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Kinematic and Dynamic Simulation of Ground Motion: Implications for Seismic Hazard Assessment Verbesserung der seismischen Gefährdungsabschätzung durch kinematische und dynamische Modellierung seismischer Bodenbewegung

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Chapter 1

Concepts of Finite Difference Modeling

The most fundamental equation underlying the theory of seismology is the equation of motion which describes the propagation of waves radiated from earthquakes sources and relates forces in the medium to measurable ground motion [4, 75]. In its most general form and for an inhomogeneous anisotropic medium it can be written as

$$\rho \ddot{u}_i = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i, \tag{1.1}$$

where ρ is the density of the material, $\ddot{u}_i$ denotes the second derivative of the three elements of the ground displacement vector $\mathbf{u}$ with respect to time, σ_{ij} is the ijth component of the stress tensor and f_i denote the components of the body forces. The indices i and j stand for the spatial directions.
In an earthquake almost all Earth materials show a linear proportionality between stress σ_{ij} and strain ϵ_{kl}. For a linear elastic medium, the empirical relationship between stress and strain is known as Hooke's law. In case of general isotropy only two elastic moduli are needed to describe the proportionality. A common representation is done using Lamé parameters λ and μ which are material dependent and may vary with location:

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}. \tag{1.2}$$

δ_{ij} indicates the Kronecker delta function, and the indices i, j, and k stand for the three spatial directions.

Using this simplified formulation of Hooke's law, and a relationship between strain and displacement

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{1.3}$$

valid for infinitesimal strain,
Combinig equations 1.1 – 1.3 yields the wave-equation for an inhomogeneous isotropic medium:

$$\rho \ddot{u}_i = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} + \lambda_{,i} u_{j,j} + \mu_{,j}(u_{i,j} + u_{j,i}) + f_i \tag{1.4}$$

This equation can be rewritten as a system of second order partial differential equations. A common representation for Finite Difference applications is the velocity-stress formulation [78, 134, 135] which is also used by the Finite Difference method throughout this thesis. There are several advantages of this formulation over the formulation expressed in displacements (e.g. [74]). The scheme is stable for all values of Poisson's ratio, grid dispersion and grid anisotropy are small, both, surface and buried sources can easily be implemented, and the free-surface boundary condition is easily satisfied [76]. The equations of motions are given by

$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + f_x, \tag{1.5}$$

$$\rho \frac{\partial v_y}{\partial t} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + f_y, \tag{1.6}$$

$$\rho \frac{\partial v_z}{\partial t} = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + f_z, \tag{1.7}$$

where v_x, v_y, and v_z denote the particle velocity components, σ_{xx}, σ_{yy}, and σ_{zz} are the normal stresses, and σ_{xy}, σ_{xy}, and σ_{yz} are the shear stresses. The constitutive laws are then expressed as

$$\frac{\partial \sigma_{xx}}{\partial t} = \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) + 2\mu \frac{\partial v_x}{\partial x}, \tag{1.8}$$

$$\frac{\partial \sigma_{yy}}{\partial t} = \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) + 2\mu \frac{\partial v_y}{\partial y}, \tag{1.9}$$

$$\frac{\partial \sigma_{zz}}{\partial t} = \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) + 2\mu \frac{\partial v_z}{\partial z}, \tag{1.10}$$

$$\frac{\partial \sigma_{xy}}{\partial t} = \mu \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right), \tag{1.11}$$

$$\frac{\partial \sigma_{xz}}{\partial t} = \mu \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right), \tag{1.12}$$

$$\frac{\partial \sigma_{yz}}{\partial t} = \mu \left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right). \tag{1.13}$$

In order to solve the equations numerically, in a Finite Difference scheme the modeling volume V is discretized into nx · ny · nz grid points with distance dx to the neighboring grid point. Given a continuous solution of the wave-equation in the volume V the solution at those discrete points can be obtained by indirectly approximating the continuous solution. This is achieved by the approximation of derivatives. Traditionally, Taylor series expansions of the solution in the neighborhood of point x_i are used. Another possibility for obtaining the approximation is by the computation of the tangent line to the function [72, 128]. The discretized form of equations 1.5 – 1.13 can be found in [76].

1.1 Stability of a System

A crucial issue for explicit Finite Difference numerical methods is stability. Stability of a system is in practical terms connected to an energy limit and reflects the fact that the total energy in a physical system should not change.
Mathematically this is achieved when the Courant-Friedrichs-Lewy (CFL) stability criterion

$$\frac{dt \cdot v_{\mathrm{max}}}{dx} \leq c \tag{1.14}$$

is met (e.g. [103]), where v_{max} is the maximum (compressional wave) velocity in the modeling volume V, dx is the grid spacing, dt the sampling interval in time, and c some constant number which has to be determined empirically. In this study, for c a value of 0.45 is used [103]. This is a conservative estimate. Other authors suggest to use a slightly larger value of 0.5 (e.g. [71, 119]).