Chapter 1

Introduction

Frequency planning for GSM cellular radio networks is the topic of this thesis. We present results which were obtained in the context of a cooperation between the Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB) and the German GSM 1800 network operator E-Plus Mobilfunk GmbH & Co. KG. This cooperation started in September 1995, and has since then been extended several times.

Our focus was primarily on fast frequency planning heuristics for the use in the regular radio planning process at E-Plus. New planning methods were developed at ZIB and integrated into E-Plus' software environment. In 1997, our software was first used successfully in practice, and, in the meantime, it has been extended to better meet practical needs. We also studied approaches to provide quality guarantees for heuristically generated frequency plans.

GSM is a second generation digital cellular radio system. Among others, GSM provides telephony service: a mobile phone may establish a communication link with any other party reachable through a public telephone network. This is achieved by means of a radio link to some stationary antenna which is part of a large infrastructure, see Figure 1.1. Since the introduction of GSM, radio telephony has grown from a costly service used by few professionals to a mass market with penetration rates as high as 70 % in Finland and Iceland, for example. In more and more countries, the mobile cellular phone subscribers outnumber the fixed-line telephone subscriptions.

Frequency planning is a key issues in fully exploiting the radio spectrum available to GSM. It has a significant impact on the quantity as well as on the quality of the radio communication services. Roughly speaking, radio communication requires a radio signal of sufficient strength which is not suffering too severely from interference by other signals. In a cellular system like GSM, these two properties, strong signals and little interference, are in conflict. The problem of finding a “good” frequency plan is sketched in the following and described in full detail later.

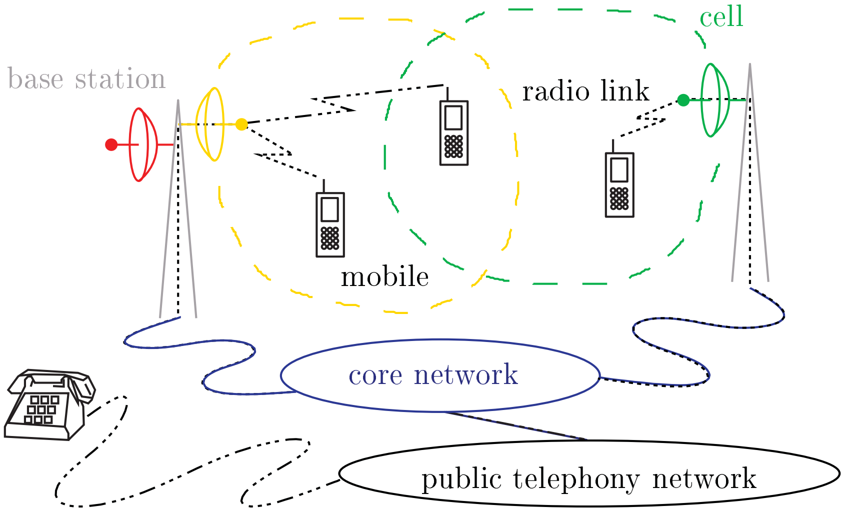

Figure 1.1: GSM in principle

Every base station operates a number of elementary transceivers, each of which uses some frequency to transmit on. A network operator has usually between 30 and 120 evenly spaced out frequencies available to satisfy the demand of several thousand transceivers. The reuse of frequencies is therefore unavoidable, but this reuse is limited by interference and by so-called separation requirements. Significant interference may occur between transceivers using the same frequency (co-channel) or directly neighboring frequencies (adjacent channels). Separation requirements are given for pairs of transceivers and impose that the assigned frequencies have a specified minimum separation in the electromagnetic spectrum. Furthermore, not every frequency is necessarily available for all transceivers. In summary, the problem to be solved is the following.

> Given are the transceivers, the set of generally available frequencies, the local unavailabilities, as well as three square matrices specifying the necessary minimum separation, the potential co-channel, and the potential adjacent channel interference values. One frequency has to be assigned to every transceiver such that the following holds. All separation requirements are met, and all assigned frequencies are locally available. The optimization goal is to find a frequency assignment resulting in the least possible interference.

We are primarily interested in minimizing the sum over the incurred co- and adjacent channel interferences here, but other goals of practical interest exist as well. Striving for "minimum interference" assignments is in a sense a luxury to be paid for with frequencies. If only few frequencies are available to a GSM operator, then the emphasis is likely on providing some acceptable frequency plan at all. But the optimization aspect gains importance when feasible assignments can be obtained "easily." E-Plus is currently in the latter position. The network contains roughly 8000 base stations, and 115 frequencies are available.

New assignments have to be computed on several occasions. Some examples are: the network is modified or expanded, the characteristics of a transceiver are changed, or significant unpredicted interference is reported and has to be resolved.

Several commercial software packages exist which allow to document the network configuration, to plan radio coverage, and to predict interference in addition to frequency planning. GSM infrastructure manufacturers develop such tools, but also independent companies such as AIRCOM International (Asset), COSIRO GmbH (Fun), Lociga Plc. (Odyssey), L&S Hochfrequenztechnik GmbH (CHIRplus), or Metapath Software International Limited (PlaNet). At the time when the cooperation with E-Plus started, however, the optimization of frequency assignments with respect to interference was often only poorly supported. This has certainly improved since then.

In the following, we deal with a broad spectrum of topics ranging from the technical background of the GSM frequency planning problem over alternative mathematical models and heuristic planning methods to quality assessments for the generated frequency plans. In addition to this introduction, the thesis comprises seven chapters and an appendix containing a compilation of mathematical notation used in the following. The content of each chapter is now briefly stated.

In *Chapter 2*, we give a survey of GSM and explain the technical conditions to be taken into account during frequency planning. We also describe how the input data is generated and stress the importance of reliable interference predictions for the success of automatic frequency planning.

In *Chapter 3*, the frequency planning problem (as sketched above) is formalized as a combinatorial minimization problem. We investigate the computational complexity of the model beyond stating its $\mathcal{NP}$-hardness, and we discuss extensions of the model as well as alternative models.

In *Chapter 4*, seven heuristic frequency planning methods are described. Depending on the point of view, five or six of them can be used (in combination) for generating frequency assignments in practice. In accordance with the objective of the cooperation with E-Plus, our focus is on fast methods rather than on more elaborate, but slower methods.

In *Chapter 5*, the previously described planning methods are compared on the basis of realistic frequency planning problems. In this comparison, we include the currently best performing method we know of as a reference. An analysis of the realistic planning scenarios is provided, and we explain how to use the described heuristics in order to obtain time savings and quality improvements in practice.

In *Chapter 6*, a lower bound on the amount of unavoidable co-channel interference is computed for each planning scenario. These bounds are obtained by solving large semidefinite programs (which are challenges to the currently existing solvers). Based on these bounds, quality guarantees are provided for the frequency assignments from the preceding chapter. Moreover, we introduce a relaxed version of our frequency planning problem. The solutions for the relaxed problem can sometimes be turned into feasible assignments for the original problem. Exploiting this connection, we point out room for further development of heuristics.

The relaxed version of frequency planning leads us to the study of the mathematical MINIMUM K-PARTITION problem and its semidefinite relaxation (which we considered so far mostly as a "black box" providing lower bounds).

In *Chapter 7*, we mostly review results on a polytope, which is obtained as the convex hull of the feasible solutions to an integer linear programming formulation of the MINIMUM K-PARTITION problem. Particular emphasis is on the hypermetric inequalities.

In *Chapter 8*, we first give an introduction to semidefinite programming and then study the semidefinite relaxation for the MINIMUM K-PARTITION problem. In particular, we describe a large class of valid inequalities for the solution set of the semidefinite relaxation (a shifted version of hypermetric inequalities), and we prove that neither the linear programming relaxation of the integer linear programming formulation nor the semidefinite programming relaxation is always stronger than the other.