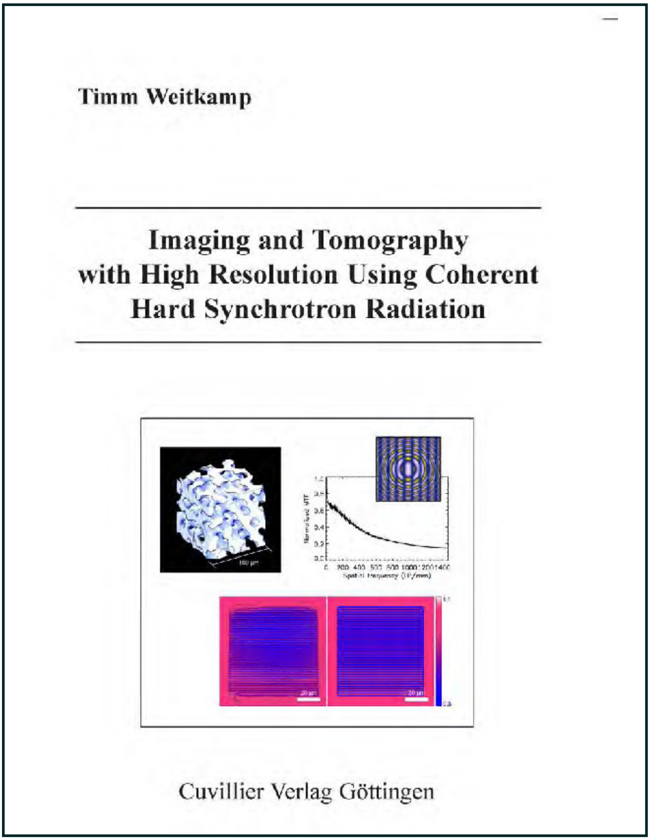
Timm Weitkamp (Autor)

Imaging and Tomography with High Resolution Using Coherent Hard Synchrotron Radiation



https://cuvillier.de/de/shop/publications/3416

Chapter 1

Interaction of X Rays with Matter

In hard X-ray transmission imaging, a beam of electromagnetic radiation with wavelengths in the range between approximately 0.2 and 2 Å, corresponding to photon energies between 6 and 60 keV, i. e., a *hard X-ray* beam, traverses an object under study, and the intensity distribution in a plane normal to the beam propagation direction somewhere downstream of the object is measured with a two-dimensional position-sensitive detector (figure 1.1). This intensity distribution, or *image*, is determined by the interaction of the X rays with the sample material. In this chapter we will briefly discuss the interaction processes that are important in hard X-ray imaging.

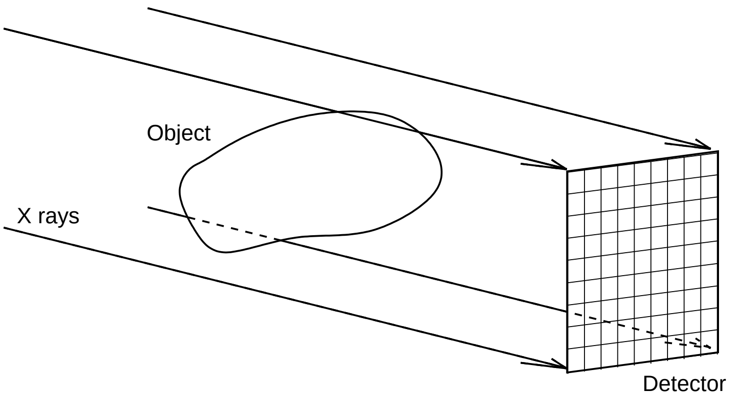


Figure 1.1: Principle of hard X-ray transmission imaging.

The main processes by which X rays interact with matter are photoelectric absorption and elastic as well as inelastic scattering of X rays by electrons.[1] Multiple elastic scattering in the forward direction, which occurs in all materials regardless of their atomic ordering, is coherent because it does not change the wavelength of the radiation nor the phase relation between parts of the wave scattered on scatterers at different locations. It gives rise to such macroscopic phenomena as refraction and surface reflection. Scattering, elastic or

[1] Scattering by atomic nuclei or by collective excitations in crystalline solids (phonons, magnons and other quasi-particles) will be neglected. So will electron-positron pair production, which occurs only at photon energies above 1 MeV.

inelastic, into other directions will be regarded only as a contribution to the attenuation of the forward beam or as noise.

1.1 Forced Damped Oscillator Model

A relatively simple model that we will use to introduce the concept of a complex refractive index and the phenomenon of anomalous dispersion is that of the atomic electrons seen as classical oscillators, with the natural frequency of each oscillator representing an atomic absorption frequency. This treatment goes back to COMPTON, who adapted the theory developed earlier for the visible wavelength region by LORENTZ. We follow JAMES [Jam82] in the presentation of the fundamental results.

We describe an atomic electron as a classical oscillator of natural angular frequency ω_q (where q denotes the orbital shell, and ω_q corresponds to the absorption resonance) under a restoring force $-m\omega_q^2 x$, i. e., a harmonic oscillator (m: mass of the electron; x: displacement). The oscillation is damped because an accelerated charged particle radiates energy. The damping factor γ can be derived from classical electrodynamics. Let the electron further be in a linearly polarized external plane wavefield, $E(z,t) = E_0 \exp(\mathrm{i}\omega(z/c - t))$. Figure 1.2 illustrates the model. The equation of motion for the electron is

$$\ddot{x} + \gamma\dot{x} + \omega_q^2 x = -\frac{e}{m}\,E_0 \mathrm{e}^{\mathrm{i}\omega(z/c-t)}\,, \tag{1.1}$$

Solutions of equation 1.1 are of the form

$$x(t) = -\frac{e}{m}\frac{1}{(\omega_q^2 - \omega^2) - \mathrm{i}\gamma\omega}\,E(z,t)\,. \tag{1.2}$$

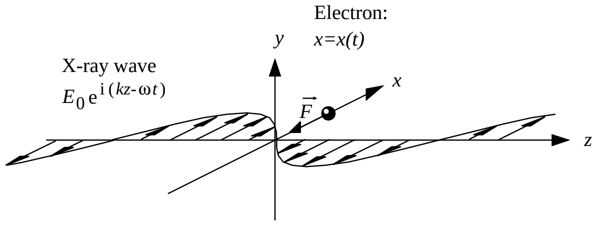


Figure 1.2: Illustration of the LORENTZ model.

The displacement $x(z,t)$ of the electrons of type q leads to a polarization

$$P_q = -n_q e x\,. \tag{1.3}$$

The contribution of electrons of this type to the dielectric susceptibility, χ_q, is then

$$\chi_q = \frac{P_q}{\epsilon_0 E} \quad \text{(definition of the dielectric susceptibility)} \tag{1.4}$$

$$= -\frac{n_q e^2}{\epsilon_0 m}\,\frac{1}{\omega_q^2 - \omega^2 - \mathrm{i}\gamma\omega} \quad \text{(substituting eq. 1.2 into eq. 1.3)}\,. \tag{1.5}$$

Separating the real and imaginary parts, we get

$$\chi_q = -2\delta_q + \mathrm{i}2\beta_q \,, \tag{1.6}$$

$$\delta_q = \frac{n_q e^2}{2\epsilon_0 m} \frac{\omega^2 - \omega_q^2}{(\omega^2 - \omega_q^2)^2 + \gamma^2\omega^2} \,, \tag{1.7}$$

$$\beta_q = \frac{n_q e^2}{2\epsilon_0 m} \frac{\gamma\omega}{(\omega^2 - \omega_q^2)^2 + \gamma^2\omega^2} \,. \tag{1.8}$$

Figure 1.3 shows δ_q and β_q as the oscillator model predicts them around the resonance frequency ω_q. The strong variation of the dielectric susceptibility near resonance frequencies is called *anomalous dispersion*.

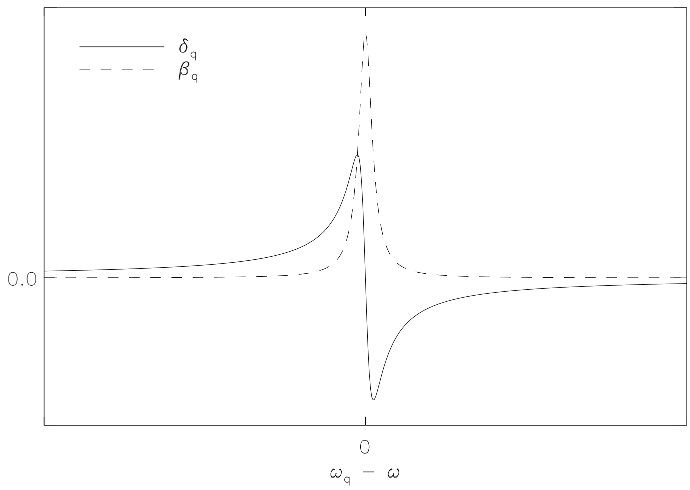


Figure 1.3: At the resonance frequency ω_q of atomic electrons in shell q, the oscillator model predicts a peak in β_q and sign reversal of δ_q. These strong fluctuations of the real and imaginary part of the dielectric susceptibility near a resonance frequency are called anomalous dispersion.

1.2 Complex Refractive Index

The refractive index is defined as the ratio of the phase velocities in vacuum, c, and in the medium, v_q. For a medium consisting of q-type electrons, it is

$$\frac{c}{v_q} = \sqrt{1 + \chi_q} \approx 1 - \delta_q + \mathrm{i}\beta_q \,, \tag{1.9}$$

where we have used the fact that both δ_q and β_q are much less than unity for X-ray wavelengths.