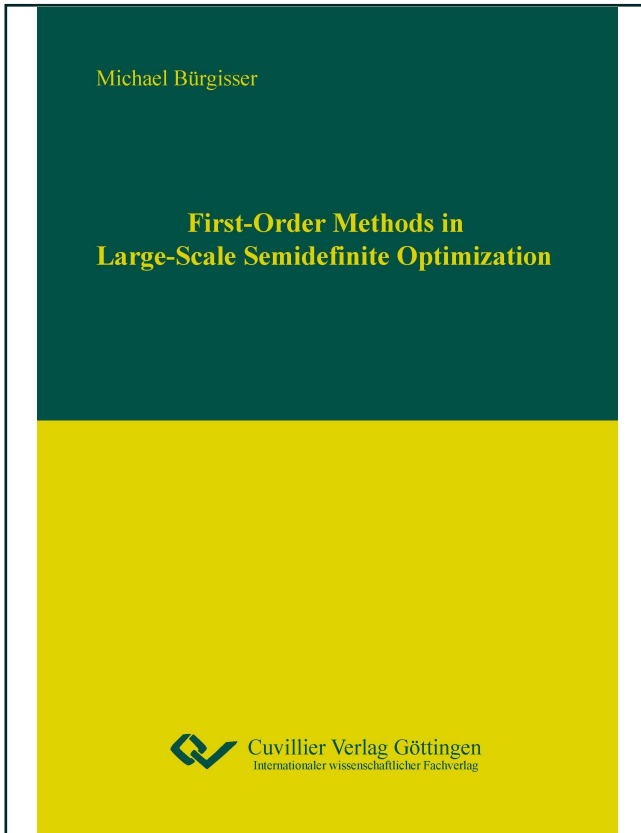




Michael Bürgisser (Autor)

First-Order Methods in Large-Scale Semidefinite Optimization



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