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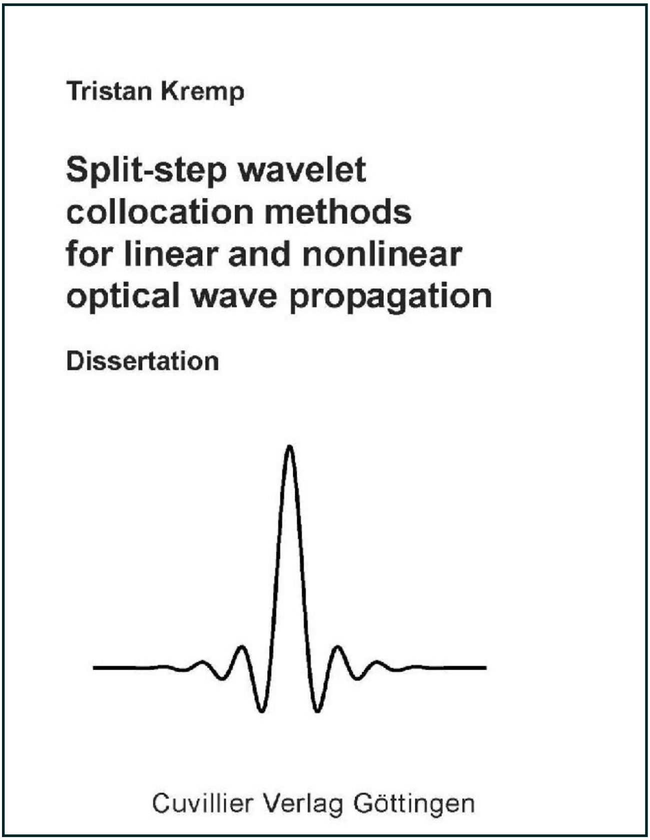
Split-step wavelet collocation methods for linear and nonlinear optical wave propagation



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Chapter 1

Introduction

With the growth of internet traffic, the demands on photonic transmission systems are subject to a permanent rise. *Wavelength division multiplexing* (WDM) is a key technology for making use of the available bandwidth of optical transmission fibers. For an optimum design of a WDM system (Fig. 1.1), all the parameters of the transmitter, transmission medium and receiver have to be chosen such that a maximum amount of information can be transmitted without detection errors. Therefore, extensive numerical simulations have to be performed to test the influence of all these parameters, and to find an optimum configuration.

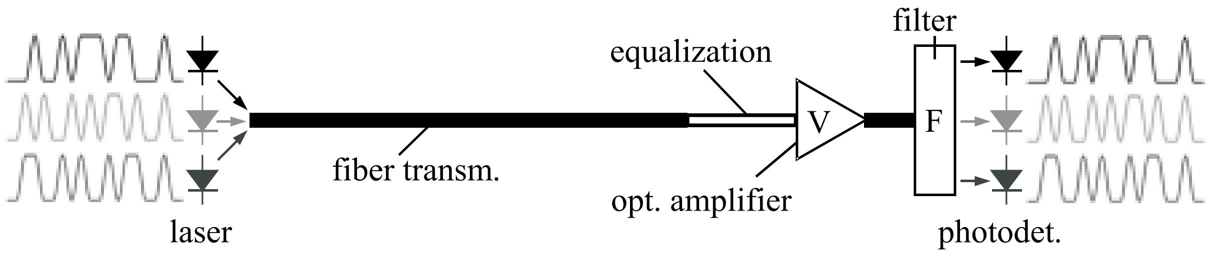

Fig. 1.1: Wavelength division multiplexing (WDM) system.

Due to the long transmission distances ranging from 50 km to thousands of kilometers, a good deal of the total computational effort concentrates on waveguide propagation, which is modeled by the *nonlinear Schrödinger equation* (NLSE). As the simulation time can be several days or even weeks, powerful numerical tools with an optimum complexity are extremely important.

The standard numerical technique used by most professional simulation tools is the *split-step Fourier method* (SSFM). As a pseudospectral method, this technique is exponentially convergent and therefore extremely accurate for sufficiently small numerical propagation steps. However, due to the use of the fast Fourier transform, the SSFM has the complexity $\mathcal{O}(N \log_2 N)$, with N as the number of time discretization points during propagation. This complexity is larger than the optimum $\mathcal{O}(N)$ complexity. For a realistic simulation of state-of-the-art WDM systems, the computing power of single-processor units is not sufficient. For numerical simulations, the size of the investigated WDM systems must be reduced, such that with the available clock rate, satisfying results can be obtained within a tolerable computation time. Hence, the clock rate determines the size of the practically computable WDM systems. According to Moore's law, which is assumed to hold for another two decades, the clock rate doubles approximately every 18 months. Hence, the dimension N of the simulated problems will further increase, and the factor $\log_2 N$, which makes the SSFM slower than a possibly faster algorithm with an optimum $\mathcal{O}(N)$ complexity, will become a more and more critical drawback in the future.

Hence, there is an increasing demand for accurate, fast $\mathcal{O}(N)$ algorithms for such long-distance integrations of the NLSE. Using the methods of *multiresolution analysis* (MRA), a new *split-step wavelet collocation method* (SSWCM) with optimum complexity $\mathcal{O}(N)$ for a fixed accuracy, and allowing a substantial reduction in computation time compared to the SSFM, is developed in the present work. It is structured as follows:

In Chap. 2, important publications in the field of wavelet methods for nonlinear optical pulse propagation and for the general solution of partial differential equations are discussed.

The NLSE describing nonlinear optical pulse propagation is derived from Maxwell's equations in Chap. 3.

In Chap. 4, the so-called *collocation method* is developed from the method of weighted residuals in the case of Dirac weight functions. The symmetric split-step integration technique, which is used for all the methods compared in this work, is illustrated. The combination of the collocation method for the discretization of the differential equation with the split-step integration technique is introduced and named *split-step collocation method* (SSCM). It is shown that the SSFM is a special case of the SSCM using harmonic basis functions. To reduce the complexity, a basis of translated interpolating functions is introduced. Since these are the scaling functions generating compactly supported interpolating wavelets, the corresponding propagation algorithm is called *split-step wavelet collo-*

cation method (SSWCM).

In Chap. 5, the concept of *multiresolution analysis* (MRA) is developed, based on the scaling equation of a so-called *scaling function*. The functions which span the complement spaces of the MRA are the *wavelets*. The *fast wavelet transform* (FWT) is introduced as a discrete transform with linear $\mathcal{O}(N)$ complexity to separate the components of a signal. Using such a discrete wavelet representation, the so-called *standard* and *nonstandard form* of general linear operators is developed. Augmenting the SSWCM with an FWT, and formulating the linear propagation operator of the NLSE in nonstandard form, we arrive at the *split-step multiresolution wavelet collocation method* (SSMRWCM).

In Chap. 6, the results using these new techniques are presented and compared to the standard SSFM:

- For the propagation of single impulses (solitons), the SSFM is compared to the SSCM using weighted orthogonal polynomials as basis functions (Gauss-Hermite functions). For a fixed accuracy, the SSCM is almost one order of magnitude faster than the SSFM.

- For the propagation of WDM pulse sequences, the SSFM and SSWCM are compared with respect to computation time and accuracy. Due to its $\mathcal{O}(N)$ complexity, the SSWCM requires less than 40 % of the computation time of the SSFM for the simulation of a large WDM system with 64 channels. This substantial reduction in computation time makes it possible to simulate the influence of the input power on the bit error rate and the maximum spectral efficiency of large WDM systems. Results for 16 and 32 channels are presented. For these very time-consuming simulations, the computation time saving is more than 70 % compared to the SSFM.

- To obtain a complexity which is independent from the number of WDM channels, a channel-separated split-step wavelet collocation method (C-SSWCM) is developed and compared to a corresponding channel-separated split-step Fourier method (C-SSFM). As a result, the C-SSWCM requires less than 20 % of the computation time of the C-SSFM for the simulation of a large WDM system with 64 channels.

- The SSMRWCM is advantageous, if signals vary strongly in only a limited range of the time discretization window. Due to the adaptivity of the method, the SSMRWCM provides a significant speed improvement in this case.

The appendices A to C give details on orthogonal collocation and split-step integration, and provide some mathematical supplements, e. g., error measures and perturbation theory.