = \begin{bmatrix} \underline{J}_1(\omega) & \underline{J}_2(\omega) \\ -\underline{J}_2^*(\omega) & \underline{J}_1^*(\omega) \end{bmatrix} \tag{2.24}$$

In that case, $\underline{\mathbf{J}}_U(\omega)$ is a complex rotation matrix and the inverse is given by $\underline{\mathbf{J}}_U^{-1}(\omega) = \underline{\mathbf{J}}_U^{T*}(\omega)$. If $\underline{\mathbf{J}}_U$ is not a function of ω the system is referred to as *polarization transformer*.

2.2 Stokes Representation

An alternative way for describing polarization effects in optical systems is the *Stokes representation*. In many cases the Stokes representation gives more insight into evolution of a signal's polarization propagating through a system.

2.2.1 Monochromatic Optical Signals in Stokes Space

The Stokes calculus was invented to describe the polarization property of light. It uses four-element vectors for expressing this property of an optical signal. The elements of these vectors are called *Stokes parameters* ($S_0..S_3$) and are originally defined by the intensities of a signal after passing through differently oriented polarizers:

$$\vec{S} \stackrel{def}{=} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I \\ I_0 - I_{90} \\ I_{+45} - I_{-45} \\ I_{lcirc} - I_{rcirc} \end{bmatrix} \tag{2.25}$$

S_0 corresponds to the signal intensity. S_1 is the intensity difference between 0° and 90° linear polarization. S_2 is the intensity difference between +45° and −45° linear polarization. S_3 is the intensity difference between left-handed and right-handed circular polarization (measured through a quarter-waveplate and a polarizer). This definition is applicable for monochromatic signals in the same way as for broadband signals. The application to broadband signals (e.g. modulated data signals) often causes confusion and will be discussed in the next section. In the following paragraphs I will focus only on the Stokes calculus for monochromatic signals.

The Stokes vector can be expressed by means of the Jones vector:

$$\vec{S} = \begin{bmatrix} \left|\underline{J}_x\right|^2 + \left|\underline{J}_y\right|^2 \\ \left|\underline{J}_x\right|^2 - \left|\underline{J}_y\right|^2 \\ \frac{1}{2}\left|\underline{J}_x + \underline{J}_y\right|^2 - \frac{1}{2}\left|\underline{J}_x - \underline{J}_y\right|^2 \\ \frac{1}{2}\left|\underline{J}_x + j\underline{J}_y\right|^2 - \frac{1}{2}\left|j\underline{J}_x + \underline{J}_y\right|^2 \end{bmatrix} \tag{2.26}$$

A Stokes vector can be converted to the Jones representation using the following relationship (note that absolute phase information is lost):

$$\phi = \arg\left(\frac{1}{2}S_2 + j\frac{1}{2}S_3\right) \tag{2.27}$$

$$\underline{\vec{J}} = \begin{bmatrix} \sqrt{\frac{1}{2}S_0 + \frac{1}{2}S_1}\; e^{+j\phi/2} \\ \sqrt{\frac{1}{2}S_0 - \frac{1}{2}S_1}\; e^{-j\phi/2} \end{bmatrix} \tag{2.28}$$

In many cases absolute intensities are not of interest and the *normalized Stokes vectors* are used:

$$\vec{s} \overset{def}{=} \frac{1}{S_0} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{2.29}$$

Since S_0 equals $\sqrt{S_1 + S_2 + S_3}$ for monochromatic signals, the normalized Stokes vectors are three-element unity vectors. If interpreted as vectors pointing into three-dimensional cartesian space, these vectors form a spherical surface, called Poincaré sphere. Hence, the polarization of a monochromatic signal can also be expressed as a point on the surface of the Poincaré sphere.

2.2.2 State of Polarization

The polarization of a monochromatic optical signal is commonly referred to as *state of polarization* (SOP). The SOP can either be expressed using Jones vectors or Stokes vectors. In Jones space the SOP can be expressed by the relationship between the two Jones vector elements, that is power distribution χ and relative phase ϕ:

$$\chi = \arctan\left(\left|\underline{J}_y\right| / \left|\underline{J}_x\right|\right) \qquad \phi = \arg(\underline{J}_x \cdot \underline{J}_y^*) \tag{2.30}$$

The corresponding normalized Jones vector is given by

$$\underline{\vec{j}} = \begin{bmatrix} \cos(\chi) \cdot e^{+j\frac{1}{2}\phi} \\ \sin(\chi) \cdot e^{-j\frac{1}{2}\phi} \end{bmatrix} \tag{2.31}$$

In Stokes space the SOP is fully described by the normalized Stokes vector $\vec{s}$. In addition to that, the Stokes vector $\vec{S}$ includes signal power which is not a polarization property.