Chapter 1

Introduction

The properties of electron systems that are quantum mechanically confined in one or more spatial directions are of considerable research interest both for a fundamental physical understanding as well as for the progress of semiconductor technology. The discovery of the quantum Hall effect [vK80] and the fractional quantum Hall effect [Tsu82] in two-dimensional electron systems (2DESs) are famous examples of the new physics resulting from the reduced dimensionality. Both discoveries have been awarded with the Nobel prize and the former led to a new definition of the resistance standard. While the quantum Hall effect was discovered in 2DESs in silicon MOSFETs (Metal Oxide Semiconductor Field Effect Transistor), the observation of the fractional quantum Hall effect was made possible by the high electron mobilities achieved by molecular beam epitaxy (MBE) of the AlGaAs and GaAs compounds. Here, 2DESs are realized at the interface between different semiconductors with similar lattice constant that are epitaxially grown on top of each other with atomic layer precision. Its properties make the AlGaAs/GaAs material system ideal for basic research in semiconductor physics.[Sto84] In technological applications it is mainly used where ultra fast processing times are required. The incompatibility to Si and the lack of a natural oxide serving as an insulator hinder a very large scale integration (VLSI) of GaAs circuits. Here, the SiGe material system offers the possibility to combine the advantageous properties of Si/SiO_2 with its VLSI capability and the design freedom of heterostructures due to the band offsets.[Sch97] The availability of high-mobility SiGe heterostructures also gives rise to a renewed interest in the fundamental research on the unique properties of the material system. In particular, only a rudimentary understanding of the splitting of the two occupied conduction band valleys in high magnetic fields has been reached so far. Even after more than 20 years of intense fundamental research, new and totally unexpected properties of 2DESs are discovered, as, for instance, the recent observation of novel zero-resistance states induced by microwave irradiation.[Man02, Zud03]

One- and zero-dimensional systems have been realized, in a bottom-up approach by self-organization or in a top-down approach starting from 2DESs by introducing an additional lateral confinement potential through structured gate electrodes or etching. These systems again open up new fields both in fundamental research as well as for possible technological applications.

A variety of experimental tools has been used for the investigation of low-dimensional systems. The majority of the experimental results were obtained using magneto-transport measurements and spectroscopic methods. These experiments probe the excitation spectrum of the system and conclusions about the ground state properties can hence only be drawn indirectly. A direct relation to the systems ground state is given for thermodynamic equilibrium quantities. Investigations of thermodynamic properties included magnetocapacitance [Smi85, Mos86, Ash93, Dol97, MR02], specific heat [Gor85], compressibility [Eis94] and the magnetization.[Sto83, Eis85b, Wie97, Mei99, Har01] However, direct measurements of the oscillatory behavior of the magnetization as a function of the magnetic field or the carrier density turns out to be a challenging experiment due to the small absolute number of electrons.

In this work the thermodynamic equilibrium magnetization of low-dimensional electron systems has been investigated experimentally using a micromechanical cantilever technique. In particular the magnetization oscillations of 2DESs formed in MBE-grown AlGaAs/GaAs and SiGe/Si heterostructures have been studied as a function of magnetic field, temperature and tilt angle between 2DES normal and magnetic field. Additionally, the magnetization of quantum wire arrays prepared starting from GaAs 2DESs has been investigated. Special attention has been paid to the influence of electron-electron interaction on the de Haas-van Alphen effect. Detailed information about the density of states of the electron systems was gained by comparison of the experimental data with model calculations.

This thesis is organized as follows. In Chapter 2 a brief introduction to the properties of 2DESs subjected to a strong magnetic field is given. A thermodynamic approach to calculate the magnetization from a model density of states is discussed. The experimental technique and the preparation of the cantilever magnetometers is explained in Chapter 3 and Chapter 4, respectively. In Chapter 5 the experimental results are presented. This chapter is divided into three main parts. In the first section the magnetization of modulation-doped AlGaAs/GaAs heterostructures is discussed. In the second section the magnetization of a SiGe/Si modulation-doped quantum well is investigated. The last section focusses on quantum wires prepared from AlGaAs/GaAs 2DESs. The experimental results are compared to model calculations and discussed with respect to the influence of the electron-electron interaction. Chapter 6 summarizes the results of this work.

Chapter 2

Fundamental theoretical concepts

In this chapter the basic theoretical concepts needed to explain the magnetization oscillations in low-dimensional electron systems will be introduced. A more detailed introduction to the physical concepts of low-dimensional electron systems can be found in Ref. [Kel95]. First of all the energy spectrum of a free electron in two dimensions with a uniform perpendicular magnetic field will be discussed. This model will be extended by a density of states (DOS) approach allowing for a more realistic modeling of a 2DES in a magnetic field. Secondly, a thermodynamic approach to calculate the orbital magnetization from the energy spectrum will be presented, additionally introducing the concept of exchange-enhanced energy gaps for spin- or iso-spin systems in a semi-phenomenological way. The influence of a magnetic field component parallel to a 2DES on the orbital magnetization will be discussed. Finally, the effect of an additional lateral confinement in one dimension will be discussed for the case of a parabolic confinement potential. To illustrate the fundamental background and to interpret the data in the experimental part, programs were developed in this thesis, which calculate the dHvA effect based on DOS models. The graphs in this section present theoretical data which were calculated using realistic parameters that model the experimental data in Chapter 5.

The kinetics of conduction band electrons in a semiconductor crystal can be described as the motion of quasi-free electrons with an effective mass m^* accounting for the periodic potential modulation of the crystal lattice in a parabolic approximation. In the samples investigated in this work m^* is assumed to be either $m^* = 0.067\ m_e$ for electrons in GaAs or $m_t^* = 0.19\ m_e$ for the transversal mass of 2D electrons in Si in the plane normal to the symmetry axis. The 2DESs investigated in this work were realized by MBE growth. The confinement potential in the growth direction is determined by the conduction band energies of the different materials in the grown layer sequence. The carrier density is controlled independently via modulation doping. In the following the growth direction is assumed to be along the z-axis. By adjusting layer sequence and doping one can achieve heterostructures in which only the lowest subband of the z-confinement E_{0z} is occupied at low temperatures. Thus, a 2DES with quasi-free motion in the (x, y)-plane is realized. The zero-field DOS of such a spin-degenerate electron system is given by

$$D_0(E) = 0 \quad \forall \quad E < E_{0z}, \quad D_0(E) = m^*/\pi\hbar^2 \quad \forall \quad E \geq E_{0z}. \tag{2.1}$$

2.1 2DES in a perpendicular magnetic field

In the effective mass approximation the Hamiltonian for noninteracting electrons in a uniform magnetic field $\vec{B}$ is given by

$$\hat{H} = \frac{\left(\hat{p} + e\vec{A}\right)^2}{2m^*} + V\left(\vec{r}\right), \tag{2.2}$$

with the momentum operator $\hat{p} = -i\hbar\nabla_{\vec{r}}$, and the vector potential $\vec{A}$, which determines the magnetic field through

$$\nabla \cdot \vec{A} = 0, \quad \nabla \times \vec{A} = \vec{B}. \tag{2.3}$$

$V\left(\vec{r}\right)$ is the external potential which we will neglect in the following, i.e. we assume $V\left(\vec{r}\right) = 0$. The case of $V\left(\vec{r}\right) = V\left(x\right)$ will be discussed in Sec. 2.5.

In a magnetic field $\vec{B} = B\vec{e}_z$ along the z-axis the vector potential in the Landau gauge is given by $\vec{A} = xB_z\vec{e}_y$ and the Schrödinger equation becomes

$$\left[\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} + \frac{ieB_z}{\hbar}x\right)^2 + \frac{2m^*}{\hbar^2}E\right]\psi\left(x, y\right) = 0. \tag{2.4}$$

Since $\left[\hat{p}_y, \hat{H}\right] = 0$ in the Landau gauge, one can choose

$$\psi\left(x, y\right) = \phi_x(x)e^{ik_y y} \tag{2.5}$$

for the motion in the (x, y)-plane. Eq. (2.4) therefore becomes the equation of a harmonic oscillator

$$\left[-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial x^2} + \frac{m^*\omega_c^2}{2}\left(x - x_0\right)^2\right]\phi_x(x) = E_{xy}\phi_x(x), \tag{2.6}$$

with the cyclotron frequency

$$\omega_c = \frac{eB}{m^*}, \tag{2.7}$$

as eigenfrequency. Here $x_0 = -\hbar k_y/m^*\omega_c = -k_y l_B^2$ is the guiding center coordinate of a cyclotron orbit and $l_B = (\hbar/eB)^{1/2}$ is the magnetic length. The eigenfunctions are

$$\phi_{x,j}(x) = \left(2^j j!\sqrt{\pi}x_0\right)^{-1/2} \exp\left\{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2\right\} H_j\left(x/x_0\right) \ , \tag{2.8}$$

with the hermite-polynomials H_j. The corresponding eigenenergies, i.e. the energies of the Landau levels, are given by

$$E_j = E_{0z} + \left(j + \frac{1}{2}\right)\hbar\omega_c, \quad j = 0, 1, 2, \ldots \quad . \tag{2.9}$$

The energy eigenvalues are degenerate with respect to $\vec{k}$. Since in a sample with area $A = L_x \cdot L_y$ the distance between two guiding centers in the Landau gauge is $\Delta x_0 = \Delta k_y l_B^2 = (2\pi/L_y)(\hbar/eB)$, the number of states with the same energy is $N = L_x/\Delta x_0 = AeB/h$. The degeneracy of a Landau level per unit area is hence

$$N_L = \frac{eB}{h} \cdot g_s \cdot g_v \ , \tag{2.10}$$

where g_v is a valley degeneracy factor which gives the number of equivalent energy bands ($g_v = 2$ in Si/SiGe 2DES and $g_v = 1$ in GaAs) and $g_s = 2$ for a spin degenerate system. For a given carrier density n_s the filling factor is defined as

$$\nu = n_s/(eB/h) \ . \tag{2.11}$$

Due to the Landau quantization in a perpendicular magnetic field B the energy independent zero-field DOS of Eq. (2.1) condenses into a set of discrete levels

$$D\left(E\right) = N_L \sum_{j=0}^{\infty} \delta\left(E - E_j - E_{0z}\right) \ . \tag{2.12}$$

2.2 Thermodynamic properties of a 2DES

In order to achieve a more realistic description of a 2DES one has to account for the effects of finite temperature and the residual disorder in the sample. The disorder leads to a broadening of the ideally δ-peak shaped Landau levels. The finite temperature leads to an equilibrium occupation of the states following the Fermi-Dirac distribution

$$f(E, \chi, T) = \left[1 + \exp\left(\frac{E - \chi}{k_B T}\right)\right]^{-1}. \tag{2.13}$$

Here, χ denotes the chemical potential of the system. The thermodynamic equilibrium quantities of such a system can now be calculated from the DOS. In our case we are interested in the magnetization

$$M = -\left.\frac{\partial F}{\partial B}\right|_{N,T}, \tag{2.14}$$

as a function of the magnetic field B at fixed particle number $N = n_s A$. M can be derived by self-consistently calculating the free energy F from

$$n_s = \int_0^{\chi} f\left(E, \chi, T\right) D(E) dE, \tag{2.15}$$

$$F = \chi N - k_B T A \int D(E) \ln\left[1 + \exp\left(\frac{\chi - E}{k_B T}\right)\right] dE. \tag{2.16}$$

In the following we present calculations modeling the magnetic properties for characteristic cases which are relevant for the experiments performed in this work. In particular these are the magnetization of a 2DES in a perpendicular magnetic field and in tilted magnetic fields as well as the magnetization of quantum wires in a perpendicular field. The calculations are performed on the basis of parameters modeling the experimental data.

In Fig. 2.1 (a) the DOS and the chemical potential χ are shown for Gaussian broadened Landau levels at different temperatures. The corresponding magnetization M per electron as a function of B as calculated from Eqs. (2.14)-(2.16) is depicted in Fig. 2.1 (b). Here, E_{0z} is taken to be independent of the magnetic field and chosen to be zero. The period $\Delta(1/B)$ of the oscillations is related to the carrier density n_s according to

$$\Delta\left(\frac{1}{B}\right) = g_s \cdot g_v \cdot \frac{e}{h n_s} = g_s \cdot g_v \cdot 0.242 \cdot \frac{1}{n_s[10^{11}/\mathrm{cm}^2]}\ [\mathrm{T}^{-1}]. \tag{2.17}$$

For the ideal 2DES χ jumps discontinuously between two adjacent Landau levels at even filling factor. The jump in χ crosses an energy gap $\Delta E = \hbar\omega_c = 2\mu_B^* B$ in the single particle spectrum, where the effective Bohr magneton $\mu_B^* = e\hbar/2m^*$ is