# Contents

**Introduction**

1. The Motion of the Human Knee as a Dynamical Contact Problem
   1.1. The Human Knee
   1.2. Basics of the Mathematical Model
   1.3. Strong and Variational Problem
   1.4. Conservation Properties and Persistency Condition

2. Newmark Methods
   2.1. Classical Newmark Method
   2.2. Contact–Implicit Newmark Method
   2.3. Contact–Stabilized Newmark Method
   2.4. Improved Contact–Stabilized Newmark Method
   2.5. Numerical Comparison

3. A Perturbation Result – Viscosity and Physical Energy Norm
   3.1. Elasticity
   3.2. Viscoelasticity
   3.3. Interpretation of the Stability Condition
   3.4. Approximations of the Signorini Condition

4. Consistency – Bounded Variation
   4.1. Newmark Methods in Function Space
   4.2. Consistency Error in Physical Energy Norm
   4.3. Consistency Error in a Discrete Displacement Norm
   4.4. Consistency Error for Permanent Active Contact

5. Convergence Theory
   5.1. Discrete Perturbation Results
      5.1.1. A Discrete Perturbation Result in Discrete Displacement Norm
      5.1.2. Discrete Stability Condition
      5.1.3. A Discrete Perturbation Result in Physical Energy Norm
   5.2. Convergence
      5.2.1. Convergence in Physical Energy Norm
Contents

5.2.2. Convergence in Discrete Displacement Norm 108

6. Adaptive Timestep Control 111

6.1. Towards an Asymptotic Error Expansion 111

6.1.1. Conical Derivative 112

6.1.2. Extension of Extrapolation Techniques 114

6.1.3. Construction of a Higher-Order Scheme 116

6.1.4. Discussion of Consistency Order 124

6.2. Timestep Control 127

6.2.1. Error Estimator in the Absence of Contact 128

6.2.2. Error Estimator in the Presence of Contact 129

6.2.3. Combined Timestep Strategy 131

6.3. Global Discretization Error 134

7. Numerical Results 137

7.1. Hertzian Contact Problem 137

7.2. Motion of the Human Knee 142

Conclusion 149

A. Inequalities 151

B. Interpretation of Contact Stresses 153

B.1. An Abstract Trace Theorem 153

B.2. Localization on Active Contact Boundaries 154

B.3. Localization on Critical Contact Boundaries 156

B.4. Interpretation of Discrete Contact Stresses 158

List of Symbols i

Bibliography xii

Danksagung xiii

Zusammenfassung xv

ii