

Contents

Introduction	1
I The single domain problem	5
1 The advection-diffusion-reaction problem	7
2 Imposing Dirichlet conditions in a weak sense	11
2.1 The weak formulation	12
2.2 Discretization and stabilization	14
2.3 Stability	18
2.4 A priori analysis	20
2.5 A posteriori analysis	25
2.6 Numerical results	31
2.7 Conclusions	38
II The three-field formulation	39
3 The three-field formulation	41
3.1 The continuous three-field formulation	42
3.2 The discrete version of the three-field formulation	51
3.3 Connection to mortar elements	54
4 A stabilized three-field formulation	57
4.1 A discrete stabilized scheme	57
4.2 Analysis of the stabilized scheme	61
4.3 Numerical results	68
4.4 Conclusions	76
5 The three-field formulation for the Oseen Equations	79
5.1 The Oseen equations	79
5.2 The three-field formulation	80

III	Nonoverlapping domain decomposition methods	85
6	A preconditioned Schur complement method	87
6.1	The continuous case	87
6.2	The discrete case	97
6.3	Numerical results	101
6.4	Conclusions	109
7	An alternating Schwarz algorithm	111
7.1	The continuous formulation	111
7.2	Discretization	116
7.3	Numerical results	117
7.4	Conclusions	122
8	Comparison of some nonoverlapping domain decomposition methods	123
8.1	Fourier analysis	123
8.2	Numerical results	136
9	Summary and Outlook	141
IV	Appendix	143
A	Functional Analysis	145
A.1	Some basic results	145
A.2	Closed Range Theorem and applications	147
B	Function spaces	153
B.1	Smooth functions	153
B.2	Lebesgue spaces	155
B.3	Distributions, weak derivatives and Sobolev spaces	155
B.4	Trace theorems and Sobolev spaces of fractional order	157
B.5	Some fundamental equalities and inequalities	159
B.6	Finite Element spaces	160
	Bibliography	165
	Notation	173
	Index	179