Contents

1 Introduction

Robotics have quickly been matured by increasing efforts of the untired researchers [19, 28]. However, a lot of works still have to be done related to the growing applications in the future. For instance, applications in hazardous environment or applications that are almost impossible to be done by human beings such as in nuclear power plants [24, 12], mining industries [8], space robots [1], virtual reality [17], unmanned vehicle operations [22], medical applications [16], cell/micro-organism applications [13], semiconductor industries [27] and so on. The research direction concerning the above mentioned works is known as robot teleoperation. In a common setting, in a teleoperation system as shown in Fig. 1, the operator will exert a force on the master manipulator which in turn, results in a displacement or velocity that is transmitted to the slave side as the order or command. In order to sense the manipulated object, some informations have to be returned from the slave side to the operator side. These information could be distance measurement, velocity measurement, force measurement or their combination. By sending the information back to the master side, the human operator will be able to feel what happened in the environment for example tactile senses. However, it may cause instability in the system if the model is not exactly known or the delay presents in the communication channel. These problems have been the main challenges faced by researchers for many years. Another problem that is also considered is about how to provide the capability to give the operator the feeling of what happened in the remote environment which is known as *transparency*. As stated by many researchers, stability and transparency are usually conflicting [15] and many works failed to compromize these two situations [10]. According to [26], in the mid of 1940s Goertz

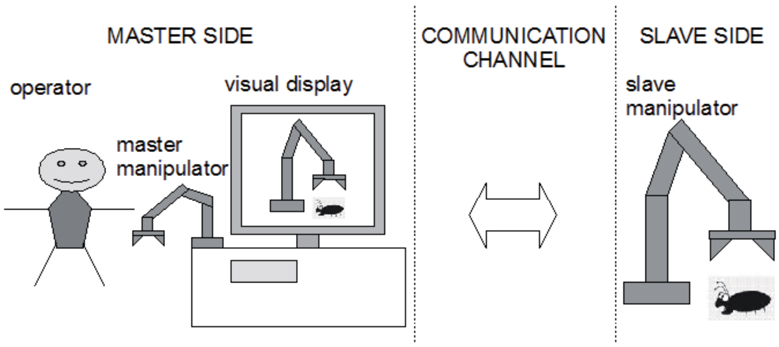

Figure 1: Illustration of a teleoperation system

developed robotic systems that was considered to be the first teleoperation system. After some years vacant, in the early of 1960s the research in teleoperation systems started increasing interests. Some preliminary concept to deal with the teleoperation problem were proposed, for instance, *move and wait scheme* [5], supervisory control [6], software languages [7, 25] and predictive visual display [3]. From 1980s, many stabilization methods for teleoperation systems started to appear, such as Lyapunov-based theorem [18], impedance scheme [23], hybrid scheme [9], scattering theory and passivity-based control [2, 20]. Since the works of [14], the passivity-based approach started its fame until now. However, in spite of being well-known and considered to be standard for so many teleoperation applications, the scenario of using local passivity based controller plus wave variables transformation have some disadvantages especially due to wave reflection (see [14, 30]). Some efforts have been done to compensate this issue and possibly growing in the future. One of the outcomes comes from [20] with their impedance matching scheme. However, this kind of impedance matching block can give serious effect to the position tracking.

In most of these works, however, only few are devoted to solve the long delay in the telecommunication channel. Moreover, it was summarized by

Imaida et. al [11], using the above control schemes, it is always difficult to control the teleoperation systems when delay is longer than one second. Considering these, in this chapter, we will deal specifically with the long delay issue. Using a simple phylosophy that if we are able to transform the teleoperation system into FDE form then we will able to stabilize the teleoperation system naturally. Therefore, we are going to start by building a concept for the FDE stabilization. Firstly we will deal with the simplest stability condition of scalar FDE systems. It is found that a simple algebraic condition will ensure the boundedness of the solution. Furthermore, the systems is Input to State Stable (ISS). From these facts, we extend the concept into the higher dimensional systems in order to develop the similar conditions. Finally, the stabilization of teleoperation systems with arbitrary long communication delay will be proposed. Numerical studies of many classes of teleoperation systems will also be presented to verify the effectivenes of the method. A bit further, we also pursue the transparency issue of our proposed scheme. Here we propose a new definition of transparency and how to achieve it as well as to find out the relationship between the stability and transparency.

Throughout this chapter the following notations are used. Suppose there is a given constant $\tau \geq 0$, $\mathbb{R}$ and $\mathbb{R}^n$ a real number and n-dimensional vector space over $\mathbb{R}$, respectively. Define $C([a,b],\mathbb{R}^n)$ a function that maps the interval $[a,b]$ into $\mathbb{R}^n$ with norm $\|x\|_\infty = \sup_{t\in[a,b]} \|x(t)\|$, where $\|\cdot\|$ is Euclidean norm. A function $f : [0,s) \to [0,\infty)$ is said to be class $\mathscr{K}_\infty$ function if it is increasing, continuous and zero at zero. We call it class $\mathscr{K}_\infty$ if $\lim_{r\to\infty} f(0,r) \to \infty$. Finally we define $f : [t,s) \to [0,\infty)$ as class $\mathscr{K}\mathscr{L}_\infty$ function if for fixed t the function is increasing while for fixed s the function is monotonically decreasing to zero.

2 Stabilization under uncertain delay

Let us consider the following delayed nonlinear system that is in the form of FDE

$$\Sigma : \begin{cases} x'(t) = f(t, x(t), x(\cdot), u(t)), & t \in [0, +\infty) \\ x(t) = \varphi(t), & t \in [-\tau, 0) \end{cases} \tag{1}$$

where $f \in [-\tau,+\infty) \times \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^n$ is locally Lipschitz continuous function on $\mathbb{R}^n$, $\varphi(t)$ is smooth differentiable function, $x(\cdot)$ means $x(t-\tau)$ and $u(t)$ on $\mathbb{R}^n$ is the input to the system. It is noted that the above class covers very general nonlinear delayed systems.

For developing stabilization tool, we are going to start with the following lemma.

Lemma 2.1. *Suppose*

$$\dot{s}(t) = \alpha s(t) + \sum_{i=1}^{N} \beta_i s(t - \tau_i) \tag{2}$$

and

$$\alpha + \| \sum_{i=1}^{N} \beta_i \| \leq 0, \tag{3}$$

then every solution of (2)$\rightarrow 0$ *as* $t \rightarrow \infty$*. Moreover, the solution is bounded by*

$$s(t) \leq \left(\exp(-\int_0^t \alpha(\eta)d\eta) + \sum_{i=1}^{N} \int_0^t (\beta_i(\zeta)\exp(-\int_\zeta^t \alpha(\eta)d\eta)d\zeta) \right) \max_{-\tau \leq \zeta \leq 0} s(\zeta). \tag{4}$$

Proof. The characteristic equation of (2) is given as

$$\lambda = \alpha + \sum_{i=1}^{N} \beta_i e^{-\lambda h_i}.$$

In order to be stable, we need to show that if under (3), the real parts of all of possible eign values are negative. Let us set $\lambda = \sigma + j\omega$, then

$$\begin{aligned} \sigma + j\omega =& \alpha + \sum_{i=1}^{N} \beta_i e^{-\sigma h_i} e^{-i\omega h_i} \\ =& \alpha + \sum_{i=1}^{N} \beta_i e^{-\sigma h_i} (\cos \omega h_i - i \sin \omega h_i). \end{aligned}$$

Here we require that

$$\sigma - \alpha = \sum_{i=1}^{N} \beta_i e^{-\sigma h_i} \cos \omega h_i \leq 0$$

Let us assume that $\sigma \geq 0$ then we have

$$-\alpha \leq \sigma - \alpha \leq \sum_{i=1}^{N} \|\beta_i\| e^{-\sigma h_i} \leq \| \sum_{i=1}^{N} \beta_i(t) \|$$

$$\alpha + \| \sum_{i=1}^{N} \beta_i \| \geq 0$$

that contradicts (3). Therefore, every solution of (2)$\rightarrow 0$ as $t \rightarrow \infty$.

To find the bound (4), let us set

$$Q(t) = p(t)\{s(t) + \delta(q_1(t) + q_2(t) + \cdots + q_N(t))\}$$

where

$$\begin{aligned}
p(t) &= \exp(-\int_0^t \alpha(\eta) d\eta) \\
q_1(t) &= -[p(t)]^{-1} \exp(-\int_0^t \beta_1(\zeta) p(\zeta) d\zeta) \\
q_2(t) &= -[p(t)]^{-1} \exp(-\int_0^t \beta_2(\zeta) p(\zeta) d\zeta) \\
&\cdots \\
q_N(t) &= -[p(t)]^{-1} \exp(-\int_0^t \beta_N(\zeta) p(\zeta) d\zeta),
\end{aligned}$$

while δ is a constant to be determined later. Differentiation on $p(t)$ and $q_i(t)$ will give

$$\begin{aligned}
\dot{p}(t) &= -\alpha(t) p(t) \\
\dot{q}_1(t) &= \alpha(t) q(t) - \beta_1(t) \\
\dot{q}_2(t) &= \alpha(t) q(t) - \beta_2(t) \\
&\cdots \\
\dot{q}_N(t) &= \alpha(t) q(t) - \beta_N(t).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\dot{Q}(t) &= -\alpha(t) p(t) \left\{ s(t) + \delta \sum_{i=1}^{N} (q_i(t)) \right\} + \dot{s}(t) p(t) + \delta \alpha(t) p(t) \sum_{i=1}^{N} (q_i(t) - \beta_i(t)) \\
&\leq p(t) \sum_{i=1}^{N} \beta_i(t) (s(t - \tau_i) - \delta).
\end{aligned}$$

Choosing δ as

$$\delta = \max_{-\tau \leq \zeta \leq t} s(\zeta)$$

where $\tau = \max \tau_i$, will make $\dot{Q}(t) \leq 0$, thus Q is a decreasing function. By rearranging $Q(t_2) - Q(t_1)$ where $-\tau \leq t_1 < t_2$, the following bound applies

$$\begin{aligned}
s(t_2) \leq & \exp(\int_{t_1}^{t_2} \alpha(\eta)d\eta)s(t_1) + \left(\int_{t_1}^{t_2} \sum_{i=1}^{N} \beta_i(\zeta) \exp(\int_{\zeta}^{t_2} \alpha(\eta)d\eta)d\zeta \right) \delta \\
\leq & \left(\exp(\int_{0}^{t_2} \alpha(\eta)d\eta) + \int_{0}^{t_2} \sum_{i=1}^{N} \beta_i \exp(\int_{\zeta}^{t_2} \alpha(\eta)d\eta)d\zeta \right) \delta \\
\leq & \left(\exp(\int_{0}^{t} \alpha(\eta)d\eta) + \sum_{i=1}^{N} \int_{0}^{t} \beta_i(\zeta_i)d\zeta_i \right) \delta.
\end{aligned} \tag{5}$$

Therefore the inequality bound (4) follows. □

Remark 2.1. *The necessary condition for the boundedness of solution of (2) is* $\alpha \leq 0$.

Remark 2.2. *The lemma can be read as, if we can find* $\alpha \leq 0$ *such as negative enough to overcome* $\|\sum_{i=1}^{N} \beta_i\|$ *then we can guarrantee that the solution of (2) is bounded.*

Corollary 2.1. *With finite nonzero input, i.e.,* $\|u(t)\|_\infty \leq \infty$ *the system (1) under Lemma 2.1 is ISS.*

Proof. Using input (2) will be

$$\begin{aligned}
\dot{s}(t) &= \alpha(t)s(t) + \sum_{i=1}^{N} \beta_i(t)s(t-\tau_i) + u(t) \\
\dot{s}(t) &\leq \alpha(t)s(t) + \sum_{i=1}^{N} \beta_i(t)s(t-\tau_i) + \|u(t)\|_\infty.
\end{aligned} \tag{6}$$

Therefore, by following the step as before we will arrive at

$$\begin{aligned}
s(t) \leq & \left(exp(\int_{0}^{t} \alpha(\eta)d\eta) + \int_{0}^{t} \beta(\zeta) exp(\int_{\zeta}^{t} \alpha(\eta)d\eta)d\zeta \right)^{\frac{1}{2}} \|s(\zeta)\|_\infty \\
& + \left(\int_{0}^{t} exp(\int_{\zeta}^{t} \alpha(\eta)d\eta)d\zeta \right)^{\frac{1}{2}} \|u(t)\|_\infty \\
\leq & \pi(t, \|s(t)\|_\infty) + \rho(\|u(t)\|_\infty).
\end{aligned} \tag{7}$$

where we have defined

$$\pi(t,\|s(t)\|_\infty)=\left(\exp(\int_0^t \alpha(\eta)d\eta)+\int_0^t \beta(\zeta)\exp(\int_\zeta^t \alpha(\eta)d\eta)d\zeta\right)^{\frac{1}{2}}\|s(\zeta)\|_\infty$$

and $\rho(\|u(t)\|_\infty)=\exp(\overline{\alpha}t)\|u(t)\|_\infty$. One can see that $\pi(t,\|s(t)\|_\infty)$ belongs to Class $\mathscr{K}\mathscr{L}_\infty$ function while $\rho(\|u(t)\|_\infty)$ belong to Class $\mathscr{K}_\infty$. Therefore, the systems is ISS. □

Theorem 2.1. *If*

$$2\left\langle x(t),f(x(t),x(\cdot))\right\rangle \leq \alpha(t)\|x(t)\|^2+\beta(t)\|x(\cdot)\|^2. \tag{8}$$

and

$$\alpha(t)+\beta(t)\leq 0. \tag{9}$$

then system (1) with zero input is stable under the bound (4).

Proof. Let us set $s=x^T(t)x(t)$, by differentiation we get

$$\begin{aligned}\dot{s}=&2x^T(t)\dot{x}(t)\\ \leq&\alpha(t)\|x(t)\|^2+\beta(t)\|x(\cdot)\|^2\\ =&\alpha(t)s(t)+\beta(t)s(\cdot).\end{aligned} \tag{10}$$

Therefore Lemma 2.1 follows. □

Definition 2.1. *The system (1) is said to be* FDE-stable *if (8) and (9) hold.*

3 Teleoperation stabilization

In this section we aim to answer the question on how to solve a teleoperation system problem via feedback *FDE-stabilization*. In many works, teleoperation systems usually have many different representations. Therefore, in order to be well-arranged, when *teleoperation system* is mentioned, it will mean that its manipulator part is modelled in the following form

$$\Sigma_i:\begin{cases}\dot{x}_i=F_i(x_i,u_i)\\ y_i=H_{xi}^T(x_i,u_i), \quad i\in\{m,s\}\end{cases} \tag{11}$$

where m,s refer to master and slave, respectively, Σ_i means the subsystem, x_i denotes the state variables of the subsystem, $F_i(\cdot)\in\mathbb{R}^n$ and $H_{xi}(\cdot)\in\mathbb{R}^m$

with $m \le n$ are the smooth differentiable functions. The matrices $F_i(\cdot)$ and $H_i(\cdot)$ refer to dynamical matrix and output matrix, respectively. It should be noted that (11) is very general equation as the above equation will include all of nonlinear or linear manipulator systems. For some classes, teleoperation systems can also be written in affine form as

$$\Sigma_i : \begin{cases} \dot{x}_i = F_i(x_i) + G_i(x_i)u_i \\ y_i = H_{xi}^T(x_i) + H_{ui}^T u_i, \quad i \in \{m, s\} \end{cases} \tag{12}$$

where $G_i(\cdot) \in \mathbb{R}^n$ and $H_{ui}(\cdot) \in \mathbb{R}^m$ with $m \le n$ are the smooth differentiable functions.

Definition 3.1. ***(Solving teleoperation problem via FDE-stabilization)*** *For a given delayed nonlinear teleoperation systems (11), find state feedback controllers for each subsystem*

$$u_i = \gamma_i(x, x(\cdot), v) \tag{13}$$

such that the overall closed loop system is FDE-stable.

The above problem seems not an easy one since there is no direct relationship between teleoperation system and FDE. However, it is turned to be clear when we merge together the separated system (*e.g.,* master and slave manipulators) into a single system. Let us define

$$\begin{aligned} x(t) &= [x_m(t)\, x_s(t)] \\ x(\cdot) &= [x_m(t - T_2(t))\, x_s(t - T_1(t))] \\ T &= [T_1,\ T_2]^T \end{aligned} \tag{14}$$

and also $u = [u_m,\ u_s]^T$ and $v = [v_m,\ v_s]^T$ where

$$\hat{F}_i(x, x(\cdot), v) = F(x, u(x(\cdot), v)) \tag{15}$$

then the teleoperation system (11) can be re-written as

$$\begin{aligned} &\dot{x} = F(x, u(x(\cdot), v)) = \hat{F}(x, x(\cdot), v) \quad t \in [0, +\infty) \\ &x = \phi(t) = x(t - T), \quad t \in [-\max\{T_m, T_s\}, 0). \end{aligned} \tag{16}$$

Equation (16) shows that the teleoperation systems indeed belong to FDE. Therefore, we have successfully translated the teleoperation stabilization problem into feedback FDE-dissipasiton.

In order to stabilize the teleoperation systems, we propose the following algorithm:

1. Transform the original system in form of system (1).

2. Consider $\langle x, f(x, x(\cdot), u(x, x(\cdot), v(t)))\rangle$ then set our condition of interest satisfying (8).

3. Set $u(x, x(\cdot), v(t))$ until (9) is met.

4. (Extra step) Using this $u(x, x(\cdot), v(t))$, minimize $\alpha(t)$ to speed up the response. For any bounded external input $v(t)$, the system will be ISS.

4 Applications

4.1 Linear case

Consider linear manipulators arranged as master and slave with communication delay as follows

$$\Sigma_i : \begin{cases} M_i\ddot{x}_i + C_i\dot{x}_i + N_i x_i = \tau_i + u_i(x_i, y_j(t - T_j)) \\ y_i = H_i(x_i), \quad i, j \in \{m, s\}, i \neq j \end{cases} \tag{17}$$

where $\tau = \{\tau_{human}\ \tau_{env}\}$ defined to be the external forces/torques from respectively, human and environment acting on the manipulators. It can be seen that all of the matrix are constants. If we interpret M_i, C_i and N_i are respectively, inertia matrix, Coriolis and centrifugal term, and gravity and external force term, it is very common in many literatures to express (17) in simpler form as follows

$$\Sigma_i : \begin{cases} M_i\ddot{x}_i + C_i\dot{x}_i = \tau_i + u_i(x_i, y_j(t - T_j)) \\ y_i = H_i(x_i), \quad i, j \in \{m, s\}. \end{cases} \tag{18}$$

For simplicity, we can augment (18) into a single system including delay. Defining,

- $x = [x_m,\ x_s]^T$
- $M := \mathrm{diag}\{M_m, M_s\}$,
- $C := \mathrm{diag}\{C_m, C_s\}$,
- $\tau := \mathrm{col}\{\tau_m, \tau_s\}$,
- $T \in \{T_m, T_s\}$,
- $u := \mathrm{col}\{u_m, u_s\}$ and