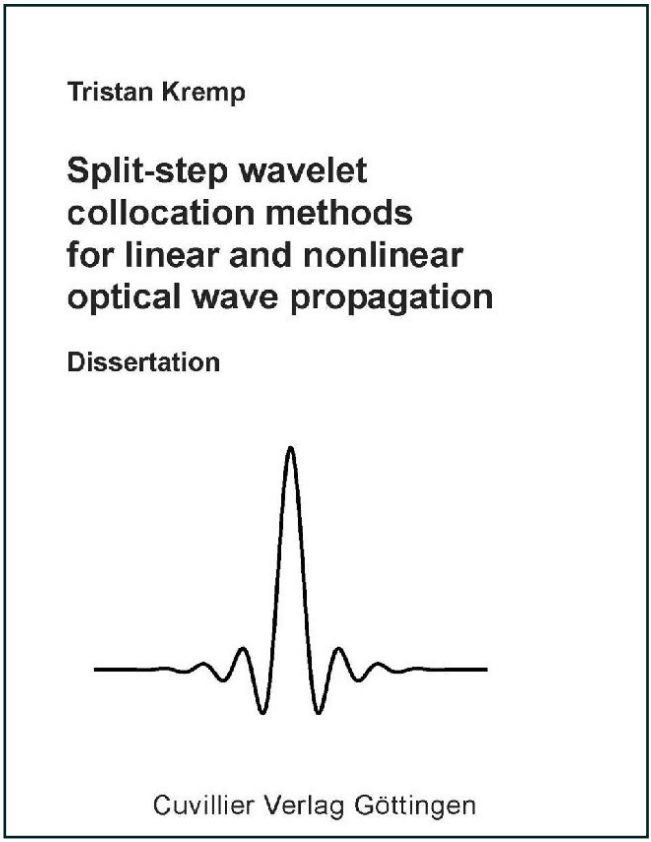
Tristan Kremp (Autor)

Split-step wavelet collocation methods for linear and nonlinear optical wave propagation



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