Chapter 1

Introduction

1.1 Why Supersymmetry ?

Today the Standard Model has become a successful theory describing physics at subnuclear scales which has been tested by many collider experiments to a high level of accuracy [1, 2]. The Higgs bosons predicted by the Standard Model has not been directly observed by todays experiments.

Despite its great success, there still remains several serious problems, such as the arbitrariness of the particle spectrum and gauge group, the large number of free parameters, and maybe the most severe one is the inability to turn on gravity described by the general theory of relativity. These suggest that the Standard Model is not the final answer of nature but rather an effective description valid up to the electroweak scale of order $\mathcal{O}(100\,GeV)$. Thus, the Standard Model has to be extended.

Various efforts have been made over the last two decades to go beyond the standard model and correspondingly, solve the above problems. The most prominent one and still promising until now is the *supersymmetric extension* of the Standard Model which is reviewed, for example, in [3].[1] It has $N = 1$ global supersymmetry because extended supersymmetries ($N \geq 2$) cannot accommodate the chiral structure of the Standard Model. As supersymmetry is not observed in nature, it must be broken at low energy if it is to play any role at all. This leads to a mass split between bosonic and fermionic partners of the supersymmetry breaking scale. The determination of this scale should explain why the supersymmetric partners of the Standard Model particles could be heavy enough to escape detection in accelerator experiments around the electroweak scale $\sim 100\,GeV$ so far. One of interesting aspects of this theory is that all three gauge couplings unify at a scale $\sim 10^{16}\,GeV$, see *e.g.* [4].

There are various ways to break supersymmetry, however only two of them are of phenomenological interest, namely, supersymmetry has to be either spontaneously or softly broken. Since the supersymmetric extension of the Standard Model only has a global supersymmetry, spontaneous breaking poses a new problem, namely the presence of a massless fermion called *Goldstone fermion*. This is a consequence of the supersymmetric Goldstone theorem, see *e.g.* [5]. So if the global supersymmetry is spontaneously broken, the supersymmetric extension of the standard model would be ruled out. Thus, the only way out is a soft breaking of global supersymmetry. This can be done by adding non-supersymmetric terms to the theory which do not generate

[1] See next section for a discussion of supersymmetry.

any quadratic divergences [6]. In addition, there is an alternative way to motivate the relevance of softly broken supersymmetric theories. Ultimately, one has to couple the supersymmetric standard model to gravity. This in turn requires the promotion of global supersymmetry to a local supersymmetry which is called *supergravity*.[2] Furthermore, spontaneous local supersymmetry breaking in the limit $M_{Planck} \to \infty$ but with the gravitino mass remains fixed, yields the soft supersymmetry breaking terms [7]. This motivates many theorists today to study supergravity as a candidate beyond the Standard Model.

Let us turn to extended supersymmetric theories. Since these theories cannot accommodate the chiral structure, it seems that the extended supersymmetries are not phenomenologically interesting. Furthermore, *the no go theorem* which states that any supersymmetric theory with N supersymmetries either all or none of them are spontaneously broken, demands that extended supersymmetric theories must be broken at the same supersymmetry breaking scale which is phenomenologically impossible. However, in the last decade few examples have been appeared which show that these theories can spontaneously be broken to $N = 1$ [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].[3] These examples indicate that the no go theorem can be avoided and in addition, yield a hope for phenomenological studies. Still the resulting $N = 1$ theories cannot accommodate the chiral structure because their parental theories are extended supersymmetric theories. Nevertheless, extended supersymmetries remain an interesting study, in particular to see how one can evade the no go theorem and study the general aspects of their breaking. In this thesis, we address some general aspects of spontaneous breaking $N = 2 \to N = 1$ in supergravity as an example of spontaneous breaking of extended supersymmetric theories.

1.2 What is Supersymmetry ?

In this section we briefly consider the structure of rigid (global) supersymmetry in four dimensional Minkowski space. The interested reader is referred to the literature for further details [20, 21, 22, 5].

By definition, supersymmetry transforms bosons into fermions and vice versa. In order to realize of such transformations one introduces *supersymmetry generators* (or supercharges) Q, acting as:

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \ , \quad Q|\text{fermion}\rangle = |\text{boson}\rangle \ , \tag{1.1}$$

where we have split up the Hilbert space into bosonic states $|\text{boson}\rangle$, and fermionic states $|\text{fermion}\rangle$. Such a boson-fermion symmetry has far-reaching consequences. First it affects the statistic of the transformed state and changes it by a half-unit. Thus, the supersymmetry generators themselves have spin one-half and form spinor representations of the Lorentz group, contrary to the usual generators of symmetry transformation which have integer spin. The second important implication of such transformation is that every particle has a *superpartner*. The notation for the bosonic superpartners of the known fermions are labeled by the prefix 's-' (*e.g.* slepton, squark), whereas the fermionic superpartners of the known bosons are denoted by the suffix '-ino' (*e.g.* gaugino, gravitino). The members of a supersymmetric theory are arranged in a so

[2]See section 1.3 for a discussion of supergravity

[3]See also section 1.4.

called *supermultiplet* which has the same number of bosonic and fermionic degrees of freedom.

Such supercharges Q which form spinor representations of the Lorentz group satisfy an *anticommutation* relation [23]

$$\{A, B\} \equiv AB + BA \quad , \tag{1.2}$$

and moreover, do not contradict the theorem of Coleman and Mandula [24], which states that for every non-trivial relativistic field theory , under some very mild assumption all the symmetries of the S-matrix commute with the generators of the Poincaré group. This is because the essential assumption that they make, is that the symmetry generators form a closed algebra under *commutation* relations, thus restricting themselves to Lie groups of symmetry transformation.

It was proven in [25] that a set of commutation and anticommutation relations between Poincaré generators and supercharges (usually called *Poincaré superalgebra*) is the only graded Lie algebra of symmetries of the S-matrix consistent with relativistic quantum field theory. Furthermore, the Poincaré superalgebra together with other generators of the Lie group G which is the symmetry of the S-matrix form an algebra which admits a $\mathbb{Z}_2$ graded structure. Such an algebra is usually called *supersymmetry algebra*. To see the meaning of this graded structure, let us first call the generators which satisfy the commutation relation (Lie algebra) even and the supercharges Q to be odd. Then these even and odd generators must satisfy the rules:

$$\begin{aligned} [\mathrm{even}, \mathrm{even}] &= \mathrm{even} \quad , \\ \{\mathrm{odd}, \mathrm{odd}\} &= \mathrm{even} \quad , \\ [\mathrm{even}, \mathrm{odd}] &= \mathrm{odd} \quad . \end{aligned} \tag{1.3}$$

To make it clear, let us denote P_a the four momentum and J_{ab} the Lorentz group generators respectively, with $a = 0, ..., 3$ and in addition there are some supercharges $Q^{\widehat{A}}$, where $\widehat{A} = 1, ..., N$. Therefore the expression of the supersymmetry algebra which has the $\mathbb{Z}_2$ graded structure (1.3) is the following:

$$\begin{aligned} [J_{ab}, J_{cd}] &= -\mathrm{i}\big(\eta_{bc}\, J_{ad} + \eta_{ad}\, J_{bc} - \eta_{bd}\, J_{ac} - \eta_{ac}\, J_{bd}\big) \quad , \\ [J_{ab}, P_c] &= \mathrm{i}\big(\eta_{ac}\, P_b - \eta_{bc}\, P_a\big) \quad , \\ \{Q^{\widehat{A}}, Q^{\widehat{B}}\} &= -2\,(\gamma C)^a P_a\, \delta^{\widehat{A}\widehat{B}} - 4\, C\, Z^{\widehat{A}\widehat{B}} \quad , \\ [P_a, P_b] &= [P_a, Q^{\widehat{A}}] = 0 \quad , \\ [J_{ab}, Q^{\widehat{A}}] &= -\frac{\mathrm{i}}{2}\gamma_{ab}\, Q^{\widehat{A}} \quad , \end{aligned} \tag{1.4}$$

where $Q^{\widehat{A}}$ are four spinors, C is a charge conjugation matrix defined by

$$C\, \gamma_a\, C^{-1} = -\gamma_a^{\mathrm{T}} \quad , \tag{1.5}$$

with the superscript T stands for the transpose, γ_a are the Dirac matrices, $2\gamma_{ab} \equiv [\gamma_a, \gamma_b]$, and the metric $2\eta_{ab} = \{\gamma_a, \gamma_b\} = 2\,\mathrm{diag}(+1, -1-1, -1)$.[4] In (1.4) we have also introduced the antisymmetric quantities $Z^{\widehat{A}\widehat{B}}$ called central charges and they commute with all the generators defined above. Due to their antisymmetry, it is easy to see that

[4]See also appendix A.

the central charges are trivially zero if there is only one supercharge. This minimal supersymmetry in four dimensions is called $N = 1$ supersymmetry. On the other hand if there are more than one supercharge present in a theory, then it is called *extended* supersymmetry.

Furthermore, since the mass squared operator defined as

$$M^2 = P^a P_a \quad , \tag{1.6}$$

commutes with all generators of the supersymmetry algebra (1.4), then the mass squared is a supersymmetric invariant.[5] Hence in Minkowski space all the particles within the same supermultiplet have to be degenerate in mass. There are two types of irreducible representation: the massive and massless representations. As we are going to see they have a rather different structure and need a separate study. In addition, we restrict ourselves in this section to study the massive representation without the central charges, *i.e.* $Z^{\widehat{A}\widehat{B}} = 0$, while the massless representation satisfies trivially this requirement.

Before proceeding further to the massless and massive representations of the supersymmetry algebra (1.4), let us first use the fact that a four spinor is reducible which means that the supercharges $Q^{\widehat{A}}$ can be decomposed into two Weyl spinors

$$Q^{\widehat{A}}_{\pm} = \frac{1}{2}(1 \pm \gamma_5) Q^{\widehat{A}} \quad . \tag{1.7}$$

Then it follows that the anticommutation relation in (1.4) reduces into

$$\begin{aligned} \{Q^{\widehat{A}}_{+}, \bar{Q}^{\widehat{B}}_{-}\} &= 2\,\sigma^a P_a\, \delta^{\widehat{A}\widehat{B}} \quad , \\ \{Q^{\widehat{A}}_{+}, \bar{Q}^{\widehat{B}}_{+}\} &= \{Q^{\widehat{A}}_{-}, \bar{Q}^{\widehat{B}}_{-}\} = 0 \quad , \end{aligned} \tag{1.8}$$

where $\bar{Q}^{\widehat{A}} \equiv Q^{\widehat{A}\dagger}\gamma_0 = Q^{\widehat{A}\mathrm{T}}\, C$ and we have chosen the following basis for γ-matrix:

$$\gamma^a = \begin{pmatrix} 0 & \sigma^a \\ \bar{\sigma}^a & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} , \quad C = \begin{pmatrix} -\mathrm{i}\sigma^2 & 0 \\ 0 & \mathrm{i}\sigma^2 \end{pmatrix} , \tag{1.9}$$

with

$$\sigma^a = (\mathbb{1}, \sigma^x) \, , \quad \bar{\sigma}^a = (\mathbb{1}, -\sigma^x) \, , \tag{1.10}$$

where $x = 1, 2, 3$ and σ^x are the standard Pauli matrices.[6]

We shall first analyze the massless case, $P^a P_a = 0$. Using Lorentz boost we can always go to the frame where $P^a = m(1, 0, 0, 1)$. The commutation relations (1.8) show that the only non-zero supercharges are $Q^{\widehat{A}}_{+2}$ and its conjugate $\bar{Q}^{\widehat{A}}_{-2}$. Furthermore, $Q^{\widehat{A}}_{+2}(Q^{\widehat{A}}_{-2})$ raise (lower) the spin of a state by a half-unit. Thus, the particle spectrum in a multiplet can be constructed by acting with $Q^{\widehat{A}}_{+2}$'s on the vacuum states $|\lambda\rangle$ where λ denotes the helicity. Below, we list some examples for $N = 1, 2, 4$.

$$N = 1 \;:\quad \begin{cases} 2|0\rangle \, , & |\pm 1/2\rangle \\ |\pm 1/2\rangle \, , & |\pm 1\rangle \end{cases}$$

[5] By definition, an element which commutes with all elements of a Lie algebra is called *Casimir element*. In our case, the mass squared operator M^2 and the central charges $Z^{\widehat{A}\widehat{B}}$ are indeed the Casimir elements of the supersymmetry algebra (1.4).

[6] See appendix A.