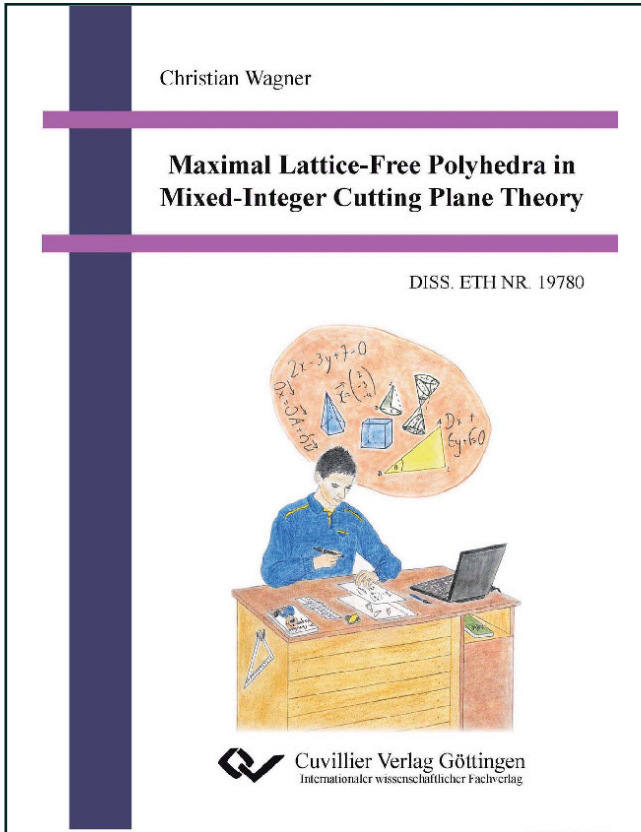




Christian Wagner (Autor)  
**Maximal Lattice-Free Polyhedra in Mixed-Integer  
Cutting Plane Theory**



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